Lecture V:

- Clusters:
  - Press-Schechter theory and $n(M)$ - Notes
  - Cluster Probes: X-ray, SZ, Opt., WL
  - Mass-obs relation, self-calibration

  } Slides
- Exposure Time Calculation for DES.

Jim Annis:
- Optical Cluster Finding
- DES Survey Strategy

Linear Perturbations in CDM:

- On large scales d, perturbation amplitude δ_m is small $\Rightarrow$ linearize:

$$\delta_m \ll 1 \quad , \quad \left(\frac{vt}{d}\right)^2 \ll \delta_m \qquad \rho_m(\vec{x},t) = \bar{\rho}_m(t)\left[1+\delta_m(\vec{x},t)\right]$$

small velocities

$$\Rightarrow \boxed{\ddot{\delta}_m + 2H\dot{\delta}_m = 4\pi G\bar{\rho}_m\delta_m}$$

$$4\pi G\bar{\rho}_m(t) = 4\pi G\bar{\rho}(t)\,\Omega_m(t) = \frac{3}{2}H^2\Omega_m$$

$$\cancel{H^2 = \frac{8\pi G}{3}\left(\rho_m(t)+\rho_{DE}(t)\right)}$$

$$\cancel{\frac{3}{2}H^2 = 4\pi G\bar{\rho}\left[\Omega_m(t)+\Omega_{DE}(t)\right]}$$

Can also write this as

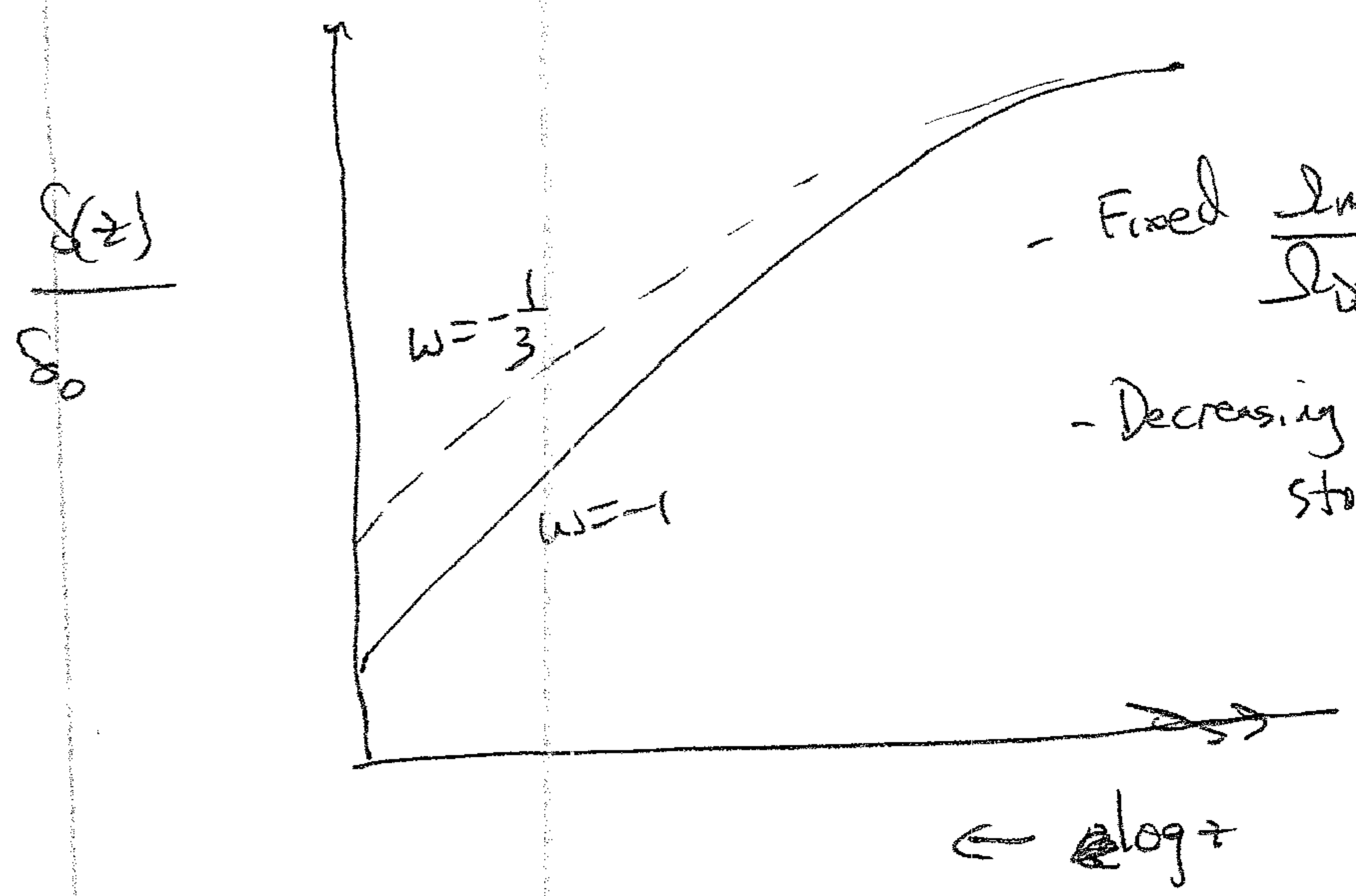
$$\boxed{\ddot{\delta}_m + 2H\dot{\delta}_m - \frac{3}{2}\Omega_m(t)H^2(t)\,\delta_m = 0}$$

