

*Astronomy and Astrophysics with  
Gravitational Waves:  
BBH Population Properties Inferred from  
LIGO/Virgo's O1 & O2*

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Midwest Supernova and Transients Workshop, Feb 25-26, 2019

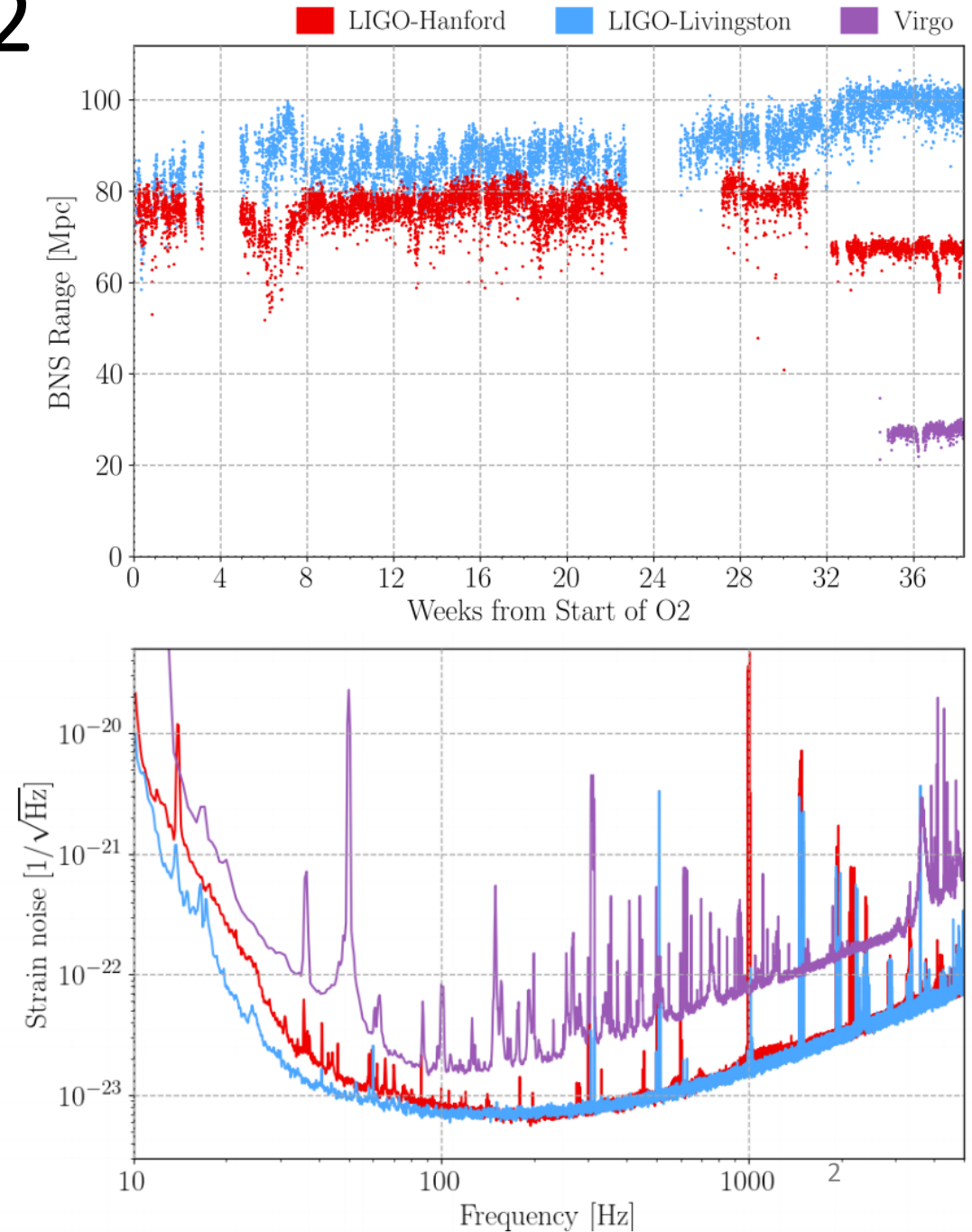
Public paper and data release

GWTC-1: <https://dcc.ligo.org/LIGO-P1800307/public>

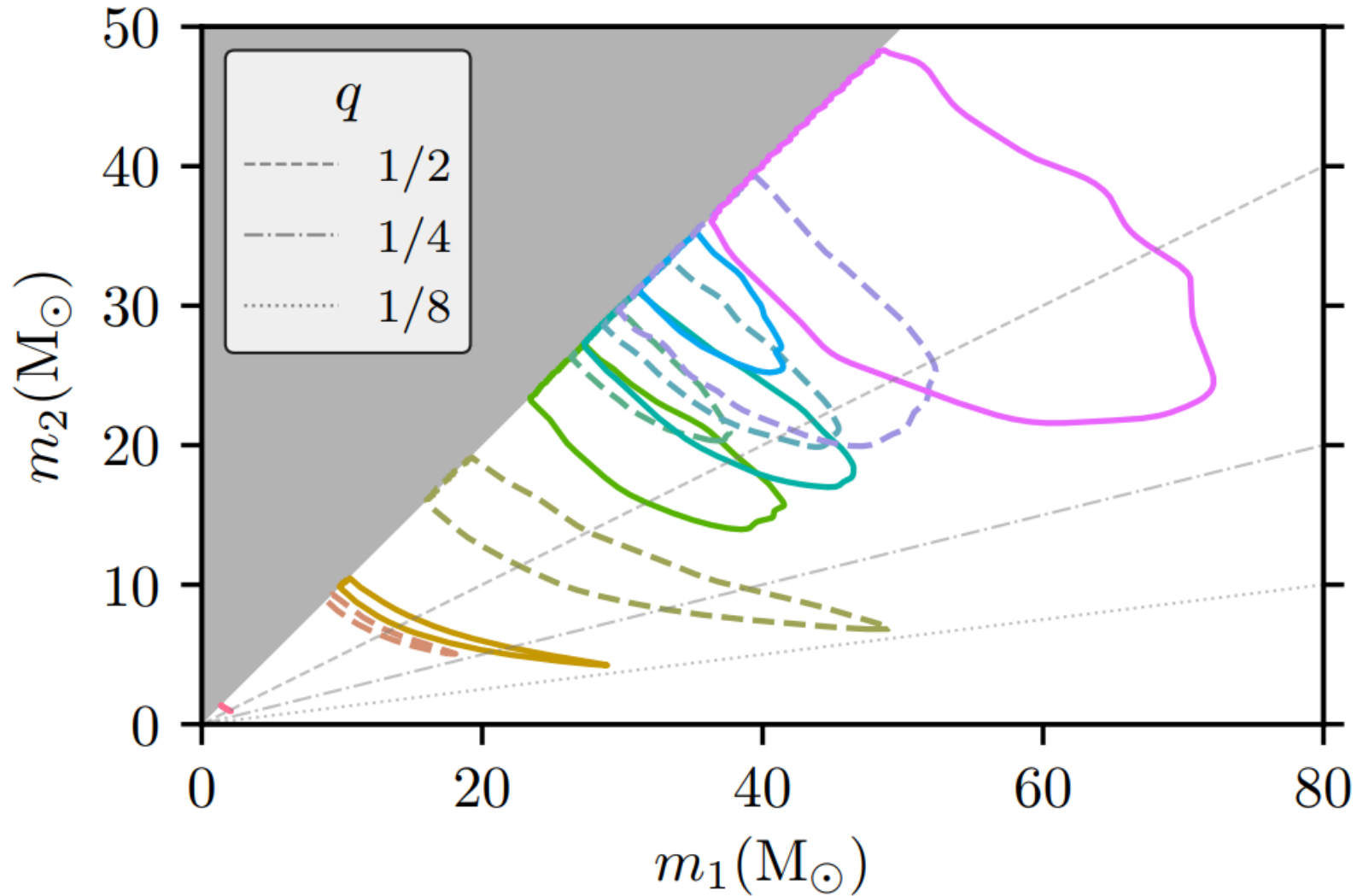
BBH Pops: <https://dcc.ligo.org/LIGO-P1800324/public>

