

LTB Models:

- Spherically symmetric, radially inhomogeneous cosmological model. Metric is (in comoving coordinates), ~~with observer at r=0~~

$$ds^2 = dt^2 - X^2(r,t)\,dr^2 - A^2(r,t)\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

FRW is special case, with

$$X(r,t) \longrightarrow \frac{a(t)}{\sqrt{1-kr^2}}$$

$$A(r,t) \longrightarrow a(t)\,r$$

- For $T_{\mu\nu}$ use non-relativistic matter ($p_m = 0$) and no dark energy or Λ.
- Observer ($r=0$) is at center of spherical symmetry, by assumption: us near center and near-symmetry needed for CMB isotropy
- Can show that $X(r,t)$ can be rewritten as

$$X(r,t) = \frac{A'(r,t)}{\sqrt{1-k(r)}} \qquad \text{where } A' = \frac{\partial A}{\partial r}$$

function associated w/ spatial curvature of t = const. hypersurfaces $= kr^2$ for FRW

- Einstein eqns. then imply:

$$\frac{2}{3}\frac{\ddot{A}}{A} + \frac{1}{3}\frac{\ddot{A}'}{A'} = -\frac{4\pi G}{3}\rho_m$$

- "Total" acceleration, given by RHS, is negative, but there can be "radial acceleration", $A'(r,t) > 0$ if $A(r,t)$ decelerates fast enough, and vice versa.

- First "FRW" equ. gives

$$\frac{\dot{A}^2}{A^2} = \frac{F(r)}{A^3} - \frac{k(r)}{A^2}$$

- $F(r)$, $k(r)$ fixed by boundary conditions.

- Can define a local Hubble rate,

$$H(r,t) \equiv \frac{\dot{A}(r,t)}{A(r,t)}$$

and local matter density, $F(r) \equiv H_0^2(r)\,\Omega_m(r)\,A_0^3(r)$,

with $k(r) \equiv H_0^2(r)\left[\Omega_m(r) - 1\right] A_0^2(r)$

where $A_0(r) = A(r,t_0)$

$$\Rightarrow \quad H^2(r,t) = H_0^2(r)\left[\Omega_m(r)\left(\frac{A_0}{A}\right)^3 + \left[1 - \Omega_m(r)\right]\left(\frac{A_0}{A}\right)^2\right]$$

familiar form, assuming $\Lambda = 0$

- Similar to ordinary FRW equations, except that H_0 and Ω_m now vary with position. Their dynamics are coupled through the Einstein equs., but their boundary conditions are independent. Can arrange $\Omega_m(r)$ and/or $H(r)$ s.t. $\bar{d}_L(z)$ is similar to that of a homogeneous, accelerating Universe.
- Next: study observable in this model, for special case of an observer at $r=0$: center of symmetry (anthropocentric)

- Can choose coordinates (gauge) s.t. $A_0 = A(r, t_0) = r$, in which case we can integrate $H^2(r,t)$ equation to get an expression for the age of the Universe:

$$t_0 - t = \frac{1}{H_0(r)} \int_{\frac{A(r,t)}{A_0(r_0)}}^{1} \frac{dx}{\left[\Omega_m(r)\, x^{-1} + \left(1 - \Omega_m(r)\right)\right]^{1/2}}$$

spatial curvature

- For any point (t, r, θ, φ), this implicitly determines $A(r,t)$ and all its derivatives

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- Light rays follow radial null geodesics : $0 = ds^2 = d\theta = d\phi$

$$0 = ds^2 = dt^2 - \frac{\left(A'(r,t)\right)^2 dr^2}{1-k(r)}$$ along the trajectory (compare p.36 for FRW)

From this, we can derive the following relations:

$$\frac{dt}{dz} = \frac{-A'(r,t)}{(1+z)\,\dot{A}'(r,t)}$$

$$\frac{dr}{dz} = \frac{\left[1 + H_0^2(r)\left[1-\Omega_m(r)\right]A_0^2(r)\right]^{1/2}}{(1+z)\,\dot{A}'(r,t)}$$

and luminosity distance

$$d_L(z) = \left(\frac{L}{4\pi F}\right)^{1/2} = (1+z)^2 A\left[r(z), t(z)\right]$$

and angular diameter distance

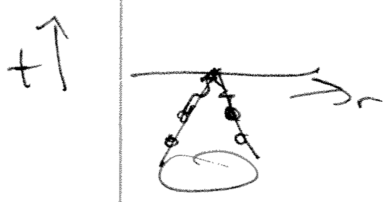
$$d_A(z) = A\left[r(z), t(z)\right] \longrightarrow a(z)\, r(z) \text{ for FRW}$$

$$= \frac{d_A}{(1+z)^2}$$ ← this result holds rather generally, independent of GR or of homogeneity, for a metric theory.