"Friction" due to expanding Universe: timescale H^{-1}

Driving force with timescale $t \sim \frac{1}{\sqrt{\Omega_m}}\frac{1}{H}$

- At early times, $\bar{\rho}_m \gg \rho_{DE}$ since $\bar{\rho}_m \sim a^{-3}$ while $\rho_{DE} \sim a^{-3(1+w)}$, $\therefore$ at early times $\Omega_m(t) = 1$ and there is a growing mode solution with $\underline{\delta_m(\vec{x},t) \propto a(t)}$.

- When DE starts to dominate, $\Omega_m(t)$ drops below 1, damping term wins over gravitational driving term, perturbation growth stops: $\delta_m \to$ const. Pert. growth effectively stops at redshift $1+z = \left(\frac{\Omega_{m,0}}{\Omega_{DE,0}}\right)^{1/3w}$



- Fixed $\frac{\Omega_m}{\Omega_{DE}}$

- Decreasing Ω_m at fixed w also stops growth earlier.

- For increasing w, $\rho_{DE}(a)$ scales more rapidly with a $\Rightarrow$ dominates earlier for fixed Ω_m/Ω_{DE} $\Rightarrow$ pert. growth stops sooner.

- For $w_{DE}=-1$, have an exact growing mode soln:

$$\delta_m(z) \propto H(z)\,\frac{5\Omega_m}{2}\int_z^\infty \frac{1+z}{(H(z))^3}\,dz$$

$$\delta_m(a) \propto \frac{5}{2} H_0^2\,\Omega_m\,H(a)\int_0^a \frac{da}{(a\,H(a))^3}$$

Characterize density field to lowest order by its variance:

smooth the field on scale R: $\delta_R(\vec{x},t) = \int \delta(\vec{x}',t)\,W(\vec{x}+\vec{x}';R)\,d^3\vec{x}'$

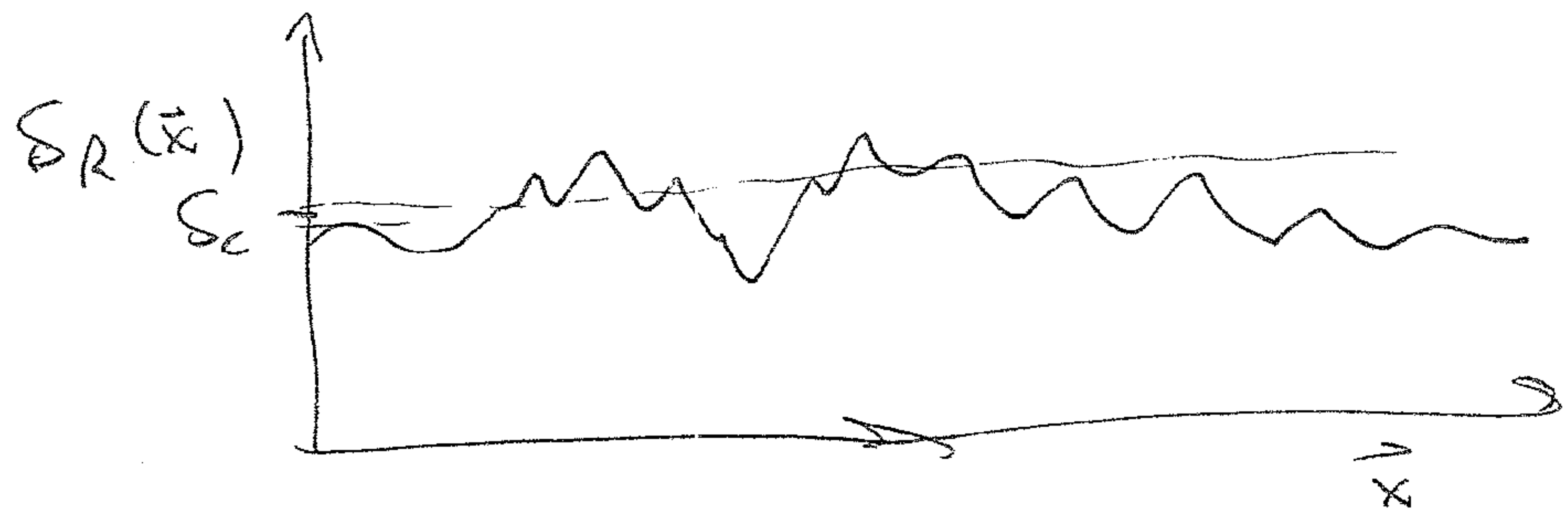
$$\sigma_R^2(t) = \langle \delta_R^2(\vec{x},t)\rangle = \frac{1}{2\pi^2}\int P_\delta(k)\,\tilde{W}^2(kR)\,k^2\,dk$$