# LIGO and Virgo During O1 and O2

- Observing runs
  - **O1 aLIGO:** Sept. 12th, 2015 - Jan. 19th, 2016
  - **O2 aLIGO:** Nov. 30th, 2016 - Aug. 25th, 2017
  - **O2 Adv. Virgo:** joined Aug 1st, 2017
- Top: BNS range for each instrument during O2
- Bottom: representative **amplitude spectral density** of the total strain noise
- Coincident analysis time: **166.6 days** (O1: 48.6 days, O2: 118 days)
- O2 data were **recalibrated and cleaned** leading to **increased sensitivity**

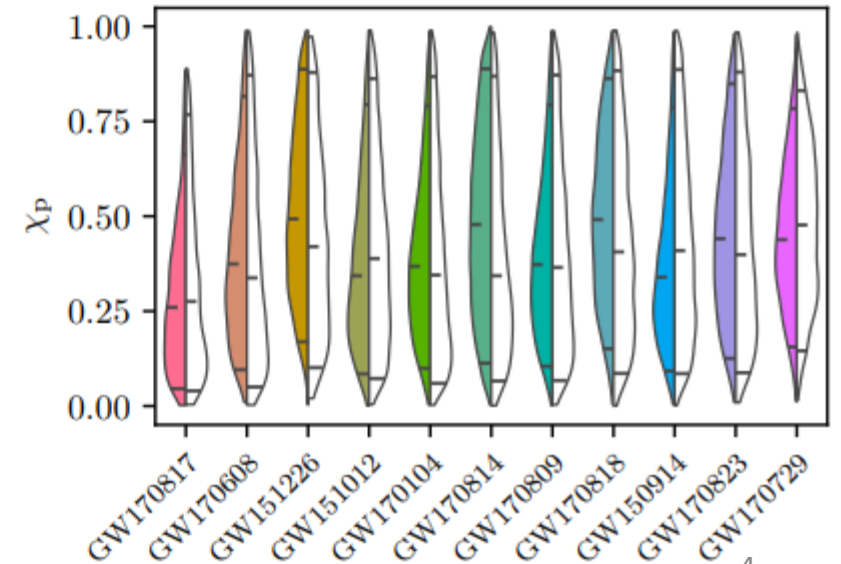
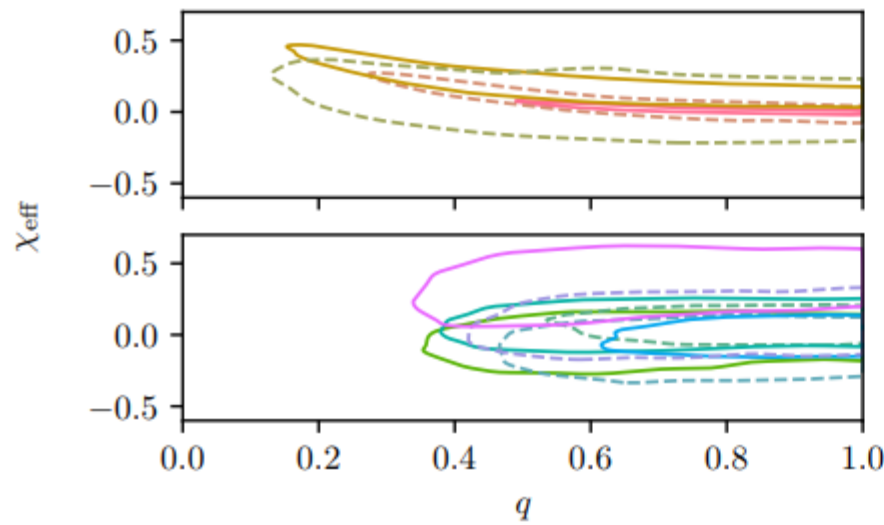
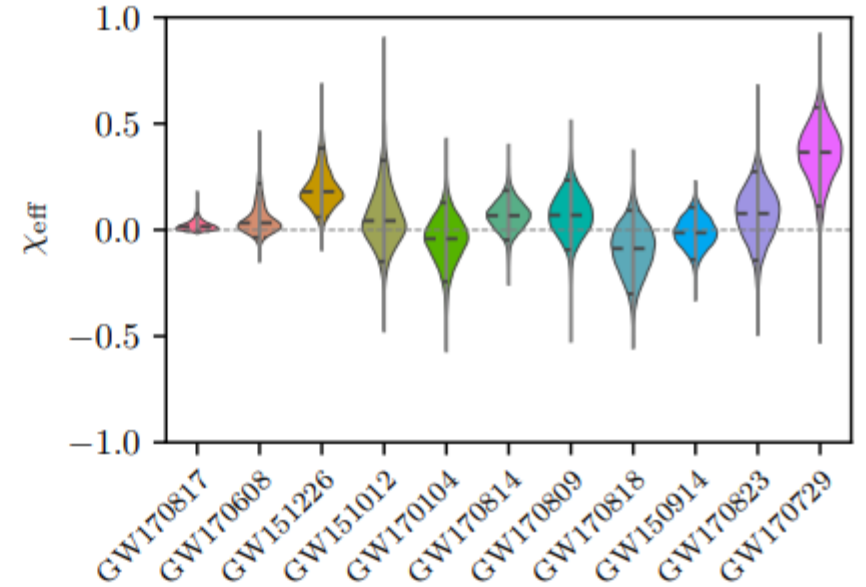
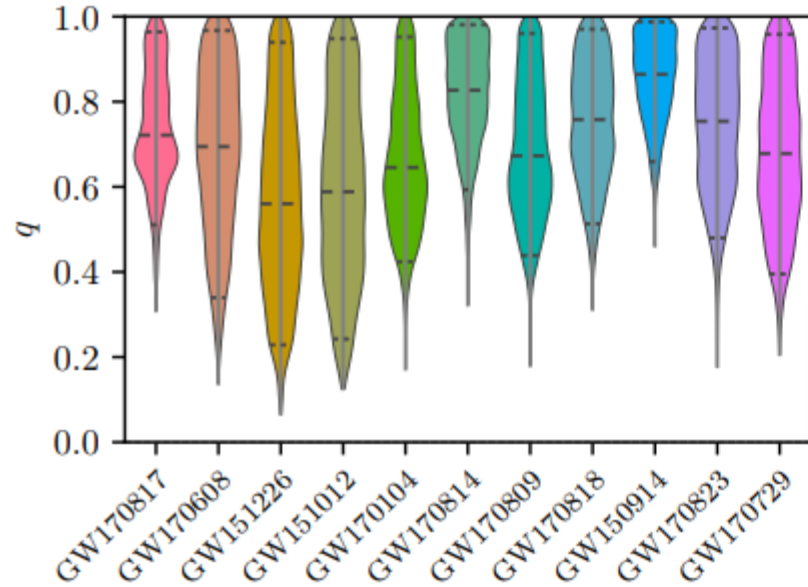


