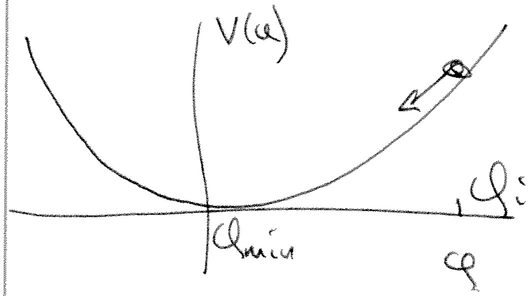
Light Scalar Fields: "Quintessence"

- Inspired by inflation, another candidate for Dark Energy is a very light, homogeneous scalar field: $\varphi(\vec{x},t) \to \varphi(t)$



$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{\partial V}{\partial \varphi} = 0$$

$$H^2 = \frac{8\pi G}{3}\left[\rho_m + \rho_\varphi\right]$$

- Example: $V(\varphi) = \frac{1}{2}m^2\varphi^2$. If $m \lesssim H_0$, then the Hubble damping term $3H\dot{\varphi}$ freezes the field near its initial value φ_i (assumed $\neq \varphi_{min}$) until recent times: field starts classically rolling when $H(t)$ drops to a value of order m. $\therefore$ This field has not yet reached its ground state (φ_{min}) because it is so light. In the future, $m \gg H$, and φ will undergo damped oscillations around $\varphi_{min} \Rightarrow \rho_\varphi \sim a^{-3}$, $P_\varphi \simeq 0$ $\left(\langle \tfrac{1}{2}\dot{\varphi}^2\rangle = V\right)$

- Recall:
$$\rho_\varphi = T_{00} = \frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}(\nabla\varphi)^2 + V(\varphi) = \frac{1}{2}\dot{\varphi}^2 + V \quad \text{for homog. } \varphi$$
$$P_\varphi = T_{ii} = \frac{1}{2}\dot{\varphi}^2 - \frac{1}{6}(\nabla\varphi)^2 - V(\varphi) = \frac{1}{2}\dot{\varphi}^2 - V$$

- If the field is slowly rolling ($m \lesssim H$) $\Leftrightarrow \dot{\varphi}^2 \ll V$, then it has negative pressure:

$$\boxed{w_\varphi = \frac{\frac{1}{2}\dot{\varphi}^2 - V}{\frac{1}{2}\dot{\varphi}^2 + V} < 0}$$

and can drive acceleration once it dominates over NR matter.

- $w(\varphi)$ determined by form of $V(\varphi)$: can be $\neq -1$ and evolves in time. $m \ll H \Rightarrow \varphi = \text{const.} \Rightarrow w = -1$. Deviations from $w = -1 \Leftrightarrow m \sim H_0$.

- More generally, $V(\varphi)$ specifies evolution of the scalar field EOS parameter as $\omega = \omega(t)$ or $\omega(a)$:

$$\rho_\varphi(a) = \Omega_\varphi \rho_{crit} \exp\left[3 \int_a^{a_0} [1+\omega_\varphi(a)]\, d\ln a\right]$$

and can be reconstructed as

$$V(a) = \frac{1}{2}\left[1-\omega_\varphi(a)\right]\rho_\varphi(a)$$

$$\varphi(a) = \int \frac{da\sqrt{1+\omega_\varphi(a)}}{aH(a)}\sqrt{\rho(a)}$$

~~Detour~~ on

- Scalar Perturbations:

- Unlike pure vacuum energy (Λ), a quintessence scalar field can vary in time and in space. Linearized fluctuations in φ follows from substituting $\varphi(\vec{x},t) = \varphi_0(t) + \delta\varphi(\vec{x},t)$ into the scalar field EOM $\Rightarrow$ in comoving coords.,

$$\ddot{\delta\varphi} + 3H\dot{\delta\varphi} + \left(\left.\frac{\partial^2 V}{\partial\varphi^2}\right|_{\varphi_0} - \frac{1}{a^2}\vec{\nabla}_x^2\right)\delta\varphi = \dot{\varphi}_0 \dot{\delta}_m$$

where $\delta_m = \frac{\delta\rho_m}{\bar{\rho}_m} = \frac{[\rho_m(\vec{x},t) - \bar{\rho}_m(t)]}{\bar{\rho}_m(t)}$ matter density perturbation