FT of W

power spectrum of δ: $\langle \delta_{\vec{k}}\,\delta_{\vec{k}'}\rangle = (2\pi)^3 P(k)\,\delta_D(\vec{k}+\vec{k}')$

Halo Mass Function: + Non-linear Perturbations

- Press-Schechter (1974) theory gives a rough estimate of the halo mass function, i.e., qualitatively agrees w/ N-body simulations, but now use more accurate fitting formulae (Jenkins etal; Sheth + Tormen; Warren, etal) Tinker, etal.
  For excursion set approach, see Zentner, 0611454
- Primordial inflation models (the simplest ones) predict a spectrum of small-amplitude density perturbations obeying Gaussian statistics, i.e., that $\frac{\delta \rho}{\rho}(\vec{x})$ is a random Gaussian field
- Smooth the density field on scale R, $\delta_R(\vec{x},t)$



Overdense regions will have characteristic mass $M_R \sim R^3 \bar{\rho}_0$ where $\bar{\rho}_0$ = present mean mass density

- Ansatz: consider regions where δ_R is above some critical value for collapse, δ_c (see below). Such regions will quickly collapse + virialize → form dark halos of mass M_R.

P-S: Fraction of mass at time t contained in halos of mass M = (twice) the probability that $\delta_R > \delta_c(t)$

fudge factor

Press-Schechter Theory

Approach: somewhat related to Gaussian peaks theory.

i.e., initial growing mode smoothed density field $\delta_R(\vec{x}, \tau_i)$ determines the distn. of NL lumps (halos) at later times. The dynamics assumed is spherical collapse, which is just used to impose a δ-threshold above which objects collapse.

Since $\delta_R(\vec{x}, \tau_i)$ is assumed Gaussian, at early times the fraction of points surrounded by a sphere of radius R within which the mean density exceeds a threshold $\delta_c = \delta_{sc}(z)$ is just

$$F(R, \tau_i) = P_G(\delta \geq \delta_c(R)) = \frac{1}{\sqrt{2\pi}\,\sigma_L(R)} \int_{\delta_c}^{\infty} d\delta_R\, e^{-\frac{\delta_R^2}{2\sigma_L^2(R)}} = \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{\delta_c}{\sqrt{2}\,\sigma_L(R)}\right)\right]$$

linear variance of $\delta_R(\tau_i)$ i.e. assuming $\delta_R^{rms} \ll 1$

Press + Schechter: identify this w/ the fraction of particles $F(>M)$ which are in NL lumps (halos) of mass $M \geq M(R) = \frac{4\pi}{3}\bar{\rho}R^3$,

numerical factor here depends on smoothing filter used in $\sigma(R)$

where $\sigma_L(R) = \sigma_L(M)$ is now the present, linearly extrapolated variance on that mass / radius scale.

- Usually choose $\delta_c = \delta_{sc}(0)$ for objects today or $\delta_{sc}(z)$ for objects at redshift z.

Problem: - $P_G(\delta) \leq \frac{1}{2}$ (since underdense regions don't collapse in linear theory)

- Note: not completely obvious this is problematic (i.e, maybe not all the mass is in halos)

- P-S: just multiplied by fudge factor $\times 2$

- Peacock + Heavens '90, Bond et al '91: derived PS expression w/ correct factor 2, so will adopt it using excursion set theory

$\Rightarrow$

Halo Mass Function:

$$n(M,z) = \frac{\bar{\rho}}{M}\left|\frac{dF}{dM}\right| = \sqrt{\frac{2}{\pi}}\,\frac{\bar{\rho}}{M}\left(\frac{\delta_c}{\sigma_L^2(R)}\right)\left|\frac{d\sigma_L(R)}{dM}\right|\exp\left(-\frac{\delta_c^2}{2\sigma_L^2(R)}\right)$$

$= \frac{dn}{d\ln M}\,dM$

where $\bar{\rho}$ = mean density ~~(at time t_0)~~

where $\delta_c = \delta_{sc}(z)$

~~$\sigma_L^2(R)$ or $\sigma_L^2(M)$ encodes~~

$\sigma_L^2(R,z)$ depends on perturbation growth $\leftrightarrow$ cosmological parameters

