ASTRONOMY 320: RELATIVISTIC ASTROPHYSICS

Winter Quarter 1999

Special Project 1

Due Thursday, 11 February 1999

1. For a spherically symmetric distribution of matter, the equation expressing the conservation of mass can be written,

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho \ , \tag{1}$$

while the equation of hydrostatic equilibrium is

$$\frac{dP}{dr} = -\frac{Gm(r)\rho}{r^2} \ . \tag{2}$$

The equation of state of an ideal electron gas can be written in the parametric form

$$P = Af(x) \ ; \qquad \rho = Bx^3 \ , \tag{3}$$

where A and B are constants.

(a) Numerically evaluate A and B, leaving μ_e as a variable. The function $f(x)$ is given in Chandrasekhar, *Stellar Structure* (1939) on p. 360 [equation (20)]; see also Shapiro & Teukolsky, *Black Holes, White Dwarfs, and Neutron Stars*, Eq.(2.3.5).

(b) Assuming a given central density, integrate equations (1) and (2) simultaneously, using equation (3) to find the mass M and radius R of the corresponding zero-temperature white dwarf configuration (express M and R in units of $M_\odot$ and $R_\odot$). By iteration, construct zero-temperature white dwarf configurations for $M = 0.1, 0.2, ...1.4 M_\odot$ for $\mu_e = 2$, *i.e.*, for He or C/O white dwarfs. Plot the run of density ρ as a function of radius r for zero-temperature white dwarfs of these masses.

(c) Using the results of (b), make a table of M and R (express M and R in units of $M_\odot$ and $R_\odot$) for $M = 0.1, 0.2, ..., 1.4 M_\odot$ for $\mu_e = 2$. Then plot the mass-radius relation for zero-temperature white dwarfs.

[Please include a copy of the program you used to obtain these results.]

2. Calculate the cooling of the white dwarf configurations that you obtained in (1), as follows.

(a) The photon luminosity L of a white dwarf can be written as

$$L = -\left[\int_0^M c_V(\rho, T)\frac{dM}{M}\right]\frac{d}{dt}\left(\frac{3kT}{2}\frac{M}{Am_u}\right) \equiv -f(M,T)\frac{d}{dt}\left(\frac{3kT}{2}\frac{M}{Am_u}\right) \ , \tag{4}$$

where $c_V(\rho, T)$ is the *dimensionless* specific heat at constant volume per ion. Assume $c_V(\rho, T) = 1$ (*i.e.*, its ideal gas value) when $\Gamma \leq 150$ [cf. Shapiro & Teukolsky, *Black Holes, White Dwarfs, and Neutron Stars*, Eqs.(4.3.15 and 4.3.16)], and $c_V(\rho, T) = f_D(\theta_D/T)$ when $\Gamma > 150$, where

$$f_D(y) = \frac{6}{y^3} \int_0^y \frac{x^4 e^x \, dx}{(e^x - 1)^2} \, ; \tag{5}$$

[*ibid.*, Eqs.(4.4.10 and 4.4.11)] (*i.e.*, use the Debye approximation for the specific heat at constant volume). However, for the Debye Temperature θ_D use $\theta_D = \Omega_p$ [*ibid.*, Eqs.(4.3.17 and 4.3.18)], *not* $\theta_D = \omega_D$ [*ibid.*, Eq.(4.4.12)].

(b) Divide equation (4) by the relationship we derived in class between the luminosity L and the central temperature T of a white dwarf [*ibid.*, Eq.(4.1.11)] to obtain a differential equation relating T and the time t. Solve this differential equation for T as a function of age τ, and thus L as a function of τ, by evaluating $f(M, T)$ at each time step using the zero-temperature white dwarf configurations that you calculated in (1). Do this for $M = 0.2, 0.6$, and $1.0 M_\odot$ white dwarfs.

(c) Using the results of (b), plot the luminosity L as a function of age τ for $M = 0.2, 0.6$, and $1.0 M_\odot$ white dwarfs.

(d) Using the results of (b), plot the discovery function ϕ as a function of $L/L_\odot$ [*ibid.*, Eq.(4.6.2)].

[Please include a copy of the program you used to obtain these results.]