- Scalar field responds to perturbations in matter (dark matter + baryons)
- If ω_φ close to -1, then $\dot{\varphi}_0 \to 0$, weaker driving term for $\delta\varphi$.
- Expect φ perturbations on scales $\lambda > m^{-1} = \left(\frac{\partial^2 V}{\partial\varphi^2}\right)^{-1/2}$ = Compton wavelength

For $m \lesssim H_0 \to \lambda \gtrsim H_0^{-1} \simeq 3000 h^{-1}$ Mpc

- This is analogous to the Jeans instability: perturbations on scales $\lambda \lesssim m^{-1}$ are dissipated w/ $c_s = c$, perturbations on larger scales can grow $\Rightarrow$ could leave an imprint on large angle (low-ℓ) CMB anisotropy (ISW), but expect largely irrelevant for structure formation.

(coefficient of $\frac{1}{a^2}\vec{\nabla}_x^2$ term

- In more general models, the quintessence sound speed c_s^2 may differ. Consider the action:

$$S = \int \sqrt{-g}\, d^4x\, \mathcal{L}(\varphi, X) = \int \sqrt{-g}\, d^4x\, p(\varphi, X)$$

where $X = \frac{1}{2}(\partial_\mu\varphi)(\partial^\mu\varphi) = \frac{1}{2}\dot{\varphi}^2 - \frac{1}{2}(\vec{\nabla}\varphi)^2$

Sound speed: $$c_s^2 = \frac{\partial p/\partial X}{\partial \rho/\partial X} = \frac{\partial p/\partial X}{\frac{\partial p}{\partial X} + 2X\frac{\partial^2 p}{\partial X^2}}$$

For ordinary, canonical scalar,

$p(X) = X \rightarrow c_s^2 = 1$.

$$w_\varphi = \frac{p}{2X\frac{\partial p}{\partial X} - p}$$ see p. 81

End detour

- Note: we're implicitly assuming $V(\varphi_{min}) \simeq 0$, i.e., that someone has solved the cosmological constant problem. Light scalar DE does not solve " " " but assumes it is solved.

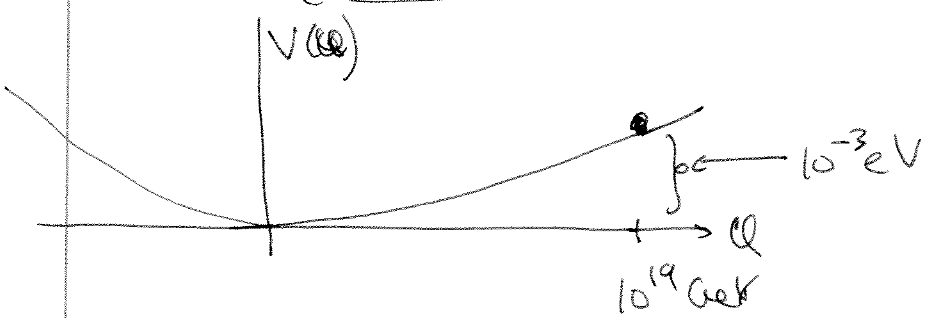
- Requirements for Scalar Field Dark Energy: Consider $V(\varphi) = \frac{1}{2} m^2 \varphi^2$

  - slow-roll: $\dot{\varphi}^2 \leq V \Rightarrow \boxed{m \lesssim H_0 \sim 10^{-33} eV}$

  - φ dominates and has correct ρ_φ: $H_0^2 \simeq \frac{8\pi G}{3} \rho_\varphi \simeq \frac{V(\varphi)}{M_{Pl}^2} \simeq \frac{m^2 \varphi^2}{M_{Pl}^2}$

$$\Rightarrow \boxed{\varphi \sim \frac{H_0 M_{Pl}}{m} \gtrsim M_{Pl} \sim 10^{19}\, GeV \sim 10^{28} eV}$$

Extremely flat potential: $\frac{m}{\varphi} \lesssim 10^{-60}$



These requirements are generic.