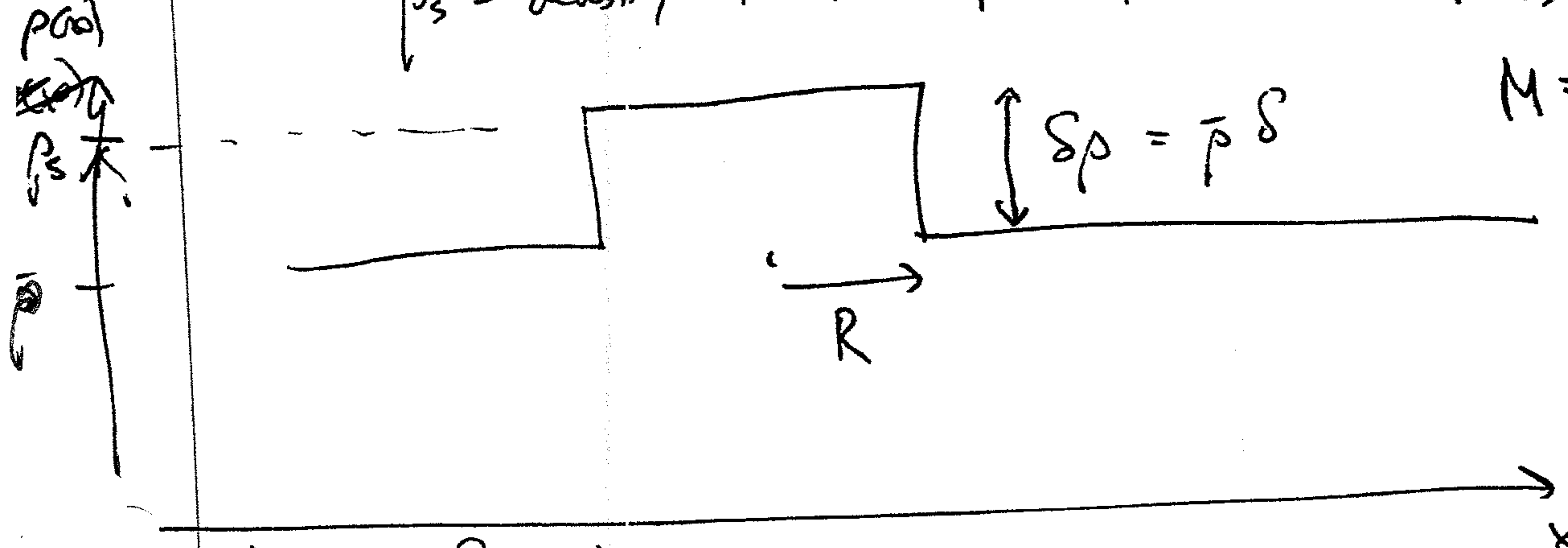
~~$\sigma_L^2(M,z) = \int d^3k\, P(k,z)\, W^2(kR)$~~

~~$P(k,z) \sim |\delta_k|^2(z)$~~

Spherical Collapse Model:

- Press-Schechter + halo models in their simplest forms rely on the spherical collapse model, an exact NL soln. for the evolution of a local, spherically symm. uniform-density (tophat) perturbation.

ρ_s = density w/in the region $= \bar\rho(1+\delta)$



Mass enclosed:

$$M = \frac{4\pi}{3}R^3\rho_s = \frac{4\pi}{3}R^3\bar\rho(1+\delta)$$

- Assume $\delta_i \ll 1$.

Uniform-density sphere in FRW universe. By spherical symm., EOM is

$$\frac{d^2R}{dt^2} = \frac{-GM}{R^2} = -\frac{4\pi G}{3}R\bar\rho(1+\delta)$$

(physical coordinates, not comoving)

(Assume $\Omega_m=1$, $\Omega_\Lambda=0$ for now).

$$\therefore\quad \ddot R/R = -\frac{4\pi G}{3}\bar\rho(1+\delta)$$

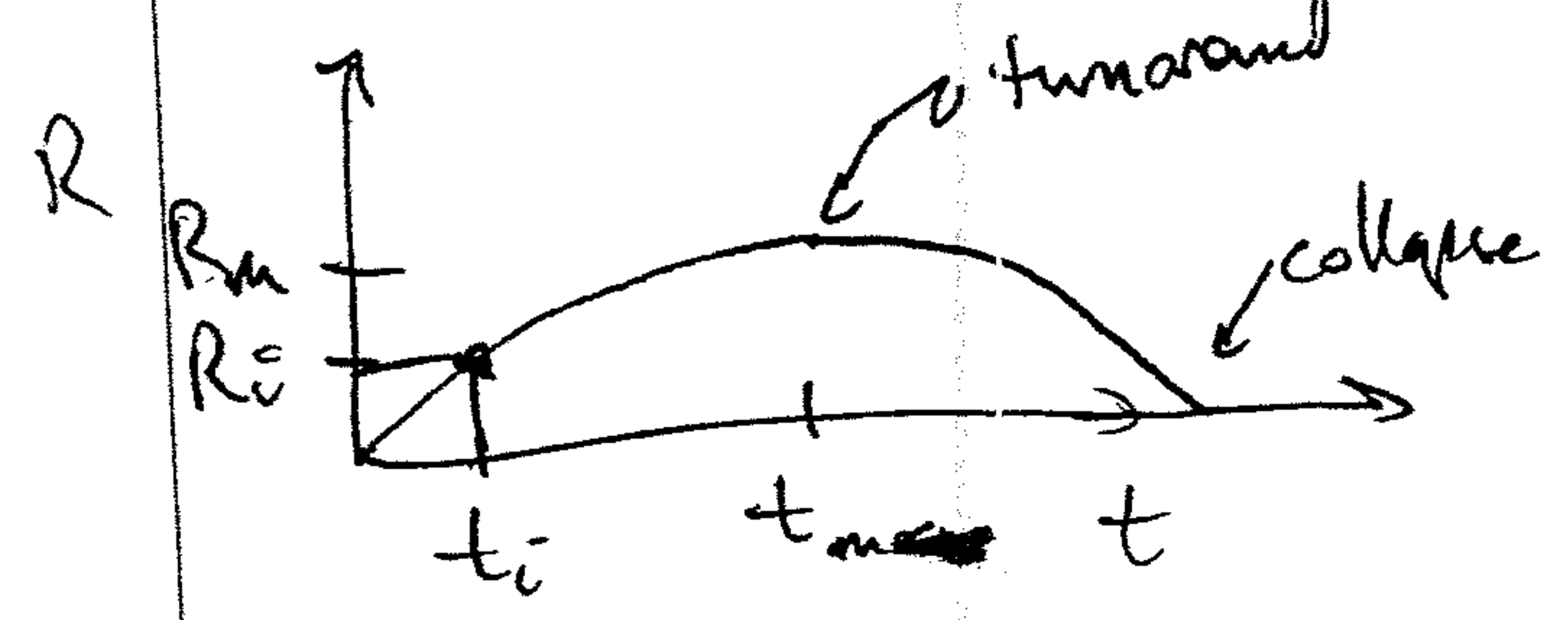
This is just the eqn. for scale factor $a(t)$ corresponding to a Universe w/ mean density $\rho_s = \bar\rho(1+\delta)$. If $\Omega_m=1$, then this "universe" must be closed, i.e., will recollapse, if $\delta_i>0$ $\leftrightarrow$ this perturbation will initially expand (when $\delta \ll 1$) and then turn around + collapse