# Event Parameters: Component Masses

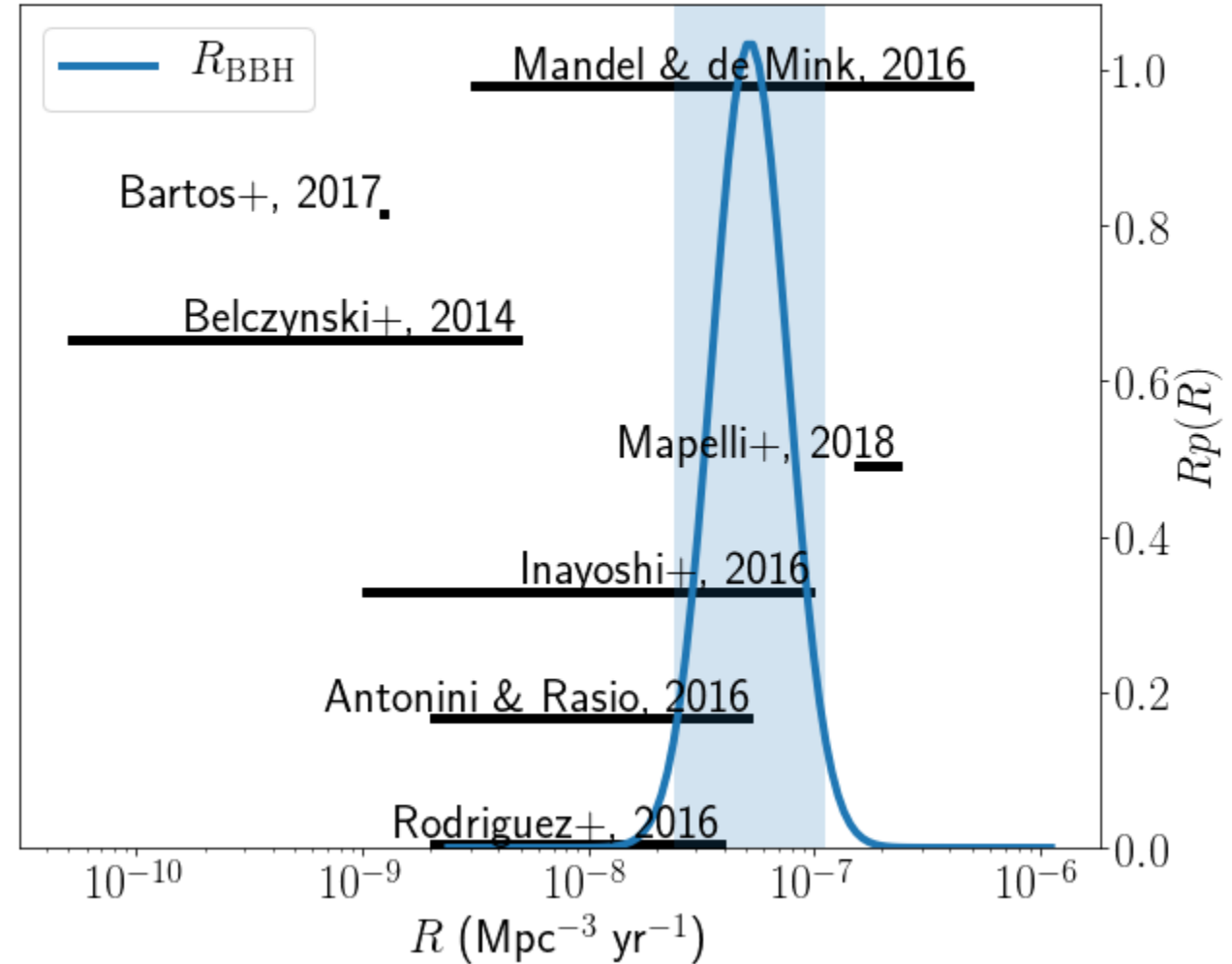
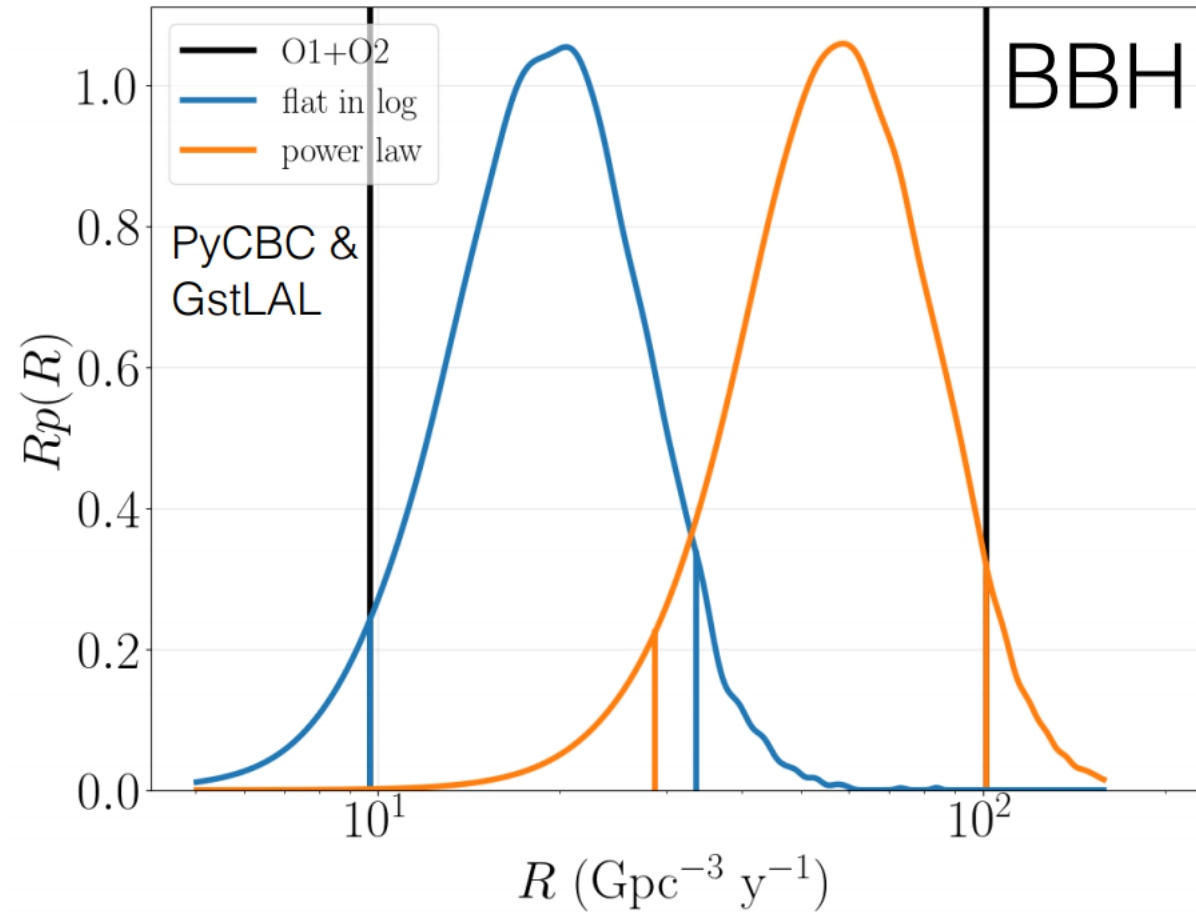


