Lecture 10: – References: Caldwell + Linder

– go over PNGB scalar field model (p. 75)

– go over scalar field density perturbations. (pp. 71–72)

Lecture 11: – Phantom Quintessence:

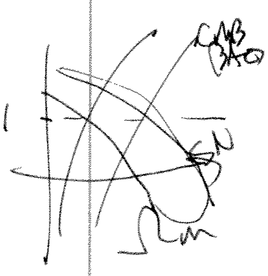
– For any $V(\varphi)$, the canonical scalar field considered above (with $\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\,\partial_\nu\varphi\, g^{\mu\nu} - V(\varphi)$) obeys the null energy condition: $\rho + p \geq 0$ i.e., $w \geq -1$.

$\therefore$ $w = -1$ is a "boundary" or "divide" for the theory and behavior vs. time. Can see this directly:

$$p_\varphi + \rho_\varphi = \left(\tfrac{1}{2}\dot\varphi^2 - V\right) + \left(\tfrac{1}{2}\dot\varphi^2 + V\right) = \dot\varphi^2 \geq 0$$

$$w_\varphi = \frac{p_\varphi}{\rho_\varphi} \geq -1.$$

– More complicated models can yield an $\boxed{\text{effective } w < -1}$: (Phantom Dark Energy)



– Modify the scalar field Lagrangian, e.g. change the sign of the kinetic term. This leads to a tachyon: runaway mode that favors increasing $\dot\varphi^2$ $\Rightarrow$ theory has instabilities. K-essence models: (Armendariz-Picon et al.) modify coefficient of $\partial_\mu\varphi\,\partial^\mu\varphi$ in Lagrangian in a more controlled way: $\mathcal{L} = f(\varphi)F[\partial_\mu\varphi\,\partial^\mu\varphi] + \ldots$ See below

– ~~Multiple fields (Linder 2004, Hu 2005) can also yield $w_{eff} < -1$?:~~

~~explicit example?~~ ~~$w_{eff} = \frac{w_1\rho_{\varphi_1}}{\rho_{\varphi_1}+\rho_{\varphi_2}} + \frac{w_2\rho_{\varphi_2}}{\rho_{\varphi_1}+\rho_{\varphi_2}}$~~

~~e.g., thawing + ...~~

- Note that $w \lesssim -1$ is consistent w/ current observations
- $\dot{\rho} + 3H(p+\rho) = 0$ $\qquad \rho \sim a^{-3(1+w)}$
- If $p + \rho < 0$, then $\dot{\rho} > 0$: energy density of a phantom component increases with time.

$$H^2 \propto \rho$$

there $H(t)$ grows with time as well. Can show (Caldwell + Kamionkowski) that $H(t)$ and $a(t)$ diverge in finite time, ripping apart everything: galaxies, stars, atoms: The Big Rip. e.g. if $w = -1.1 = \text{const.}$, then Univ terminates in a singularity in ~ 100 Gyr.

Canonical or k-essence scalar cannot cross the $w = -1$ barrier: Caldwell + Kamionkowski. But Linder seems to differ?

Generalized Scalar Field Models: Cf. Copeland, et al.

Consider generalized action (includes k-essence): [recall $S = \int d^4x \sqrt{-g}\, \frac{R}{16\pi G}$ for GR]

$$S = \int d^4x \sqrt{-g} \left[\frac{F(\varphi)}{2} R + p(\varphi, X) + \mathcal{L}_{\substack{\text{matter} \\ + \\ \text{radiation}}} \right]$$

where $X = \frac{1}{2}(\partial_\mu \varphi)(\partial^\mu \varphi) = \frac{1}{2}\dot{\varphi}^2 - \frac{1}{2}(\vec{\nabla}\varphi)^2 \underset{\text{homogeneous}}{\Rightarrow} \frac{1}{2}\dot{\varphi}^2$ — note change in sign of metric convention from before

Here p = scalar field Lagrangian = scalar pressure.

Canonical scalar: $p(X, \varphi) = X - V(\varphi)$

FRW equations become:

$$H^2 = \frac{1}{3F}\left[2X \frac{\partial p}{\partial X} - p - 3H\dot{F} + \rho_m \right]$$

$$\dot{H} = -\frac{1}{2F}\left[2X \frac{\partial p}{\partial X} + \ddot{F} - H\dot{F} + \rho_m + p_m \right]$$

$$\frac{1}{a^3}\frac{d}{dt}\left(a^3 \dot{\varphi} \frac{\partial p}{\partial X} \right) - \frac{\partial p}{\partial \varphi} - \frac{1}{2}\frac{\partial F}{\partial \varphi} R = 0$$

$$\dot{\rho}_m + 3H(p_m + \rho_m) = 0$$

EOS of φ is then:

$$w_\varphi = \frac{p + \ddot{F} + 2H\dot{F}}{2X \frac{\partial p}{\partial X} - p - 3H\dot{F}}$$

can imagine this being < -1. ?