R

~~Sta~~ Parametric Solutions:

We know a first integral is $\left(\frac{\dot R}{R}\right)^2 = \frac{8\pi G}{3}\rho_s - \frac{k}{R^2} = \frac{2GM}{R^3} - \frac{k}{R^2}$

Initial conditions: Instead of fixing $R_i = R(t_i)$, instead specify maximum radius at turnaround $= R_m = R(t_m)$ s.t. $\dot R(t_m) = 0$



$\Rightarrow \frac{2GM}{R_m^3} = \frac{k}{R_m^2}$

$\Rightarrow k = 2GM/R_m$

$$\Rightarrow (\dot R/R)^2 = \frac{2GM}{R^4}\left(R - \frac{R^2}{R_m}\right)$$

Now rescale the time coordinate: define "conformal time" by

$$d\eta = \sqrt{\frac{2GM}{R_m}}\,\frac{dt}{R} \Rightarrow \left(\frac{\dot R}{R}\right)^2 = \frac{2GM}{R^4 R_m}\left(\frac{dR}{d\eta}\right)^2$$

$$\Rightarrow \boxed{\left(\frac{dR}{d\eta}\right)^2 = R_m R - R^2}$$

Solution: $\boxed{R = \frac{R_{m}}{2}(1-\cos\eta)}$ $\therefore R = R_{max}$ at $\eta = \pi$

$$\Rightarrow t = \sqrt{\frac{R_m}{2GM}}\int R\,d\eta = \frac{R_m^{3/2}}{\sqrt{2GM}}(\eta - \sin\eta) = \frac{t_m}{\pi}(\eta - \sin\eta)$$

Overdensity:

$$1+\delta(t) = \frac{\rho_s(t)}{\bar\rho(t)} = \frac{(3M/4\pi R^3)}{1/6\pi G t^2} = \frac{9}{2}\frac{(\eta - \sin\eta)^2}{(1-\cos\eta)^3}$$

$\Omega_m = 1$

At $R = R_m$, $1+\delta(t_m) = \frac{9\pi^2}{16} = 5.6$

Linear Solns: For $\delta << 1$, $\eta << 1$, have

$$\left.\begin{aligned} \delta_L(t) &\simeq \frac{3}{20}\eta^2 \\ t &\simeq \frac{t_m}{\pi}\frac{\eta^3}{3!} = \frac{t_m}{6\pi}\eta^3 \end{aligned}\right\} \Rightarrow \underline{\delta_L(t) = \frac{3}{20}\left(\frac{6\pi t}{t_m}\right)^{2/3}}$$

This makes sense: for $\Omega_m = 1$, $\delta_L \sim a(t) \sim t^{2/3}$.

~~∴ Extrapolated linear density at t_m is $\delta_L(t_m) = \frac{3}{20}$~~

∴ Extrapolated linear density at $t = t_m$ turnaround, is

$$\delta_L(t_m) = \frac{3}{20}(6\pi)^{2/3} = 1.062 \quad (<< 4.6 = \delta_{NL})$$

Collapse: At $\eta \to 2\pi$, $R \to 0$, $t = 2t_m$, $\delta \to \infty$.

At this time, extrapolated linear density is

$$\delta_L = \frac{3}{20}(12\pi)^{2/3} = \underline{1.686} \equiv \delta_{sc}(0) \quad [\text{independent of } M]$$

We can interpret this as the critical (linear) density pert. for spherical collapse of a pert. at the present time (in an E-dS cosmology).

