CMB: Sound Waves in the Early Universe

Before recombination:
- Universe is ionized.
- Photons provide enormous pressure and restoring force.
- Photon-baryon perturbations oscillate as acoustic waves.

After recombination:
- Universe is neutral.
- Photons can travel freely past the baryons.
- Phase of oscillation at $t_{\text{rec}}$ affects late-time amplitude.

Today
Recombination & Last scattering
$z \sim 1000$
$\sim 400,000$ years
Neutral

Time
The cartoon

- At early times the universe was hot, dense and ionized. Photons and matter were tightly coupled by Thomson scattering.
  - Short m.f.p. allows fluid approximation.
- Initial fluctuations in density and gravitational potential drive acoustic waves in the fluid: compressions and rarefactions.

\[ \frac{d}{d\tau} \left[ m_{\text{eff}} \frac{d\delta_b}{d\tau} \right] + \frac{k^2}{3} \delta_b = F[\Psi] \]

\[ m_{\text{eff}} = 1 + 3\rho_b/4\rho_\gamma \]

- These show up as temperature fluctuations in the CMB

\[ \Delta T \sim \delta \rho_\gamma^{1/4} \sim A(k) \cos(kc_s t) \]  [harmonic wave]
Acoustic Oscillations in the CMB

Although there are fluctuations on all scales, there is a characteristic angular scale, ~ 1 degree on the sky, set by the distance sound waves in the photon-baryon fluid can travel just before recombination: sound horizon $\sim c_s t_{ls}$
Acoustic scale is set by the *sound horizon* at last scattering: $s = c_s t_{ls}$
Sound Waves

- Each initial overdensity (in dark matter & gas) is an overpressure that launches a spherical sound wave.
- This wave travels outwards at 57% of the speed of light.
- Pressure-providing photons decouple at recombination. CMB travels to us from these spheres.
Sound horizon more carefully

\[ s = \int_0^{t_{\text{rec}}} c_s (1 + z) \, dt = \int_{z_{\text{rec}}}^{\infty} \frac{c_s \, dz}{H(z)} \]

• Depends on
  – Epoch of recombination
  – Expansion of universe
  – Baryon-to-photon ratio (through \(c_s\))

\[ c_s = \left[ 3 \left( 1 + 3 \rho_b / 4 \rho_\gamma \right) \right]^{-1/2} \]

Photon density is known exquisitely well from CMB spectrum.
CMB Angular Diameter Distance

- Temperature (and polarization) patterns shift in and out in angular scale with the angular diameter distance to recombination.

fixed plasma conditions
- baryon-photon ratio: $\Omega_b h^2$
- matter-radiation ratio: $\Omega_m h^2$
  (expansion rate)

fixed recombination

Angular scale subtended by $s$
Geometry of three-dimensional space

K>0
K<0
K=0
Seeing the Sound Horizon

a) If universe is closed, “hot spots” appear larger than actual size

b) If universe is flat, “hot spots” appear actual size

c) If universe is open, “hot spots” appear smaller than actual size

CMB Maps
Angular positions of acoustic peaks probe spatial curvature of the Universe

Hu
Microwave Background Anisotropy Probes Spatial Curvature

Data indicates nearly flat geometry if \( w = -1 \)
CMB Results
WMAP3 Results

assuming $w=-1$
Degeneracy of the Peak Locations

- But raising the equation of state $w = p/\rho$ has the same effect as changing $\Omega_{DE}$.
Degeneracy of the Peak Locations

- Contours of angular diameter distance $H_0D_A$ at constant $\Omega_b h^2$, $\Omega_m h^2$ (peak locations and morphology)
CMB shift parameter

CMB anisotropy constraint on Angular Diameter distance to last-scattering well approximated by:

\[
R = \left( \Omega_m H_0^2 \right)^{1/2} \int_0^{z_{LS}} \frac{dz}{H(z)} = 1.715 \pm 0.021
\]

\[z_{LS} = 1089\]

WMAP5 results  Komatsu et al 2008
\textbf{SDSS only:}

\textbf{Nearby+SDSS:}

\textbf{MLCS}

\( w = -0.93 \pm 0.13 \text{(stat)} ^{+0.10}_{-0.32} \text{(syst)} \)

\textbf{SALT}

\( w = -0.92 \pm 0.11 \text{(stat)} ^{+0.07}_{-0.15} \text{(syst)} \)
Standard rulers