- Field must be also extremely weakly self-coupled:

$$V_{int} = \lambda \varphi^4 \Rightarrow \lambda M_{Pl}^4 \lesssim \rho_0 \sim H_0^2 M_{Pl}^2$$

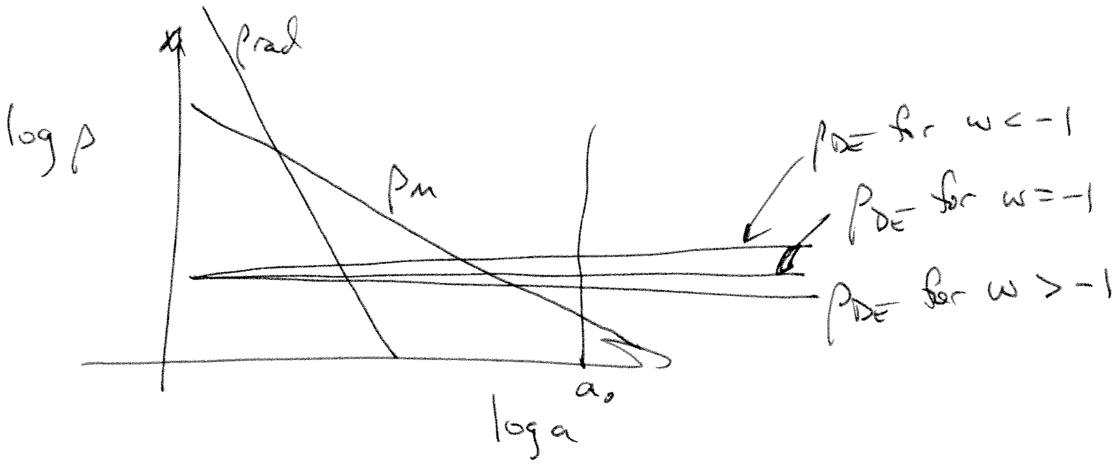
$$\Rightarrow \lambda \lesssim \frac{H_0^2}{M_{Pl}^2} \sim 10^{-120}$$

By quantum mechanics, it must also be extremely weakly coupled to other particles.

- How do we come up w/ such light, non-interacting particles in the context of particle physics? Role of Symmetry - return to this

The Coincidence Problem:

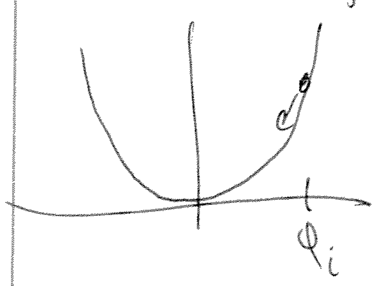
- In these models, why is dark energy just becoming dominant "today" (i.e., in the last 5 billion years)?



- We live at the "special" epoch when $\Omega_{DE} \sim \Omega_m$: at other epochs, they differ by orders of magnitude. Is there some explanation for this coincidence? i.e., what determines ρ_m/ρ_{DE} as fn. of time? Does it require fine-tuning of model parameters?

Two Approaches to this Problem:

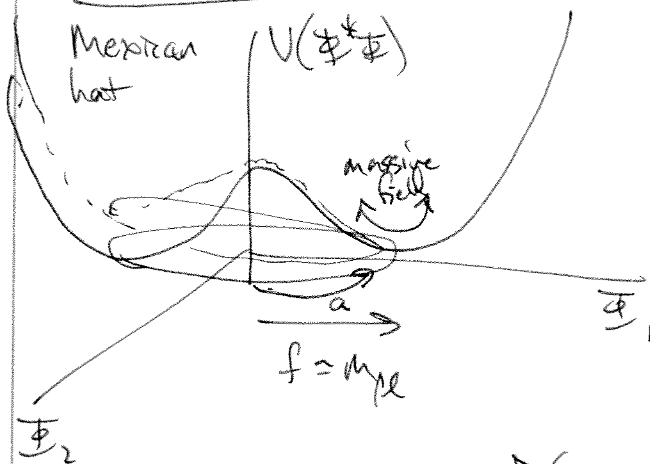
1.) Coincidence just reflects mass scales and initial conditions of the theory:



$V = \frac{1}{2}m^2\varphi^2$. Setting $\varphi_i \sim m_{pl}$ and $m \sim H_0$ determines $\rho_\varphi(t)$ and therefore $\rho_\varphi(t)/\rho_m(t)$

Example:

Pseudo-Nambu-Goldstone Boson (PNGB) model: Frieman et al PRL (1995)



Complex scalar field $\Phi = \Phi_1 + i\Phi_2$ w/ global spontaneously-broken $U(1) = O(2)$ symmetry:

$$V(\Phi) = \lambda(\Phi^*\Phi - f^2)^2$$

$$\langle\Phi\rangle = f e^{ia/f} \quad \text{ground state}$$

- Global $U(1)$ symmetry ($\Phi \to \Phi e^{i\alpha}$) of the Lagrangian is spontaneously broken by the VEV to a residual shift symmetry: $a \to a + \text{const.}$ leaves $\Phi^*\Phi$ invariant $\Leftrightarrow$ massless Nambu-Goldstone boson, a.

- Explicitly break the symmetry: tilt the Mexican hat

$$\Rightarrow \quad V(a) = \Lambda_a^4\left[1 + \cos\left(\frac{a}{f}\right)\right] \quad \text{pseudo-NG boson}$$

since a is a periodic field.

> Note: Λ_a is not the cosmological constant!

- Mass of the PNGB:

$$V(a) \sim \Lambda_a^4 + \frac{a^2}{f^2}\Lambda_a^4 + \ldots$$

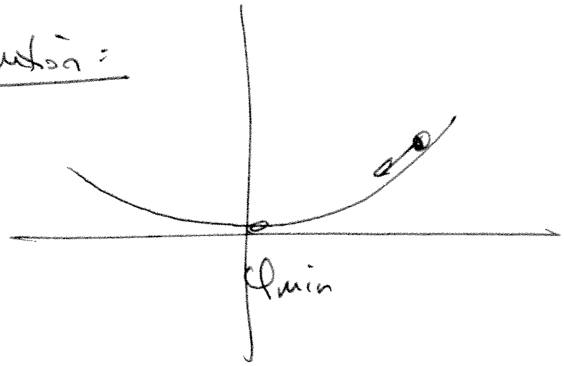
$$m_a^2 a^2 = \frac{a^2 \Lambda_a^4}{f^2} \quad \Rightarrow \quad m_a \sim \frac{\Lambda_a^2}{f}$$

- A large hierarchy between Λ_a and f is technically natural, i.e., is protected by the symmetry against large quantum corrections.(*) Set $\Lambda_a \sim 10^{-3}\,\text{eV} \sim m_\nu$, $f \sim m_{pl} \sim 10^{19}\,\text{GeV}$

$$\Rightarrow m_a \sim \frac{10^{-24}}{10^{19}}\,\text{GeV} \sim 10^{-43}\,\text{GeV} \approx H_0 \quad \text{as needed}$$

(*) since setting $\Lambda_a \to 0$ increases (restores) the symmetry

Evolution:



A.) - initially trapped: $\varphi = \text{const.}$, $w_\varphi = -1$, $m_\varphi \ll H(t)$

B.) - starts rolling: $m_\varphi \simeq H(t)$, $|\dot\varphi| < V$, $w_\varphi > -1$ and evolving away from -1.