Thus, for a pert. to collapse by redshift z_c (in an $\Omega_m = 1$ model) it must have had this linear amplitude at redshift z_c, i.e., $\delta_L(z_c) = 1.686$. Extrapolated to today, this implies

$$\delta_L(0) = \delta_L(z_c)(1+z_c) = 1.686(1+z_c) \equiv \delta_{sc}(z_c)$$

i.e. $\delta_{sc}(z_c)$ is the present linearly extrapolated ampl. for a pert. that collapsed at redshift z_c (in an $\Omega_m = 1$ Universe).

Virialization: At $\eta \to 2\pi$, $\delta \to \infty$. Unphysical.

In practice, there will be non-radial motions $\Rightarrow$ object doesn't collapse to ∞ density but instead reaches virial equilibrium at finite size:

For self-gravitating sys. in virial equilibrium,

$$U_{grav} + 2K = 0 \Rightarrow K_{vir} = -\frac{U_{vir}}{2}$$

At t_m, $K_m = 0 \Rightarrow E = U_m$

Energy conservation $\Rightarrow E = U_m = U_{vir} + K_{vir} = \frac{U_{vir}}{2}$

Now since $U \sim \frac{GM}{R} \sim \frac{1}{R} \Rightarrow \underline{R_{vir} = \frac{1}{2} R_m}$

$\Rightarrow \rho_s(\text{vir}) \sim R^{-3} = 8 \rho_s^{\text{max}}$

Now since $t_{vir} \simeq t_{collapse} = 2t_m$ and $\bar{\rho}(t) \sim t^{-2}$, have

$$\frac{\rho_s(\text{vir})}{\bar{\rho}(t_{vir})} = 8 \times \frac{\rho_s(t_m)}{\bar{\rho}(t_m)/4} = 32 \frac{\rho_s(t_m)}{\bar{\rho}(t_m)} = 32\left(\frac{9\pi^2}{16}\right) = 18\pi^2 = \underline{178}$$

$\therefore\ 1 + \delta(t_{vir}) \simeq \underline{\delta(t_{vir}) = 178}$.

[N.B. for $\Omega_m \neq 1$, $1 + \delta(t_{vir}) \simeq 178\,\Omega_m^{-0.7}$]

After virialization, $\rho_s \simeq$ const., $\bar{\rho} \sim t^{-2}$ (for $\Omega_m = 1$) $\Rightarrow \underline{\delta \sim 178\left(\frac{t}{t_{vir}}\right)}$

Lecture 4c:

DES Exposure Time Calculation

- We want to detect many clusters to $z \gtrsim 1$ optically and measure accurate photo-z's for them.

Many → volume → area + depth

$z > 1$: depth

photo z's : depth + multiple filters

- Accurate photo-z's for clusters + galaxies to $z \gtrsim 1$ are achieved for ($\sigma(z) \lesssim 0.02$) ($\sigma(z) \lesssim 0.1$)

griz(Y) ~ 24th mag.

Roughly: want to detect red cluster galaxies with

$L \gtrsim 0.4 L_*$ to $z \sim 1-1.3$.

In R-band: $L_* \simeq 2 \times 10^{10} L_\odot$ from galaxy luminosity fn. measurements

$$M_* \equiv -2.5 \log L_* \simeq -20.3 + 5 \log h$$

(modulo evolution)

$$m_R - M_R = \mu = 5 \log_{10}\left(\frac{d_L}{10\,pc}\right)$$

$$\overset{z \ll 1}{\simeq} 5 \log z - 5 \log (H_0 \cdot 10\,pc) + 5 \log \left[1 + \frac{z}{2}(1 - q_0)\right]$$

For $q_0 = -0.6$, $H_0 = \frac{1}{3000 h^{-1}\,Mpc}$, $z = 1$ (poor approx):

$\mu \simeq 44.5$ for $z = 1$, ΛCDM model.

$0.4 L_* \rightarrow M_* + 1 \simeq -19.3 + 5\log h = -20.1$

$\rightarrow \quad m_R - M_R = 44.5$

$$m_R \simeq 44.5 + M_R = 44.5 - 20.1 = \underline{\underline{24.4}}$$

- This roughly sets the limiting mag in r-band.
- Similar depth in other bands for photo-z's.

- How do we go from limiting mag to exposure time?

Ignoring cosmology and K-corrections,

$$L_x = 4\pi r^2 f_x$$

where received flux in passband x is

$$f_x = \int f_\lambda \, F_x(\lambda) \, R(\lambda) \, T(\lambda) \, d\lambda$$

$F_x(\lambda)$: filter transmission function

$R(\lambda)$: telescope + instrument efficiency = (CCD QE) × corrector optics × mirror reflectivity × vignetting

$T(\lambda)$: atmosphere transmission: depends on direction (airmass)

Roughly, $FRT \approx 0.4$ estimated for DECam + Blanco in r-band (near overhead?)