# Event Parameters: Mass Ratio and Eff. Spin

- No convincingly asymmetric mass ratio events
- Only two events with non-zero  $\chi_{\text{eff}}$
- New events are all at higher masses (e.g.  $> 20 M_{\odot}$ )
- Precession mostly follows expectation from prior



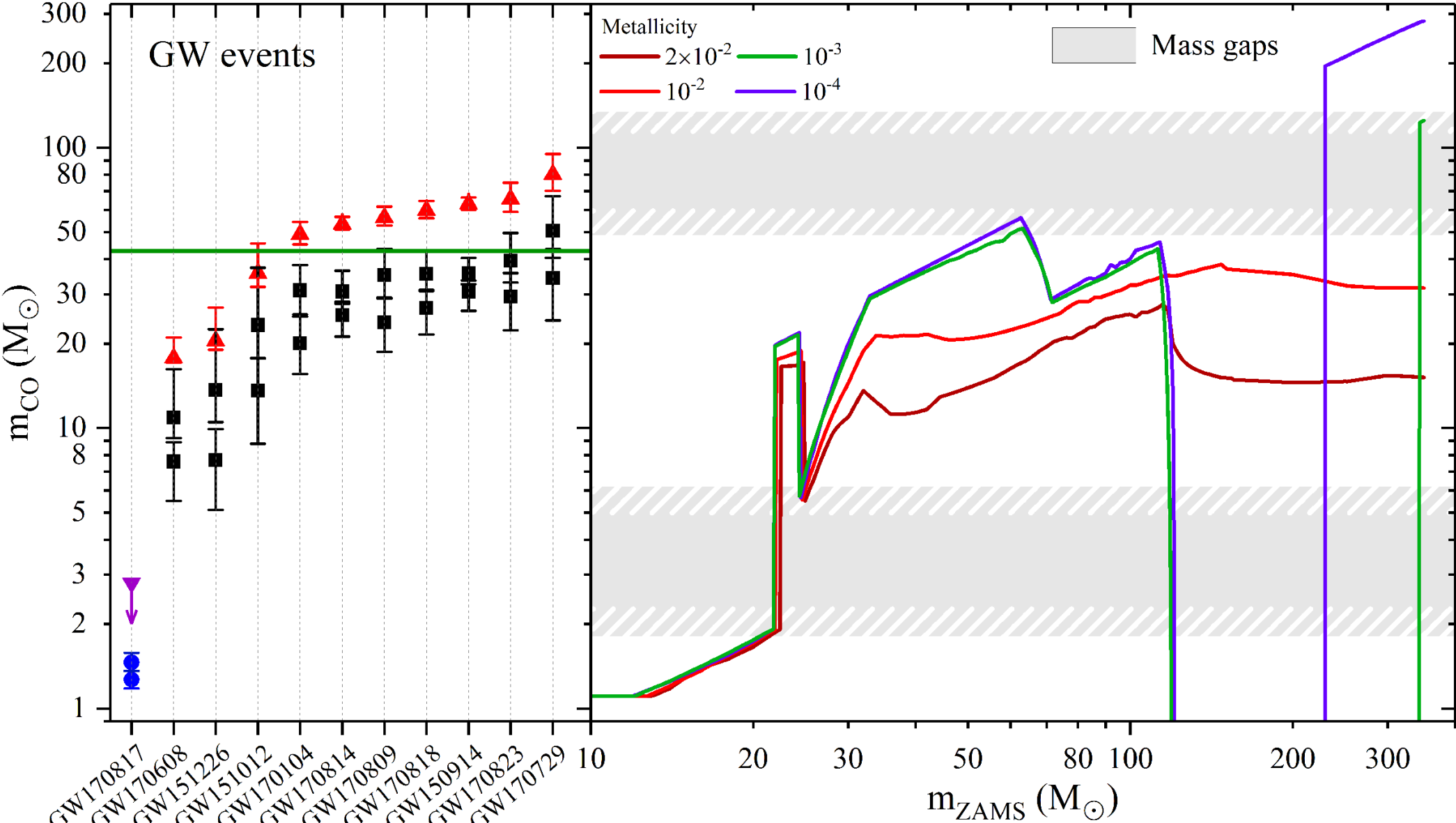
# BBH Event Rates: Observation vs. Prediction



# Population Modelling and Selection

- Perform **Bayesian model inference** on the product of a set of *mass*, *spin*, and *redshift* models using our **ten BBH event posteriors** from **O1 and O2**, specifically:
  - Probe the **GW merger rate distribution** over **mass and mass ratio**
  - Probe the **shape** of the **spin distribution** of BH in GW binaries
  - Examine models of **rate evolution with redshift**

# Some Astrophysical Context



# Mass Distributions

After four events (GW170104):  
power law (primary mass)