C.) Late times: $m_\varphi \gg H(t)$, φ oscillates around φ_{min} w/

period $\tau \ll H^{-1}$ $\Leftrightarrow$ $\rho_\varphi \sim a^{-3}(t)$, scales as

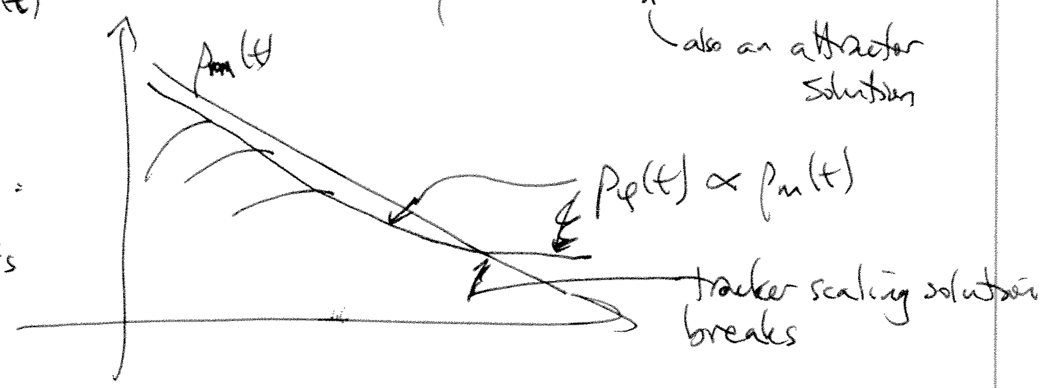
$\langle \frac{1}{2}\dot\varphi^2 \rangle_{period} = \langle V(\varphi) \rangle$ non-relativistic matter if $V(\varphi_{min}) = 0$

- Current epoch would be phase A or B
- If $V(\varphi_{min}) = 0$, then acceleration will not continue into the indefinite future, but will transition to ~~a~~ another matter-dominated epoch.

2.) Tracker Fields:

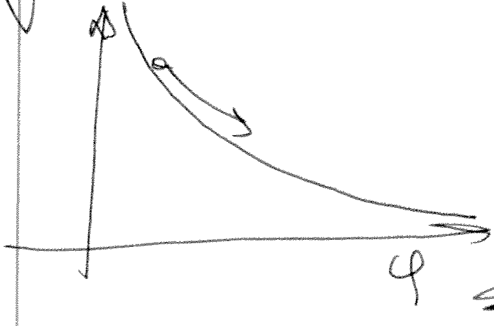
Here one arranges $V(\varphi)$ so that $\rho_\varphi(t)$ "tracks" the behavior of $\rho_m(t)$ (or $\rho_{rad}(t)$) for a long period of cosmic evolution: in this case, it's not so much a coincidence that $\rho_\varphi \simeq \rho_m$ today.

↑ also an attractor solution



Issue: from Big Bang Nucleosynthesis, we must have $\rho_\varphi \ll \rho_{rad}$ at $T \sim 1$ MeV, $t \sim 1$ sec after the Big Bang. Similarly for LSS formation, require ρ_φ to be subdominant compared to ρ_m for most of cosmic history. But need something to trigger it ~ 5 Gyr ago so that it stops tracking and starts to dominate and cause acceleration. Some models may do this naturally, e.g.

$$V(\varphi) = M^4 \left(\frac{\varphi}{M_{pl}}\right)^{-n} \quad \text{with } n > 0.$$



In this case, field is not initially frozen and now rolling, but the opposite: field initially rolls "quickly" but then gradually slows and becomes more like dark energy, $w_\varphi \to -1$.

- When ρ_φ is subdominant compared to the dominant component (matter or radiation), it has $w_\varphi \simeq \frac{(n w_d - 2)}{n+2}$

where $w_d = \frac{P_{dominant}}{\rho_{dominant}} = \begin{cases} \frac{1}{3} & \text{in radiation-dominated era} \\ 0 & \text{" matter- " "} \end{cases}$

i.e. $w_\varphi = \begin{cases} \left(\frac{n}{3} - 2\right) / (n+2) & \text{in rad. dom. era} \\ \frac{-2}{n+2} & \text{" matter " "} \quad (= -1 \text{ to } -\frac{2}{3}) \end{cases}$

- At late times, φ begins to slow and to dominate over other components and eventually $w_\varphi \to -1$
- Evolution determined by M and n.
- Viable model requires $0 < n < 1$ and $M \simeq 0.002\,eV$, $\varphi_{today} \sim M_{Pl}$
- Still need to set M to solve coincidence problem.
- other issues: generically expect φ to couple to ordinary matter at some level: time-dependent $\varphi(t)$ can cause long-range forces and ~~[illegible]~~ effective time-variation of constants of Nature.

e.g. $\varphi F_{\mu\nu} F^{\mu\nu} \to$ variation of α_{em}.