Define the AB magnitude system by

$$m_x^{AB} = -2.5 \log(f_x / f_{x,0})$$

where

$$f_{x,0} = \text{flux zero-point}$$
$$= 3.63 \times 10^{-20} \text{ erg/s/cm}^2\text{/Hz} \cdot \int F_x(c/\nu)\, d\nu$$

filter passband in frequency space

"flat spectrum"

In this system, a source w/ const. flux per unit frequency interval has zero color

r-band: ~~$\Delta\lambda = 150$ nm~~

~~$\langle \nu \rangle = 1.13 \times 10^{14}$ Hz~~

~~$\langle E_\gamma \rangle = h \langle \nu \rangle = 3.13 \times 10^{-19}$ Joules~~

~~$= 3.13 \times 10^{-12}$ erg~~

~~$h = 6.63 \times 10^{-27}$ erg s~~

~~Then, for a 0th mag star ($r_{AB} = 0$):~~

R-band: $\langle\lambda\rangle = 6000\,\mathring{A} = 6\times10^{-5}\,cm = 600\,nm$

$$\langle\nu\rangle = \frac{c}{\langle\lambda\rangle} = \frac{3\times10^{10}\,cm/sec}{6\times10^{-5}\,cm} = 5\times10^{14}\,sec^{-1}$$

$$\langle E_\gamma\rangle = h\langle\nu\rangle = 6.63\times10^{-27}\,erg\cdot sec \cdot 5\times10^{14}\,sec^{-1}$$
$$= 3.3\times10^{-12}\,erg$$

$$\cancel{\Delta\nu = \frac{c}{150\,nm} = \frac{3\times10^{10}\,cm/sec}{150\times10^{-7}\,cm} = 2\times10^{15}\,sec^{-1}}$$

r-passband $\Delta\lambda = 150\,nm$

$\lambda_1 = 525\,nm \qquad \lambda_2 = 675\,nm$

$$\Delta\nu = \frac{c}{525} - \frac{c}{675} = 3\times10^{10}\,\frac{cm}{sec}\left[\frac{1}{5.25\times10^{-5}\,cm} - \frac{1}{6.75\times10^{-5}\,cm}\right]$$

$$= 3\times10^{15}\,Hz\left[\frac{1}{5.25} - \frac{1}{6.75}\right] = 1.3\times10^{14}\,Hz$$

∴ 0th mag star has flux

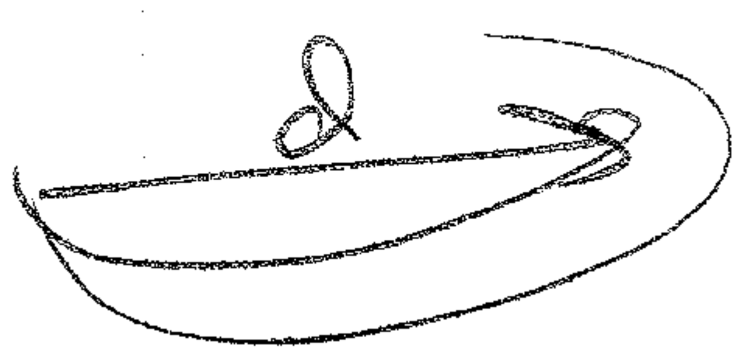
$$f_{r,0} = 3.63\times10^{-20}\ \text{erg/s/cm}^2\text{/Hz} \cdot (1.3\times10^{14}\ \text{Hz})$$

$$= 4.7\times10^{-6}\ \text{erg/s/cm}^2$$

Each γ has $\langle E\rangle = 3.3\times10^{-12}$ erg

→ Photon flux $= \dfrac{f_{r,0}}{\langle E\rangle} = 1.4\times10^{6}\ \text{s}^{-1}\text{cm}^{-2}$

$= 1.4\times10^{10}\ \text{s}^{-1}\text{m}^{-2}$ at top of atmosphere



$d = 4\text{m}$ $\quad A = \pi r^2 = 4\pi = 12.6\ \text{m}^2$

but central part is removed →

$A = 10.6\ \text{m}^2$

→ $N_{photon} = 1.5\times10^{11}\ \text{sec}^{-1}$ in r-band for 0 mag source

∴ $N_{photons}$ (0 mag source) $= 1.5\times10^{11}\ t$

where t = exposure time in seconds.

Figure in 0.4 efficiency →

N_{ph}(0 mag) $= 6\times10^{10}\ t$ are actually detected.

Assume each detected γ liberates 1 photoelectron in the CCD.

Galaxy at $r=24$:

$$\cancel{N_{ph}(24) = N_{ph}(22)}$$

$$m_1 - m_2 = -2.5\log(f_1/f_2)$$

$$f_1/f_2 = 10^{-0.4(m_1-m_2)}$$

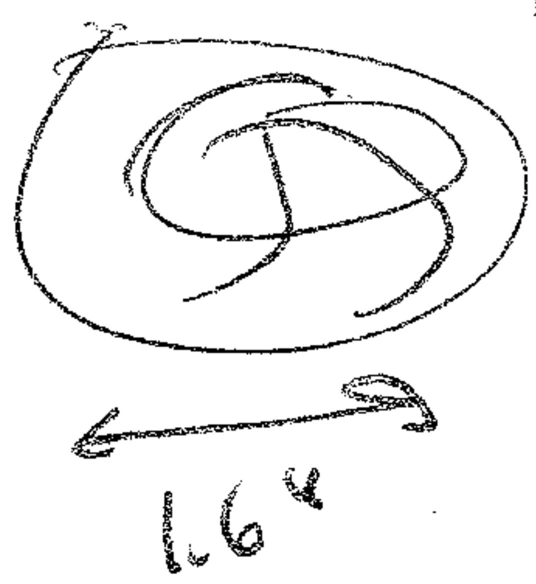
$$N_{ph}(24) = N_{ph}(0)\cdot 10^{-0.4(24)}$$