Check: for simple case, F is indep. of φ and constant $\Rightarrow$

$$w_\varphi \to \frac{p}{2X\frac{\partial p}{\partial X} - p}$$

$p = \mathcal{L} = \frac{1}{2}\dot{\varphi}^2 - V(\varphi)$

$\frac{\partial p}{\partial X} = \frac{\partial p}{\partial\left[\frac{1}{2}\dot{\varphi}^2\right]} = 1$

Denom: $2X\frac{\partial p}{\partial X} - p = \dot{\varphi}^2 - \left[\frac{1}{2}\dot{\varphi}^2 - V\right] = \frac{1}{2}\dot{\varphi}^2 + V$

$\Rightarrow w \to (\frac{1}{2}\dot{\varphi}^2 - V)/(\frac{1}{2}\dot{\varphi}^2 + V)$

Scalar Field Perturbations + Sound Speeds:

Can show that curvature perturbation satisfies

$$\ddot{R} + \frac{\dot{s}}{s}\dot{R} + \frac{c_A^2 k^2}{a^2} R = 0$$

where perturbed metric is

$$ds^2 = (1+2A)dt^2 - 2a\,\partial_i B\,dx^i dt$$

$$- a^2(t)\left[(1+2\psi)\delta_{ij} + 2\partial_{ij}E\right]dx^i dx^j$$

and $R \equiv \psi - \frac{H}{\dot{\varphi}}\delta\varphi$

Don't worry about s.

$$c_A^2 = \frac{\partial p/\partial x + \frac{3\dot{F}^2}{4FX}}{\frac{\partial \rho}{\partial X} + \frac{3\dot{F}^2}{4FX}}$$

where $\frac{\partial \rho}{\partial X} = \frac{\partial p}{\partial X} + 2X\frac{\partial^2 p}{\partial X^2}$

- For ordinary (simple) scalar field, $p = X - \frac{V(\varphi)}{A}$ so $c_A^2 = 1\ (=c)$
- If c_A^2 went negative, would have violent instability of perturbations.
- From equ. for w_E, have $w_E \to -1$ as $\frac{\partial p}{\partial X} \to 0$ which corresponds to $c_A^2 \to 0$. ∴ Crossing phantom divide may require change in sign of c_s^2. He offers a model that evades this (2005)

$f(R)$ Theories :

- So far, we've considered Λ and scalar fields as candidates for dark energy. Now want to start considering modified gravity models. ~~These are intermediate in status: can often map onto scalar field models.~~
- Recall GR action: $S_{GR} = \frac{1}{16\pi G}\int d^4x \sqrt{-g}\, R + S_{matter}$
- Simplest class of such models have action of the form:

$$S = \frac{1}{16\pi G}\int d^4x \sqrt{-g}\, f(R) + S_{matter}$$

generalizes the Einstein-Hilbert action of GR, for which $f(R) = R$. Could arise in string theory? though typically expect $f(R) \sim aR\, m_{pl}^2 + bR^2 + cR_{\mu\nu}R^{\mu\nu} + \dots$
and R^2 terms only important for $R \sim m_{pl}^2$ and we want them large now.

Notation Note: Hu uses $R + f(R)$ instead of $f(R)$ for the action

- Field eqns: vary S wrt $g_{\mu\nu}$: $\delta S / \delta g_{\mu\nu} = 0$

$$f'(R)\, R_{\mu\nu} - \frac{1}{2} f\, g_{\mu\nu} - \nabla_\mu \nabla_\nu \left[f'(R)\right] + \left(\Box f'\right) g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

where $f'(R) = \frac{\partial f}{\partial R}$.

Einstein gravity: $f(R) = R$, $f' = 1$, reduces to

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu}$$

Idea: choose $f(R)$ so that behavior changes at <u>small</u> R, since we want <u>late-time</u> acceleration, when $R \sim H^2$ is small.

$1/R$ gravity: an interesting failure: Carroll, et al. 0306438

Choose $f(R) = R - \frac{\mu^4}{R}$, μ = constant

- This model has a "self-accelerating" vacuum ($T_{\mu\nu}=0$) solution, with $R = 12H^2 = \sqrt{3}\mu^2$ $\therefore$ want $\mu \sim H_0 \sim 10^{-33}$ eV for cosmic acceleration

  R = const. $\Leftrightarrow$ de Sitter expansion

- Einstein eqns:

$$\left(1+\frac{\mu^4}{R^2}\right)R_{\mu\nu} - \frac{1}{2}\left(1-\frac{\mu^4}{R^2}\right)g_{\mu\nu}R + \mu^4\left[g_{\mu\nu}\Box - \nabla_{(\mu}\nabla_{\nu)}\right]\frac{1}{R^2} = 8\pi G T_{\mu\nu}$$