- Suppose we had an object whose length (in meters) we knew as a function of cosmic epoch.
- By measuring the angle ($\Delta \theta$) subtended by this ruler ($\Delta \chi$) as a function of redshift we map out the angular diameter distance $d_A$

$$\Delta \theta = \frac{\Delta \chi}{d_A(z)} \\
d_A(z) = \frac{d_L(z)}{(1+z)^2} \propto \int_0^z \frac{dz'}{H(z')}$$

- By measuring the redshift interval ($\Delta z$) associated with this distance we map out the Hubble parameter $H(z)$

$$c \Delta z = H(z) \Delta \chi$$
Sound horizon more carefully

\[ s = \int_0^{t_{\text{rec}}} c_s (1 + z) \, dt = \int_{z_{\text{rec}}}^{\infty} \frac{c_s \, dz}{H(z)} \]

\[ s = \frac{1}{H_0 \Omega_{m}^{1/2}} \int_0^{a_{\text{r}}} \frac{c_s}{(a + a_{\text{eq}})^{1/2}} \, da \]
CMB calibration

• Not coincidentally the sound horizon is extremely well determined by the structure of the acoustic peaks in the CMB.

\[ s = 147.8 \pm 2.6 \text{ Mpc} \quad \text{(WMAP 3rd yr data)} \]
\[ = (4.56 \pm 0.08) \times 10^{24} \text{m} \]

Dominated by uncertainty in \( \rho_m \) from poor constraints near 3rd peak in CMB spectrum.
(Planck will nail this!)
The Structure Formation Cookbook


   Primordial Inflation: initial spectrum of density perturbations

   \[ P_m(k) \sim k^n, \ n \sim 1 \]

2. Cooking with Gravity: Growing Perturbations to Form Structure

   Set the Oven to Cold (or Hot or Warm) Dark Matter

   Season with a few Baryons and add Dark Energy

   \[ P_m(k) \sim T(k)k^n \]

3. Let Cool for 13 Billion years

   Turn Gas into Stars

   \[ P_g(k) \sim b^2(k)T(k)k^n \]

4. Tweak (1) and (2) until it tastes like the observed Universe.
Cold Dark Matter Models

Power Spectrum of the Mass Density

\[ \delta(k) = \int d^3 x \cdot e^{i k \cdot x} \frac{\delta \rho(x)}{\rho} \]

\[ \langle \delta(k_1) \delta(k_2) \rangle = (2\pi)^3 P(k_1) \delta(\vec{k}_1 + \vec{k}_2) \]
Non-linear collapse
Cold Dark Matter Models

Theoretical Power Spectrum of the Mass Density

\[ \delta(k) = \int d^3x \cdot e^{i k \cdot \vec{x}} \frac{\delta \rho(x)}{\rho} \]

\[ \langle \delta(k_1) \delta(k_2) \rangle = (2\pi)^3 P(k_1) \delta^3 \left( k_1 + k_2 \right) \]

Power spectrum measurements probe cosmological parameters

\[ \delta \sim d \int x \cdot e^{i \rho k \cdot \rho x} \delta \rho(x) \]

\[ \delta \sim \Omega_{m} h = 0.2 \]

\[ \Omega_{m} h = 0.5 \]

\[ k_{eq} \sim \Omega_{m} h \]

Fig. 3.— Predicted uncertainties in the power spectrum estimated from a volume-limited (\( R_{max} = 500 h^{-1} \) Mpc) sample of SDSS North and for the Bright Red Galaxy sample (upper set of error bars). These errors assume that the true power spectrum is that of an \( \Omega h = 0.25 \) CDM model and that the BRGs are more clustered than normal galaxies. Plotted for comparison is the SDSS North errors are CDM power spectra (normalized to \( q_0 = 1 \)) for a

\[ \lambda [h^{-1} \text{ Mpc}] \]

\[ P(k) [\text{[h}^{-1} \text{Mpc}]^3] \]

\[ k [2\pi/\lambda] \]

Linear

Non-linear

SDSS BRGs

SDSS North

COBE

MAP

h/Mpc
Sound Waves again

- Each initial overdensity (in dark matter & gas) is an overpressure that launches a spherical sound wave.
- This wave travels outwards at 57% of the speed of light.
- Pressure-providing photons decouple at recombination. CMB travels to us from these spheres.
- Sound speed plummets. Wave stalls at a radius of 150 Mpc.
- Overdensity in shell (gas) and in the original center (DM) both seed the formation of galaxies. Preferred separation of 150 Mpc.