$\alpha$  : power law slope

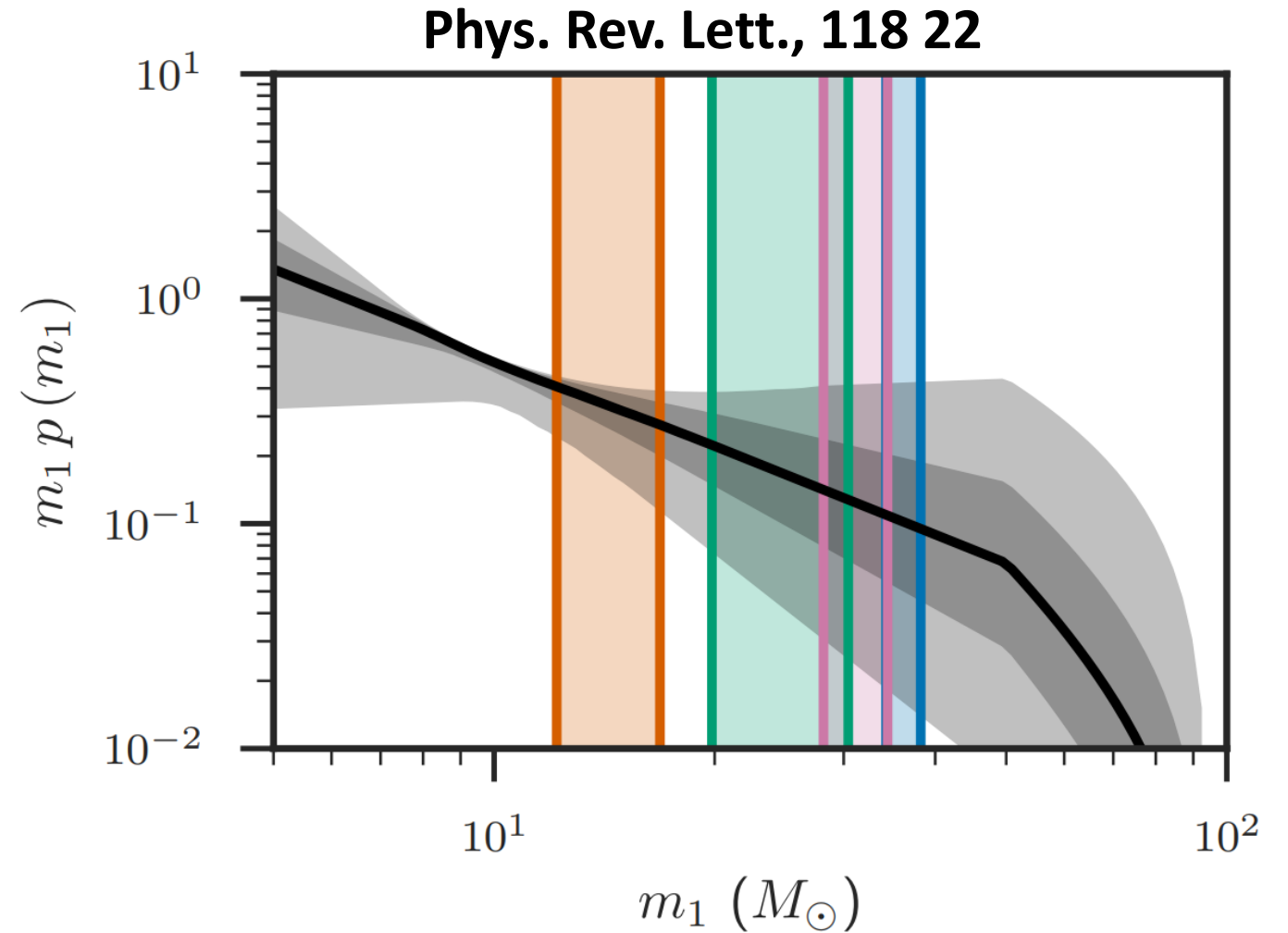
Fixed mass bounds at 5 and 100  $M_{\odot}$

O2 – Model A/B/C: power law  
(primary mass)

$\alpha$  : power law slope

$m_{\min}$  : minimum power law cutoff

$m_{\max}$  : maximum power law cutoff



$$p(m_1, m_2 | m_{\min}, m_{\max}, \alpha, \beta_q) = \begin{cases} C(m_1) m_1^{-\alpha} q^{\beta_q} & \text{if } m_{\min} \leq m_2 \leq m_1 \leq m_{\max} \\ 0 & \text{otherwise.} \end{cases}$$



# Mass Distributions

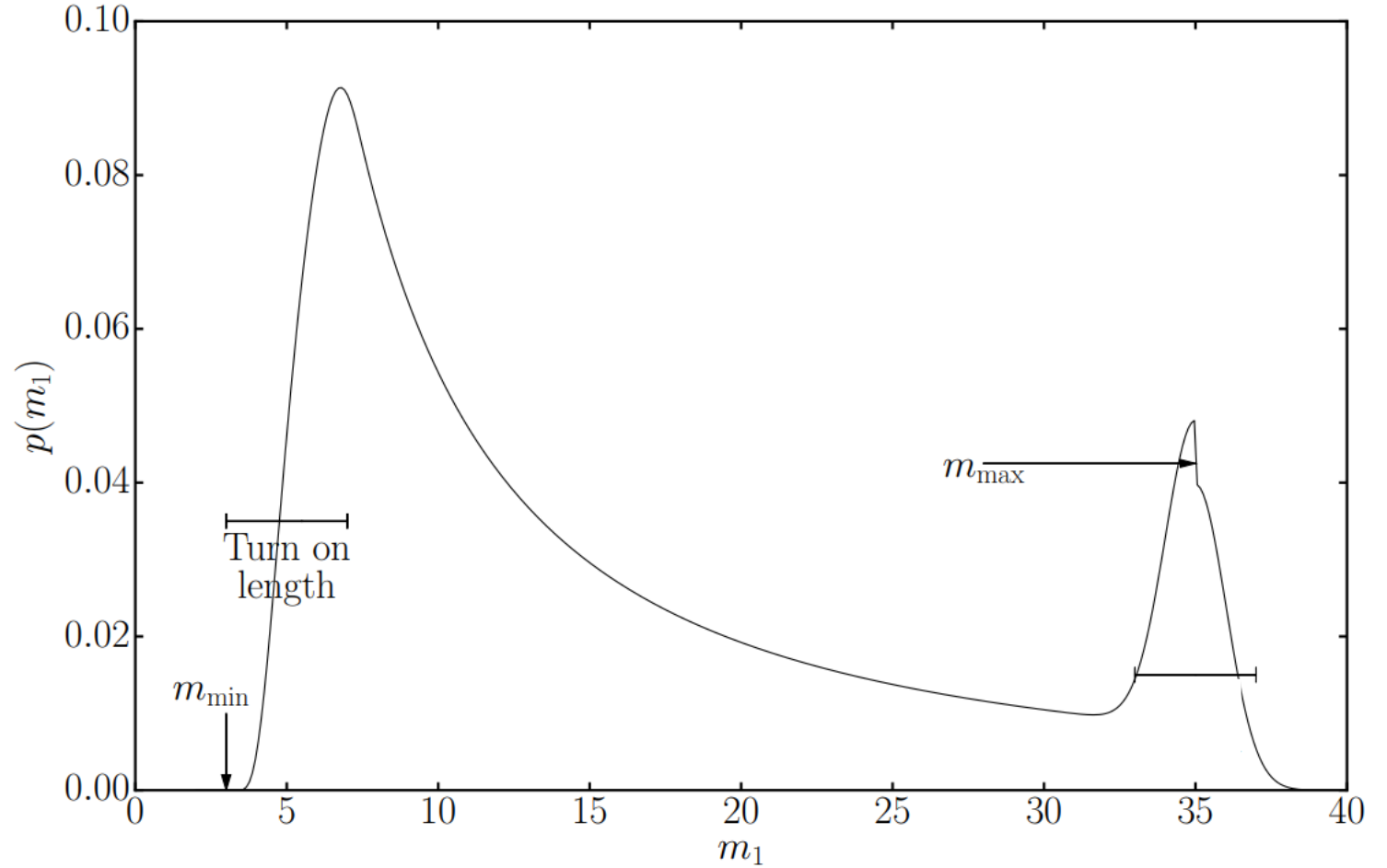
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## O2 – C: PL + Gaussian component (primary mass)

