Astronomy 411: Science of DES
Fall Quarter 2010

Problem Set 3 (Exercise 6)

Due: Friday, Nov. 19

If you have any questions, please contact me *before* the due date by email: frieman@fnal.gov.

1. The mean separation of rich clusters (masses larger than a few $\times 10^{14} M_\odot$) in the nearby universe is about $40h^{-1}$ Mpc. Use this to determine the local mean density of rich clusters, $n_{cl}(z = 0)$.

2. Assume that rich clusters have constant comoving number density, i.e., that the proper density $n_{cl}(z) = n_{cl}(0)(1 + z)^3$, and that all rich clusters can be detected in a survey out to $z = 1$. (These assumptions are *not* justified but are made here for simplicity.) Plot the mean number of rich clusters per unit redshift that you would detect as a function of redshift (to $z = 1$) in a 5000 sq. deg. survey for 3 cosmological models: (a) spatially flat ($k = 0$), with $\Omega_m = 0.2$, $w = -1$; (b) spatially flat, with $\Omega_m = 0.2$, $w = -0.9$; (c) spatially flat, with $\Omega_m = 0.3$, $w = -1$. Also plot the residuals of models (b) and (c) with respect to model (a), so that the model differences can be better seen.

3. Let's adopt model (a) as the fiducial model. Imagine carrying out this cluster counts measurement in 10 redshift bins of width $\Delta z = 0.1$ out to $z = 1$. For each Δz bin, model (a) predicts the mean counts in that bin, $\bar{N}(z) = (dN(z)/dz)\Delta z$. In the real universe, there will be fluctuations in the measured counts due to two sources: (i) Poisson statistics ($\sqrt{N}$) and (ii) large-scale structure variations in the local density of clusters (also called cosmic variance). We'll ignore (ii). Using (i), make a Monte Carlo realization of the counts in model (a) by drawing $N(z)$ in each redshift bin from a Poisson distribution with mean $\bar{N}(z)$ in that bin. Add these points to the plot you made above.

4. Now fit the Monte Carlo data with a two-parameter cosmological model: spatially flat, with parameters w and Ω_m. Carry out a likelihood analysis in this 2-d parameter space and plot the 65, 95, and 99% CL contours in this space. Derive the marginalized errors on Ω_m and w from this hypothetical data.

5. Repeat this exercise but for survey areas of 2500 and 7500 sq. deg. How does the marginalized error on w scale with survey area?