Probing DE w/ Clusters Counts

Recall FRW metric:

$$ds^2 = dt^2 - a^2(t)\left\{ \frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)\right\}$$

Volume Element

- If we have a set of objects (e.g., galaxies, clusters, dark matter halos) whose spatial number density we know/can predict, then counting them provides another cosmological test. We'll apply this espec. to galaxy cluster counts.

- ~~[illegible]~~ A proper area dA at redshift z and radial coordinate r subtends solid angle $d\Omega$ at the origin given by $dA = a(t_e)\, r\, d\theta\, a(t_e)\, r \sin\theta\, d\varphi$

$$= a_e^2 r^2 d\Omega = \frac{a_0^2 r^2 d\Omega}{(1+z)^2}$$

- The rate of proper displacement w/ z along a light ray is

$$d\ell = c\,dt = \frac{da}{\dot{a}} = \frac{dz}{(1+z)}\frac{a}{\dot{a}} = \frac{dz}{H(z)(1+z)}$$

= linear depth of a sample of objects in the redshift interval $(z, z+dz)$

- Proper volume element of the sample is then

$$d^2V_{prop} = dA\, d\ell = \frac{a_0^2 r^2(z)}{H_0 E(z)(1+z)^3}\, d\Omega\, dz$$

where $H(z) = H_0 E(z)$

- If $n_{prop}(z)$ is the proper number density of objects, then the #counts per unit redshift and solid angle are

$$\boxed{\frac{d^2N}{dz\, d\Omega} = n_{prop}(z)\frac{d^2V_{prop}}{dz\, d\Omega} = \frac{n_{prop}(z)\, a_0^2 r^2(z)}{H_0 E(z)(1+z)^3}}$$

- *If* objects are conserved (neither created nor destroyed), then

$$n_{prop}(z) = n_0 (1+z)^3 \Rightarrow \frac{d^2N}{dz\,d\Omega} = \frac{n_0\, r^2(z)}{H(z)}$$

- Convenient to define the comoving # density,

$$n_c(z) = \frac{n_{prop}(z)}{(1+z)^3}$$ = constant if objects are conserved

= # density per comoving volume,

where the comoving volume element is defined by

$$\frac{d^2V_c}{dz\,d\Omega} = \frac{r^2(z)}{H(z)} = \frac{r^2(z)}{H_0 E(z)} = \left(\frac{d^2V_{prop}}{dz\,d\Omega}\right)(1+z)^3$$

so that the counts are also given by

$$\boxed{\frac{d^2N}{dz\,d\Omega} = n_c(z)\frac{d^2V_c}{dz\,d\Omega} = \frac{n_c(z)\, r^2(z)}{H(z)}}$$

- See Fig. 2 right-hand panel of Rio lectures for examples + Fig. 9 Rio

- We will apply this to clusters, for which theory predicts $n(z)$.

- See eg. Vikhlinin etal results

$$\frac{d^2V}{dz\,d\Omega} \quad \text{scaling:} \quad \sim \frac{r^2}{H(z)}$$

$$\cancel{E(z) =} \qquad \chi = \int \frac{dz}{H(z)}$$

$$r = S_k(\chi) \approx \chi \quad \text{at low } z \ll 1$$

$$H(z) = H_0 \left[\Omega_m (1+z)^3 + \Omega_{DE} (1+z)^{3(1+w)} \right]^{1/2}$$

Expand: for $z \ll 1$

$$H(z) \cong H_0 \left[1 + (1+q_0) z \right]$$

$$q_0 = \frac{\Omega_m}{2} + \frac{(1-\Omega_m)}{2}(1+3w) \leftarrow \text{for flat}$$

$$\chi \cong \frac{1}{H_0} \left[z - (1+q_0) \frac{z^2}{2} \right]$$

To lowest order, $$\frac{d^2V}{dz\,d\Omega} \sim \frac{r^2(z)}{H(z)} \sim \frac{\chi^2(z)}{H(z)}$$

$$\frac{d^2V}{dz\,d\Omega} \propto \frac{\left[z - (1+q_0)\frac{z^2}{2} \right]^2}{1+(1+q_0)z} = \frac{z^2 \left[1 - (1+q_0)\frac{z}{2} \right]^2}{1+(1+q_0)z}$$

$z \ll 1$

$$\frac{d^2V}{dz\,d\Omega} \propto z^2\left(1 + (1+q_0)\frac{z^2}{4} - (1+q_0)z\right)\left(1 - (1+q_0)z\right)$$