$\lambda_m$ : mixture parameter

$\mu_m$   $\sigma_m$ : Gaussian parameters

$\delta m$ : low mass tapered turn on

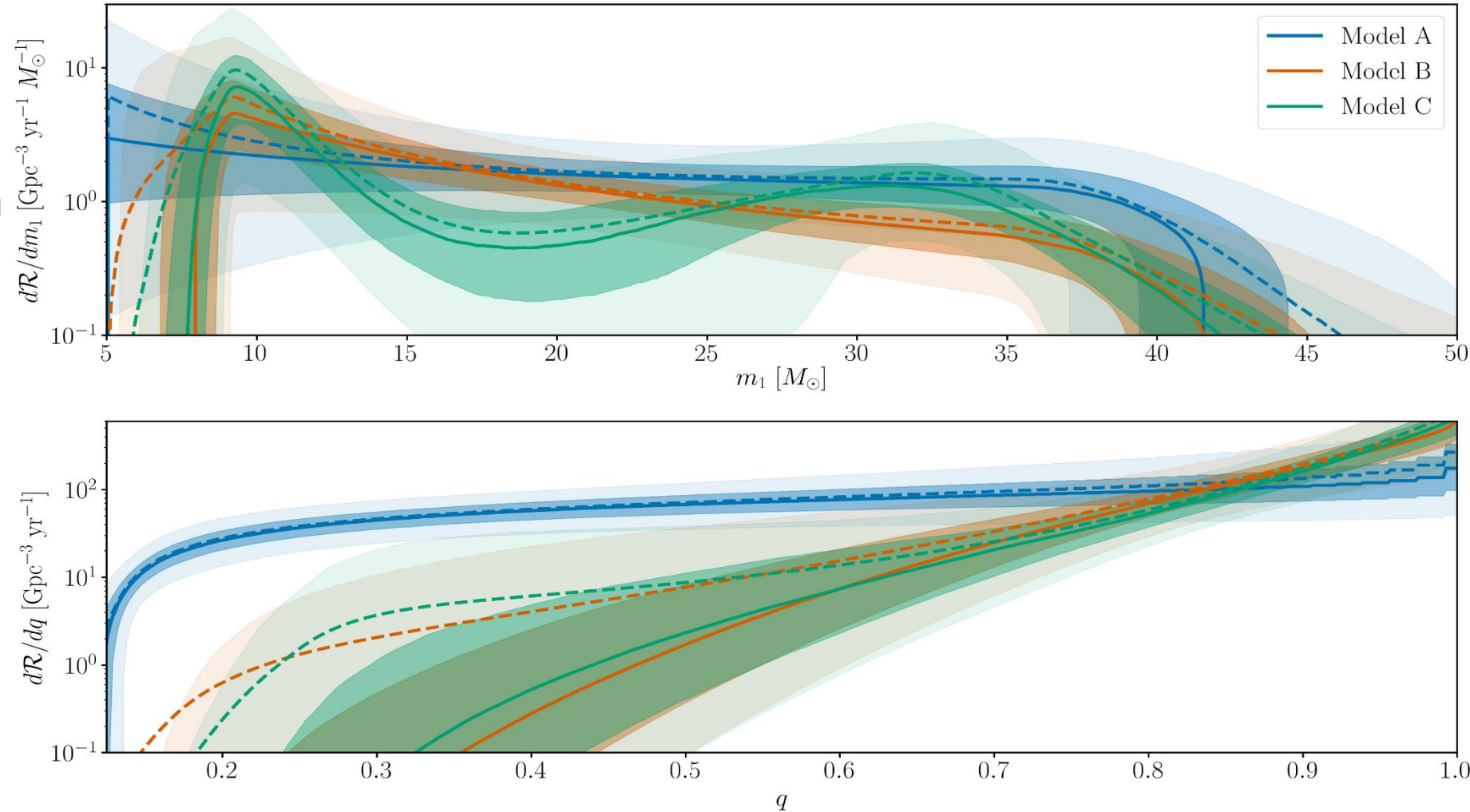


$$p(m_1|\theta) = \left[ (1 - \lambda_m) A(\theta) m_1^{-\alpha} \Theta(m_{\max} - m_1) + \lambda_m B(\theta) \exp\left(-\frac{(m_1 - \mu_m)^2}{2\sigma_m^2}\right) \right] S(m_1, m_{\min}, \delta m),$$