- Take the trace:

$$\mu^4\Box\left(\frac{1}{R^2}\right) - \frac{R}{3} + \frac{\mu^4}{R} = \frac{8\pi G T}{3} \quad \text{where } T = g^{\mu\nu}T_{\mu\nu}$$

- For pressureless source, $T = -\rho$ (note change in GR sign convention)
- For GR ($\mu=0$), $R = 8\pi G\rho$
- New eqn. is quadratic rather than linear in $R \Rightarrow 2$ diff. R = const. solutions for same ρ:
  - one solution, w/ $R \simeq 8\pi G\rho$, is violently unstable to small-wavelength perturbations i.e. can't perturb around GR
  - other soln: $R \simeq \mu^2$, is stable. However, in the solar system, this soln. looks nothing like GR/Newton, which gives $R \simeq 8\pi G\rho$, e.g. outside the sun. $\Rightarrow$ violates solar system tests of GR.

This is not too surprising: this theory does not have a consistent Minkowski space ($R=0$) solution, since f diverges as $R \to 0$.

- Can evade this problem w/ different choice for $f(R)$: will explore this next time.
- Note that $\Box \frac{\mu^4}{R^2}$ term: in GR, the trace equ., $R = 8\pi G\rho$,

  is a constraint equ. that determines R uniquely. Here, R becomes a dynamical variable, a new scalar deg. of freedom. This aspect is generic to $f(R)$ models.

Intermediate Summary:

- $f(R)$ requires mass parameter $\mu \sim H$ to explain accel. similar to quintessence
- $f(R)$ contains new scalar DOF
- Solar system constraints important
- multiple solutions for same source, one of which may be unstable

[illegible].

Equivalence of $f(R)$ theories to scalar-tensor and scalar fields:

Consider the generalized scalar field theory of p. 81 (written in slightly different form:

$$S = \frac{1}{16\pi G}\int d^4x \sqrt{-g}\left[F(\varphi)R - \frac{1}{2}h(\varphi)g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)\right]$$

Choose the special case:

$$h(\varphi) = 0$$

$$F(\varphi) \equiv f'(\varphi)$$

$$V(\varphi) = \left[-f(\varphi) + \varphi f'(\varphi)\right]$$

Then the action becomes

$$S = \frac{1}{16\pi G}\int d^4x \sqrt{-g}\left[f'(\varphi)R + f(\varphi) - \varphi f'(\varphi)\right]$$

$$= \frac{1}{16\pi G}\int d^4x \sqrt{-g}\left[f'(\varphi)(R-\varphi) + f(\varphi)\right]$$

Now if $f''(\varphi) \neq 0$, the scalar field EOM is $\varphi = R$ (note absence of kinetic terms in φ), in which case the action becomes

$$S = \frac{1}{16\pi G}\int d^4x \sqrt{-g}\, f(R)$$

$\therefore$ $f(R)$ models are equivalent to a scalar-tensor theory (w/no kinetic term). We can make this look more familiar by transforming to the "Einstein" frame: $g^E_{\mu\nu} = F(\varphi)g_{\mu\nu}$ and defining canonical scalar field $\tilde{\varphi}$ by $F(\varphi) = \exp\left[\sqrt{\frac{16\pi G}{3}}\,\tilde{\varphi}\right]$

Then the action above becomes just the usual scalar + Einstein actions

$$S = \int d^4x \sqrt{-g_E}\left[\frac{R_E}{16\pi G} - \frac{1}{2} g_E^{\mu\nu}\,\partial_\mu\tilde{\varphi}\,\partial_\nu\tilde{\varphi} - V(\tilde{\varphi})\right]$$

where

$$\cancel{V(\tilde{\varphi}) = \frac{\lambda(\varphi) f'(\lambda(\varphi)) - f(\lambda(\varphi))}{16\pi G\,[f'(\lambda(\varphi))]^2}}$$

∴ In the Einstein frame (which means coeff. of R is $\frac{1}{16\pi G}$ in action) $f(R)$ gravity looks like ~~just~~ Einstein gravity plus a scalar field. However, g_E is not the metric whose geodesics determine particle orbits, it is the Jordan-Brans-Dicke metric g. ∴ Scalar tensor theories resemble GR w/ an extra force on the particles: i.e., the scalar field couples to matter ($\mathcal{L}_m$) in the Einstein frame.

$$\boxed{V(\tilde{\varphi}) = \frac{\varphi(\tilde{\varphi})\, f'(\varphi(\tilde{\varphi})) - f(\varphi(\tilde{\varphi}))}{16\pi G\left[f'(\varphi(\tilde{\varphi}))\right]^2}}$$