Freezing + Thawing Models:

Caldwell + Linder 2005
Linder 2006, 2007

- Above 2 examples are cases of a more general classification:

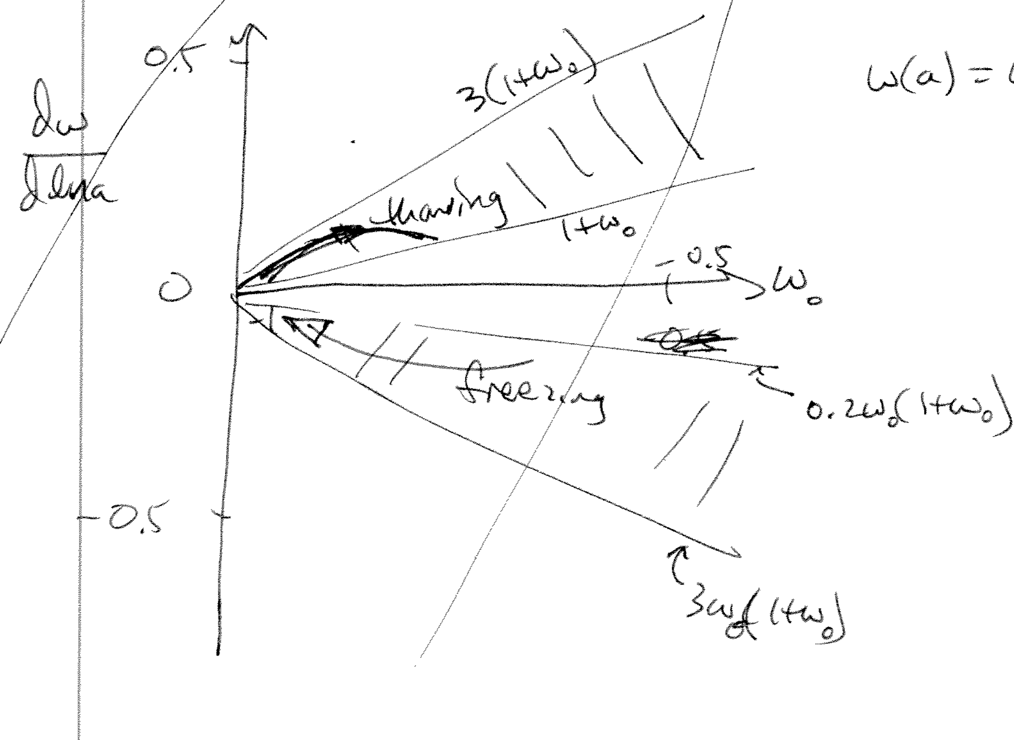
Thawing: a.) massive field, frozen early by Hubble damping, then starts to roll

$$w_i = -1 \Rightarrow w_{late} \neq -1$$

Freezing: b.) tracking field, rolls early due to steep potential, freezes late due to flattening potential

$$w_i \neq -1 \Rightarrow w_{late} \to -1$$

- Generically, in scalar field models, $w \neq -1$ and it evolves in time $\Rightarrow$ can potentially distinguish from Λ by observations that are sensitive to dw/dz (dw/da).



$$w(a) = w_0 + \underbrace{\left(\frac{dw}{d\ln a}\right)}_{=\frac{1}{H}(dw/dt)} \ln a + \ldots$$

These models tend to live in these two shaded regions and follow the evolutionary arrows.

would like: $\sigma\left(\frac{dw}{d\ln a}\right) \lesssim 2(1+w)$ to distinguish freeze from thaw.

current constraints $\Rightarrow$ would aim to shrink this in the future, to exclude classes of models.

Note: thawing models predict $\langle w \rangle \approx -1 \pm 0.05$, so must go beyond "constant w" constraints to test them.