$$p(q|m_1, \theta) = C(m_1, \theta) q^{\beta_q} S(m_2, m_{\min}, \delta m).$$

# Mass-Dependent Event Rate Distributions

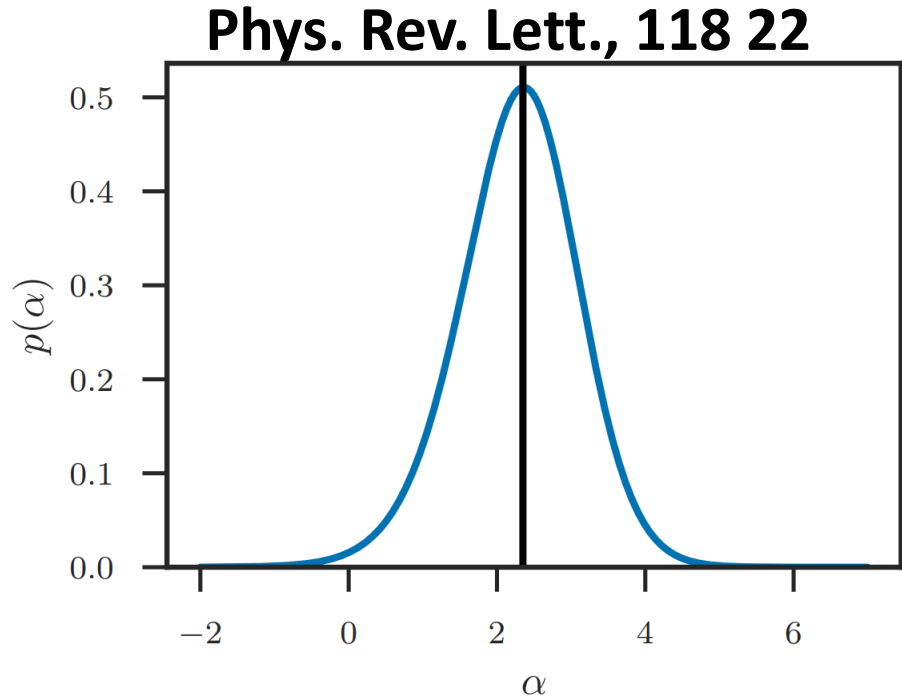
- Merger rate with mass dependence
  - Model A/B: Light more frequent than heavier
  - Model C:  $\ln\text{BF} \sim 2$  for build up (Gaussian) at high masses
- Mass ratios
  - near flat or declining
  - most asymmetric mergers disfavored