- Above equs. provide all the info. needed to calculate $d_L(z; \Omega_m(r), H_0(r))$. Because we are free to choose $\Omega_m(r)$ and $H_0(r)$, any isotropic observed $d_L(z)$ can be explained by suitable choices of these functions: no need for acceleration.

- Moreover, we can choose $\Omega_m(r)$ = constant, so the model need not contradict observed homogeneity in galaxy surveys.

- However: (a) the model is profoundly anti-Copernican (we are at the center, (b) choice of $\Omega_m(r)$, $H_0(r)$ does not match onto an ~~early~~ earlier nearly-FRW evolution from inflation, etc. ~~often~~ varying $H_0(r)$ often requires <u>"inhomogeneous big bang"</u>: Universe came into being at different times at different points: spatially varying age of the Universe.

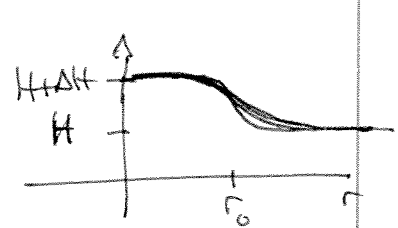


- Key point: our observations are carried out along the past light cone, ~~[illegible]~~ so observations are affected by variations of quantities in both t and r (in FRW, it's just t).

- To mimic acceleration, expansion rate should look as if it is increasing toward us along the past light cone. $\Rightarrow$

  $H_0(r)$ must <u>decrease</u> as r grows.

- Simple toy model:



$$H_0(r) = H + \Delta H e^{-r/r_0}$$

$$\Omega_m(r) = \Omega_0 = \text{constant}$$

H, ΔH, Ω_0, r_0 are free parameters. (Could add a parameter specifying "sharpness" of the transition).

- Fit SN data of Riess et al 2007:

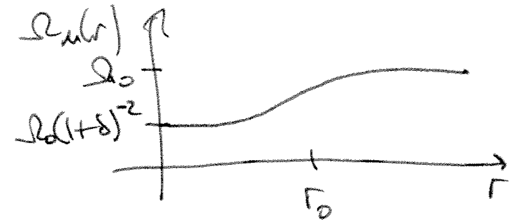
$H + \Delta H = 67 \frac{\text{km}}{\text{sec Mpc}}$ (a bit high), $\Delta H = 11$ km/sec/Mpc, $r_0 = 500$ Mpc, $\Omega_0 = 0.45$

fits as well as ΛCDM

- Alternative model of varying density but homogeneous big bang (spatially constant t), has:

$$H_0(r) = H\left[\frac{\sqrt{1-\Omega_m(r)} - \Omega_m(r)\sinh^{-1}\left(\sqrt{\frac{1-\Omega_m(r)}{\Omega_m(r)}}\right)}{\left(1-\Omega_m(r)\right)^{3/2}}\right]$$

$$\Omega_m(r) = \frac{\Omega_0}{\left(1+\delta e^{-r/r_0}\right)^2}$$



Fit SN data:

$H_0 = ??$, $\delta = 1.2$, $r_0 = 1000$ Mpc, $\Omega_0 = 0.29$

to fit SNe. $H_0(0) = 65 \rightarrow H_0(r \gg r_0) = 52$

$t_0 = 1/H = 12.8$ Gyr. Very large underdense region: don't see this in the galaxy distribution?

- Models w/ sharper transitions predict features/kinks in $d_L(z)$, which are not observed (redshift gap at $z \sim 0.2$ has been filled by SDSS).

- Other tests: CMB isotropy and blackbody spectrum (Caldwell + Stebbins) and LSS formation : largely unaddressed to date. and CMB power spectrum

but see Alnes et al.: $r_0 = 1.35$ Gpc

fit to SN + first CMB peak position:
$$\begin{cases} \frac{\Delta r}{r_0} = 0.4 \\ \Omega_m^{in} = 0.2,\ \Omega_m^{out} = 1 \\ h^{in} = 0.65,\ h^{out} = 0.5 \end{cases}$$

- Off-center observers: SN data become anisotropic. Claim that can allow up to $r_{\text{off-center}}/r_0 \lesssim 20\%$. But CMB dipole only allows $r_{\text{off-center}} \lesssim 15$ Mpc.

- More "realistic" models: Swiss-cheese universe w/ many voids. But in this case, light passage through many ~15 Mpc voids could only increase $d_L(z=1)$ by a few percent.