ASTRONOMY 320: RELATIVISTIC ASTROPHYSICS

Winter Quarter 1999

Special Project 2

Due Thursday, 11 March 1999

1. For a spherically symmetric distribution of matter, the Tolman-Oppenheimer-Volkoff (OV) equations of stellar structure in General Relativity are as follows. The equation expressing the conservation of mass is

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho \, , \tag{1}$$

while the equation of hydrostatic equilibrium is

$$\frac{dP}{dr} = -\frac{\rho m}{r^2}\left(1+\frac{P}{\rho}\right)\left(1+\frac{4\pi P r^3}{m}\right)\left(1-\frac{2m}{r}{}^{-1}\right) \, , \tag{2}$$

and the derivative of the gravitational potential Φ is given by

$$\frac{d\Phi}{dr} = -\frac{1}{\rho}\frac{dP}{dr}\left(1+\frac{P}{\rho}\right)^{-1} \, . \tag{3}$$

The equation of state both below and above neutron drip is complicated, and is best handled in tabular form. Use the results tabulated in Baym, Pethick, and Sutherland (1971).

(a) Assuming a given central density, integrate equations (1) - (3) simultaneously, finding $P(\rho)$ by interpolation in the equation of state table, to determine the mass M and radius R of the corresponding zero-temperature neutron star configuration for this particular equation of state (express M and R in units of $M_\odot$ and $R_\odot$). By iteration, construct zero-temperature neutron star configurations for $M = 0.6$, 0.8, 1.0, 1.2, and 1.4 $M_\odot$. Plot the run of density ρ as a function of radius r for zero-temperature neutron stars of these masses.

(b) Using the results of (b), make a table of M and R (express M and R in units of $M_\odot$ and $R_\odot$) for $M = 0.6$, 0.8, ..., 1.4. Then plot the mass-radius relation for zero-temperature neutron stars for this particular equation of state.

[Please include a copy of the program you used to obtain these results.]

2. Calculate the cooling of the neutron star configurations that you obtained in (1), as follows.

(a) Neutrino emission dominates the cooling of the neutron star. Use the expression for the modified URCA process given in Shaprio and Teukolsky for $\epsilon_\nu^{\rm URCA}$ (equation 11.5.22). (Ignoring neutrino emission due to crust bremsstrahlung is a relatively good approximation for this equation of state, since it is relatively soft and the resulting neutron stars therefore have a thin crust.) Calculate $L_\nu = L_\nu(T_c)$, where T_c is the core temperature of the neutron star.

(b) For the heat capacity of the neutron star, use the expression for c_V (the specific heat per particle) given in Shaprio and Teukolsky (equation 11.8.1). Note that the relativity parameter x should be evaluated for neutrons (not electrons) from the density ρ.

(c) Use the results from (a) and (b) above to obtain a differential equation relating the central temperature of the neutron star T_c and the time t. Solve this differential equation for T_c as a function of age τ, and thus L_ν as a function of τ, by evaluating the heat capacity c_V at each time step using the zero-temperature neutron star configurations that you calculated in (1). Do this for $M = 0.6, 1.0,$ and $1.4 M_\odot$ neutron stars.

(d) The photon luminosity L of a neutron star can be written as $L = L(T_c)$. Adopt the relation $T_s = T_s(T_c)$ given in Shapiro and Teukolsky (equation 11.8.11). Using this relation and the results of (c), plot the luminosity L as a function of age τ for $M = 0.6, 1.0,$ and $1.4 M_\odot$ neutron stars for this particular equation of state.

[Please include a copy of the program you used to obtain these results.]

Reference

Baym, G., Pethick C, J., and Sutherland, P. 1971, ApJ, 170, 299