Eisenstein
A Statistical Signal

- The Universe is a superposition of these shells.
- The shell is weaker than displayed.
- Hence, you do not expect to see bulls’ eyes in the galaxy distribution.
- Instead, we get a 1% bump in the correlation function.
Origin of Baryon Acoustic Oscillations (BAO)

\[ \ddot{v}_b + \frac{\dot{a}}{a} \dot{v}_b + \vec{\nabla} \Psi = C \]

\[ C = \frac{1}{\rho_b} \int \frac{d^3p}{(2\pi)^3} (-\vec{p}) C[f_\gamma(\vec{p})] \]
Collision Term

\[ C[f_\gamma(\hat{p})] = -p \frac{\partial f_\gamma^{(0)}}{\partial p} (n_e \sigma_T) (\Theta_0 - \Theta(\hat{p}) + \hat{p} \cdot \vec{v}_b) \]

\[ f_\gamma(\hat{p}) = \left[ \exp \left\{ \frac{p}{T(1+\Theta)} \right\} - 1 \right]^{-1} \approx f_\gamma^{(0)} - p \frac{\partial f_\gamma^{(0)}}{\partial p} \Theta \]
Baryon oscillations in $P(k)$

- Since the baryons contribute $\sim 15\%$ of the total matter density, the total gravitational potential is affected by the acoustic oscillations with scale set by $s$. sound horizon scale

- This leads to small oscillations in the matter power spectrum $P(k)$.
  - No longer order unity, like in the CMB, now suppressed by $\Omega_b/\Omega_m \sim 0.1$
Baryon (acoustic) oscillations

RMS fluctuation $\Delta_{\rho}(k)$

$\Delta_{\rho}(k)$

k (h/Mpc)

Wavenumber
Divide out the gross trend ...

A damped, almost harmonic sequence of “wiggles” in the power spectrum of the mass perturbations of amplitude $O(10\%)$.

$k_A = 2\pi/s$
The error due to sample variance on a power spectrum measurement, averaged over a radial bin in $k$-space of width $\Delta k$, is

$$
\left( \frac{\sigma_P}{P} \right)^2 = 2 \times \frac{(2\pi)^3}{V} \times \frac{1}{4\pi k^2 \Delta k}
$$

(2)

plus Poisson errors: multiply by $(1+1/nP)^2$

Assumes Gaussian errors (linear theory)

Fit with:

$$
P(k) = P_{ref} \left[ 1 + A k \exp \left[ -\left( \frac{k}{0.1 h \text{Mpc}^{-1}} \right)^{1.4} \right] \sin \left( \frac{2\pi k}{k_A} \right) \right]
$$
Power Spectrum
Correlation Function

Measure redshifts and angular positions

Convert to comoving separation using redshift-distance relation

\[
\frac{dx}{dz} = \frac{c}{H_0 \Omega_m^{1/2}} \frac{1}{\sqrt{(1 + z)^3 + (\Omega_m^{-1} - 1)(1 + z)^{3(1+w)}}}
\]
Dependence on $w$

Fig. 5.— The length distortion of a rod as a function of redshift, supposing the true cosmology is $\Omega_m = 0.3$, $w_{true} = -1$ and the assumed cosmology is $\Omega_m' = 0.3$, $w_{ass} = -0.9$. The dashed and
Fig. 6.— Power spectrum measurement for a simulated survey with the same parameters as Figure 2, except that the value $w = -0.8$ has been incorrectly assumed. The wavenumber of the fitted function (the dashed line) is spuriously distorted.
SDSS Galaxy Distribution

Luminous Red Galaxies
Large-scale Correlations of SDSS Luminous Red Galaxies

Redshift-space Correlation Function

\[ \xi(r) = \langle \delta(\bar{x})\delta(\bar{x} + \bar{r}) \rangle \]

Baryon Acoustic Oscillations seen in Large-scale Structure

Warning: Correlated Error Bars

Eisenstein, et al
Model Comparison

Equality scale depends on \((\Omega_m h^2)^{-1}\).

Acoustic scale depends on \((\Omega_m h^2)^{-0.25}\).

CDM with baryons is a good fit: \(\chi^2 = 16.1\) with 17 dof.
Pure CDM rejected at \(\Delta\chi^2 = 11.7\)

Fixed \(\Omega_b h^2 = 0.024\)

\(n_s = 0.98\), flat

\(\Omega_m h^2 = 0.12\)

\(\Omega_m h^2 = 0.13\)

\(\Omega_m h^2 = 0.14\)

\(\Omega_b h^2 = 0.00\)
Constraints

Galaxy pair with separations $\Delta z$, $\Delta \theta$:

$\Delta r_c = c \Delta z / H(z)$ radial comoving separation

$\Delta r_c = \Delta \theta (1 + z) d_A$ angular comoving separation

Spherically averaged correlation function probes

$$D_V(z) = \left[ (1 + z)^2 d_A^2(z) \frac{cz}{H(z)} \right]^{1/3}$$

SDSS: $D_V(z = 0.35) = 1370 \pm 64$ Mpc

$R_{0.35} = D_V(0.35) / d_A(z_{LS}) = 0.0979 \pm 0.0036$

$A = D_V(0.35) \frac{\sqrt{\Omega_m H_0^2}}{0.35 c} = 0.469 \pm 0.017$

Eisenstein etal 2005
SDSS only:

Nearby+SDSS:

MLCS

\[ w = -0.93 \pm 0.13\text{(stat)}^{+0.10}_{-0.32}\text{(syst)} \]

SALT

\[ w = -0.92 \pm 0.11\text{(stat)}^{+0.07}_{-0.15}\text{(syst)} \]
BAO from SDSS + 2dFGRS

BAO detected at low redshift
0<z<0.3 (effective redshift 0.2)
SDSS main + 2dFGRS

BAO detected at high redshift
0.15<z<0.5 (effective redshift 0.35)
SDSS LRGs

BAO from combined sample
(detected over the whole redshift range 0<z<0.5)
All SDSS + 2dFGRS

\[
\frac{D_v(z = 0.35)}{D_v(z = 0.2)} = 1.812 \pm 0.060
\]

Percival et al. 2007
Cosmological constraints: BAO

Consider two simple models:

1. $\Lambda$CDM
2. Flat, constant $\omega$

Constraint fitting $s/D_V$ with model for $s$

Constraint including distance to CMB $d_A(z_{LS})/D_V$

Constraint from $D_V(0.35)/D_V(0.2)$

Percival et al. 2007
Galaxy Clustering varies with Galaxy Type

How are each of them related to the underlying Dark Matter distribution?

BIAS

Caveat for inference of Cosmological Parameters from LSS
Galaxy Clustering as a function of Galaxy Luminosity

Based on sample of ~200,000 galaxies

Zehavi, et al.

Tegmark, et al.
Correct For Luminosity Bias

Vertical Shift: Constant Bias

\[
\frac{\delta n_{\text{gal}}(L, \bar{x})}{n_{\text{gal}}(L)} = b(L) \frac{\delta \rho(\bar{x})}{\rho}
\]
Systematic Issues for BAO

Effects of non-linearities on BAO signal

Modeling redshift distortions precisely

Effects of (non-linear) galaxy bias
Halos vs. Dark Matter

Real Space

Redshift Space
III. Baryon Acoustic Oscillations

Galaxy Angular Correlation Function in Photo-z bins

Systematics:
photo-z’s, correlated photometric errors, non-linearity, scale-dependent bias

Fosalba & Gaztanaga
III. Baryon Acoustic Oscillations

DES photo-z survey ($r=24$, 5000 deg$^2$, $\delta z/(1+z)=0.03$)

- $0.5 < z < 1.0$ ($\delta k_A/k_A = 2.60\%$)
- $1.0 < z < 1.5$ ($\delta k_A/k_A = 1.86\%$)
DES Science Program

Four Probes of Dark Energy

- **Galaxy Clusters**
  - ~100,000 clusters to z>1
  - ~10,000 with SZE measurements from SPT
  - Sensitive to growth of structure and geometry

- **Weak Lensing**
  - Shape measurements of 300 million galaxies
  - Sensitive to growth of structure and geometry

- **Baryon Acoustic Oscillations**
  - 300 million galaxies to z = 1 and beyond
  - Sensitive to geometry

- **Supernovae**
  - 15 sq deg time-domain survey
  - ~3000 well-sampled SNe Ia to z ~1
  - Sensitive to geometry

Forecast Constraints on DE Equation of State
Photometric Redshifts

- Measure relative flux in multiple filters: track the 4000 Å break

- Estimate individual galaxy redshifts with accuracy \( \sigma(z) < 0.1 \) (~0.02 for clusters)

- Precision is sufficient for Dark Energy probes, provided error distributions well measured.
DES grizY

$\sigma$ Limiting Magnitudes

$g$ 24.6
$r$ 24.1  $J$ 20.3
$i$ 24.0  $H$ 19.4
$Z$ 23.8  $Ks$ 18.3
$Y$ 21.6

+2% photometric calibration error added in quadrature

Photo-z systematic errors under control using existing spectroscopic training sets to DES photometric depth: low-risk

+Developed improved Photo-z & Error Estimates and robust methods of outlier rejection

Oyaizu, Cunha, Lima, Frieman, Lin

*Vista Hemisphere Survey