$$= 6\times 10^{10}\cdot t \cdot 10^{-9.6} = 6\times 10^{0.4}\, t$$

$$= 15t$$

Typical galaxy subtends $\simeq 1.6$ arcsec. diameter

→ Area = 2 arcsec² aperture is optimal. Even if galaxy is smaller, the PSF/seeing isn't. (0.8" FWHM)



- In r-band, ~~bright~~ dark sky photon flux is $r = 20.7$ mag/sec²

$$\cancel{N_{ph}(sky)}\quad \cancel{N_{ph}(sky)}\quad N_{ph}(sky) = 6\times 10^{10}\, t\cdot 10^{-0.4(20.7)}$$

$N_{ph}(sky) \simeq 316\ \text{sec}^{-1}\ \text{arcsec}^{-2}$ ← is this during dark ~~or grey time~~? yes

→ $N_{sky} = 633t$ in the same aperture as above

10 days past new moon, sky is ~0.6 mag brighter in r, ~2 mag brighter in g

let's assume read-noise of $5e^-$/pixel $= N_{read}$

(determined by read-out time, not exposure time)

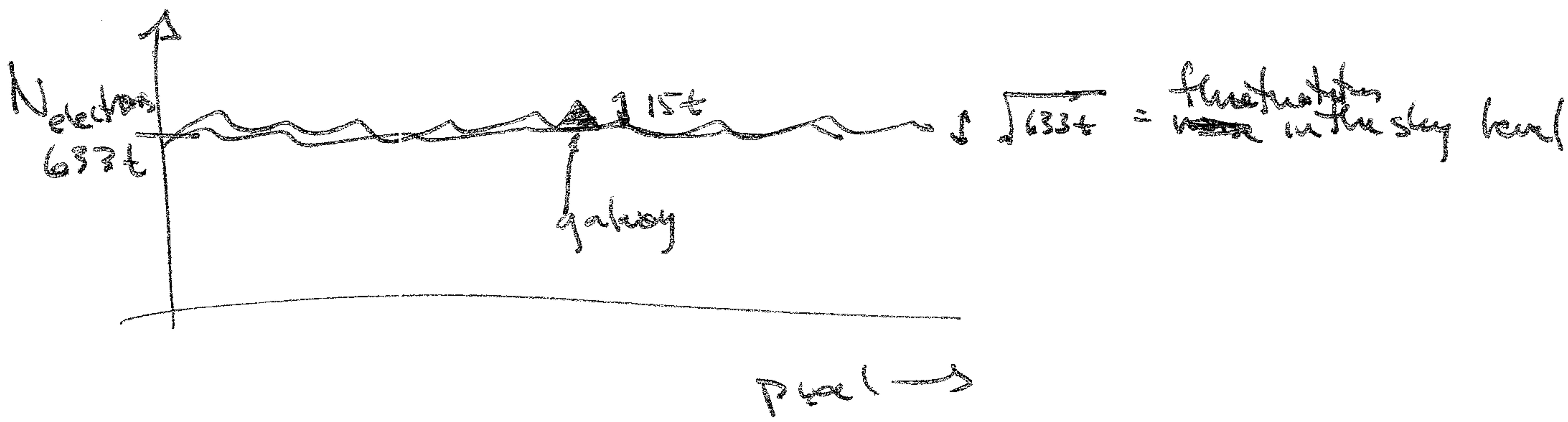
DECam has pixel size $= 0.27''$ → $n_{pix}(1.6''\ \text{ap}) = \frac{2}{(0.27)^2} = 27.4$ pixels

- Arrival of photons is a Poisson process:

Then the galaxy signal to noise is

$$\frac{S}{N} = \frac{15t}{\sqrt{15t + 633t + \underbrace{(5)(27.4)}_{137}}}$$

∴ Exposures are sky-noise dominated: can detect an object if it is above the sky noise



~~For S/N = 10 and~~ For $t \gg 1$ sec:

$$\frac{S}{N} = \frac{15t}{\sqrt{648t}} = 0.59\, t^{1/2}$$

$\frac{S}{N} \geq 10 \Rightarrow t \geq 288$ seconds

↑ minimum for the faintest ($r = 24$) galaxies

We have set $t_{total} = 400$ sec by co-adding 4×100 sec exposures.

- Observing objects overhead maximizes atmosphere transmission (minimizes airmass)

- Better seeing → deeper mag for fixed t :

$0.8'' \to 0.7'' \Rightarrow 0.15$ mag deeper

- Longer exposure → deeper : twice the exposure ⇒

$$\Delta(\text{mag lim}) = 2.5 \log_{10}(\sqrt{2}) = 0.4 \text{ mag.}$$