$$\propto z^2\left[1 - (1+q_0)z - (1+q_0)z + O(z^2)\right]$$

$$\propto z^2\left\{1 - 2(1+q_0)z\right]$$

For $\Omega_m = 0.2$, $q_0 = 0.1 + 0.4(1+3w)$

$\underline{w=-1}$: $q_0 = 0.1 + 0.4(1-3)$

$= 0.1 - 0.8 = -0.7$

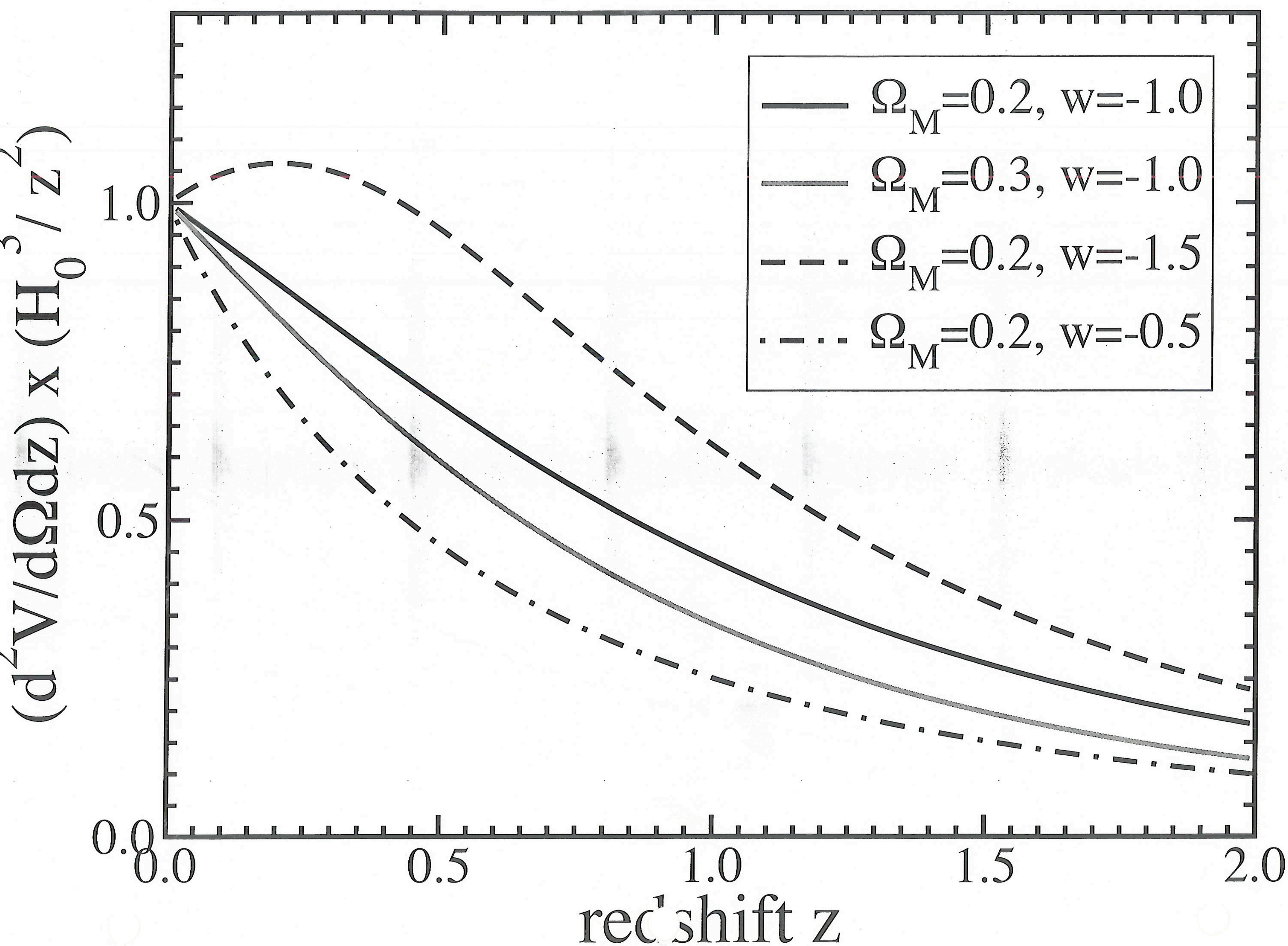
$1+q_0 = 0.3$

$w = -0.5$: $q_0 = 0.1 + 0.4(1-1.5)$

$= 0.1 - 0.2 = -0.1$

$1+q_0 = 0.9$

$$\text{Vol} \propto \begin{cases} z^2\left[1 - 0.6z\right], & w=-1 \\ z^2\left\{1 - 1.8z\right\}, & w=-0.5 \end{cases}$$


$(d^2V/d\Omega dz) \times (H_0^3 / z^2)$
0.0
0.5
1.0
0.0
0.5
1.0
1.5
2.0
redshift z
Ω_M=0.2, w=-1.0
Ω_M=0.3, w=-1.0
Ω_M=0.2, w=-1.5
Ω_M=0.2, w=-0.5

Counting Clusters

$$\frac{d^2N(z)}{dz\,d\Omega} = \frac{r^2(z)}{H(z)} \int_0^\infty f(O,z)\,dO \int_0^\infty p(O|M,z)\,\frac{dn(z)}{dM}\,dM$$

selection fn. for observable O, e.g. optical richness, X-ray luminosity, ~~SZE~~ flux, WL mass

mass-observable relation

Theoretical mass fn.

not constant with redshift

Halo Mass function, predicted by N-body simulations

approx. by Press-Schechter fn.

encodes DE dependence of perturbation growth

Cf. Vikhlinin et al.

recall $D_A(z) = r(z)(1+z)^{-1}$

$D_L(z) = r(z)(1+z)$