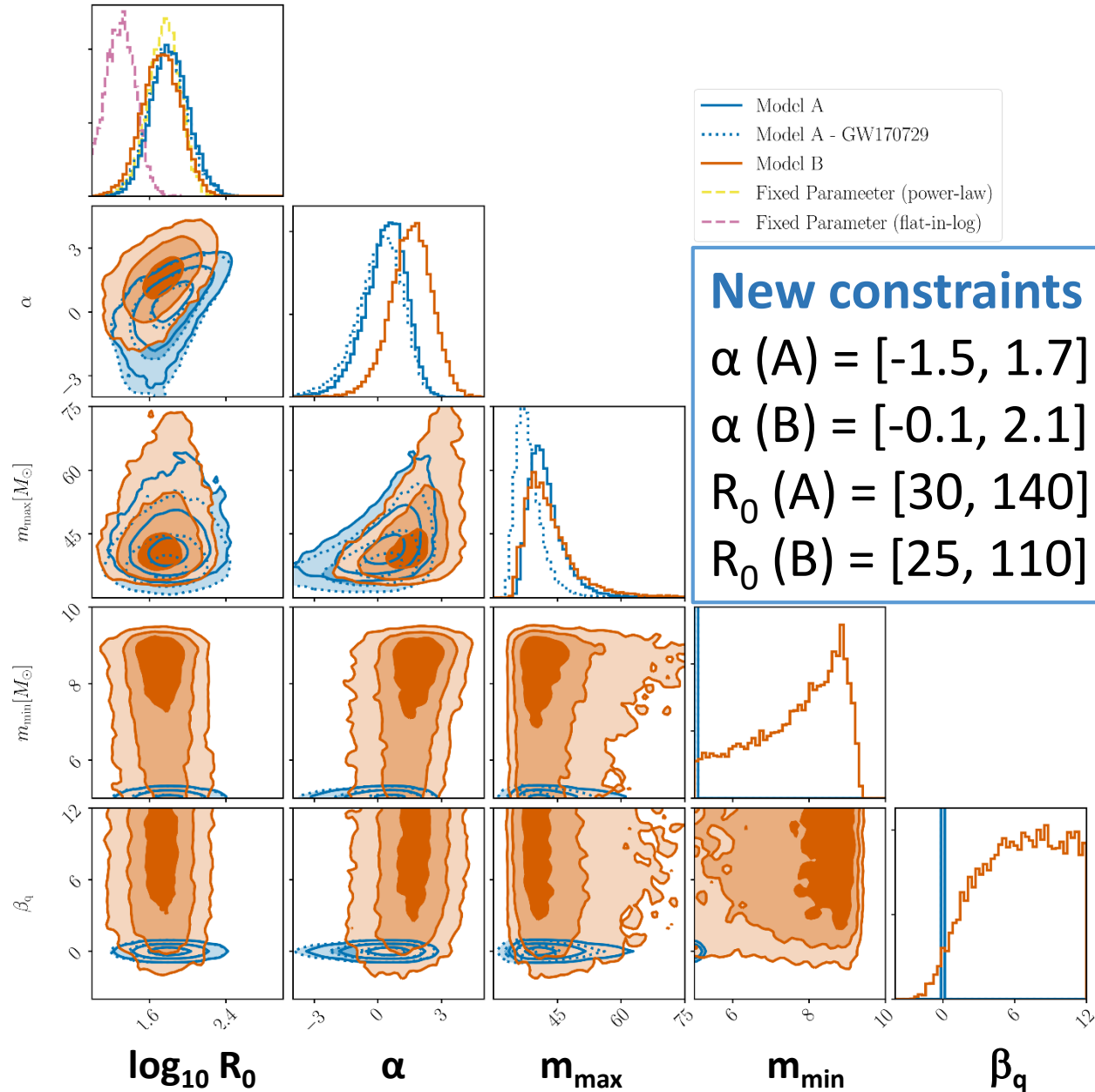
**99% mass limits on top panel:**

**Model A:  $43.8 M_\odot$    Model B:  $42.8 M_\odot$    Model C:  $41.8 M_\odot$**

# New Model Parameter Constraints



**Above:** Power law index with  
four events:  $m_{\min} = 5$ ,  $m_{\max} = 100$



**New constraints (10 events):**

$$\alpha (A) = [-1.5, 1.7]$$

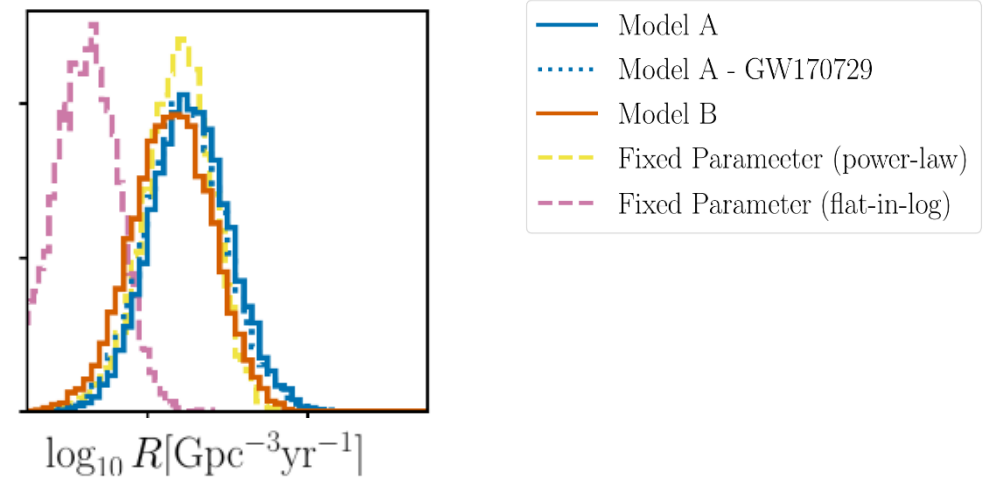
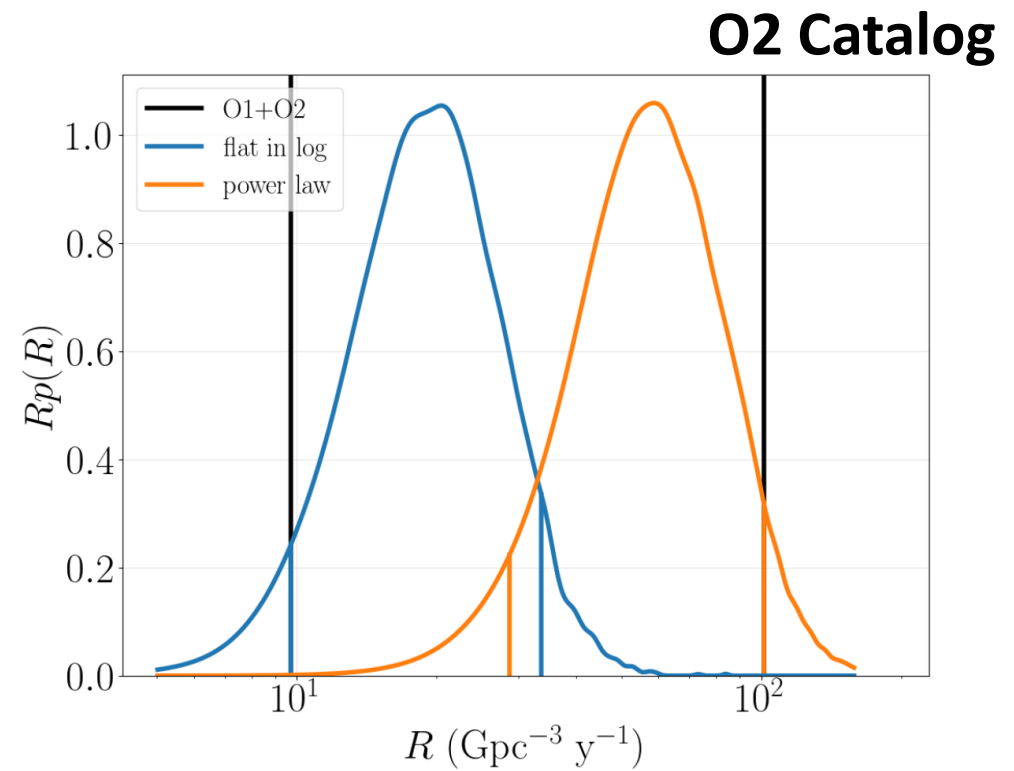
$$\alpha (B) = [-0.1, 2.1]$$

$$R_0 (A) = [30, 140] \text{ Gpc}^{-3}\text{yr}^{-1}$$

$$R_0 (B) = [25, 110] \text{ Gpc}^{-3}\text{yr}^{-1}$$

# Rate Distributions

- Current observational rates assume **uniform in comoving volume**
- Flexible models compare well with fixed param. choices in catalog

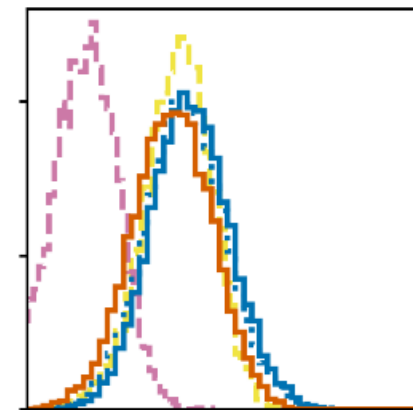
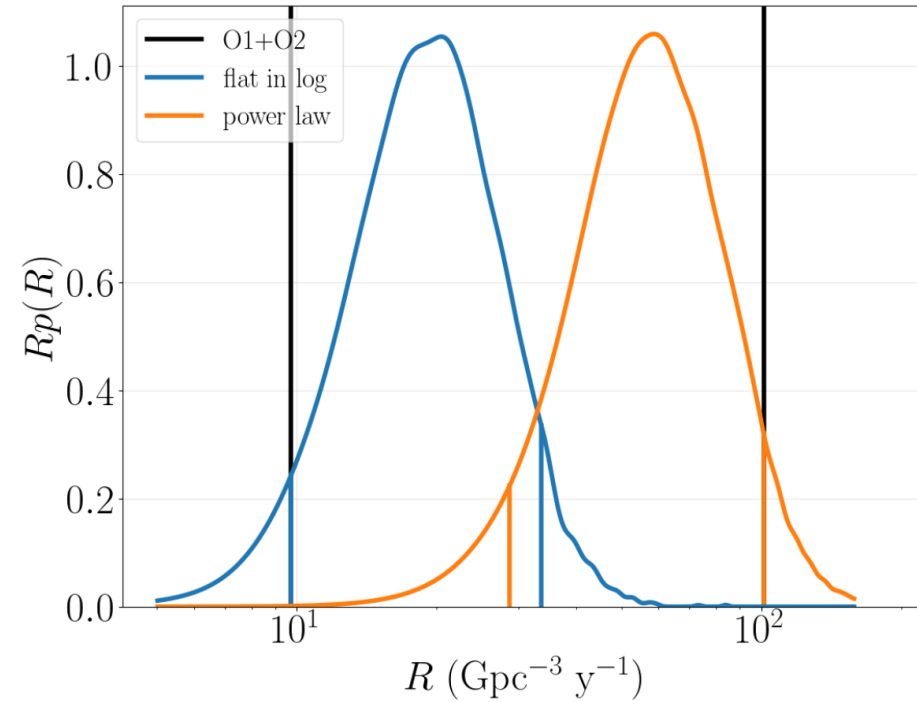


## O2 Populations

# Rate Distributions

- Current observational rates assume **uniform in comoving volume**
- Flexible models compare well with fixed param. choices in catalog
- **Rate evolution** with redshift is possible
  - Model **rate evolution** with redshift with **power law**
  - Also allow mass Model A to vary given **strong covariance between mass and redshift distribution**

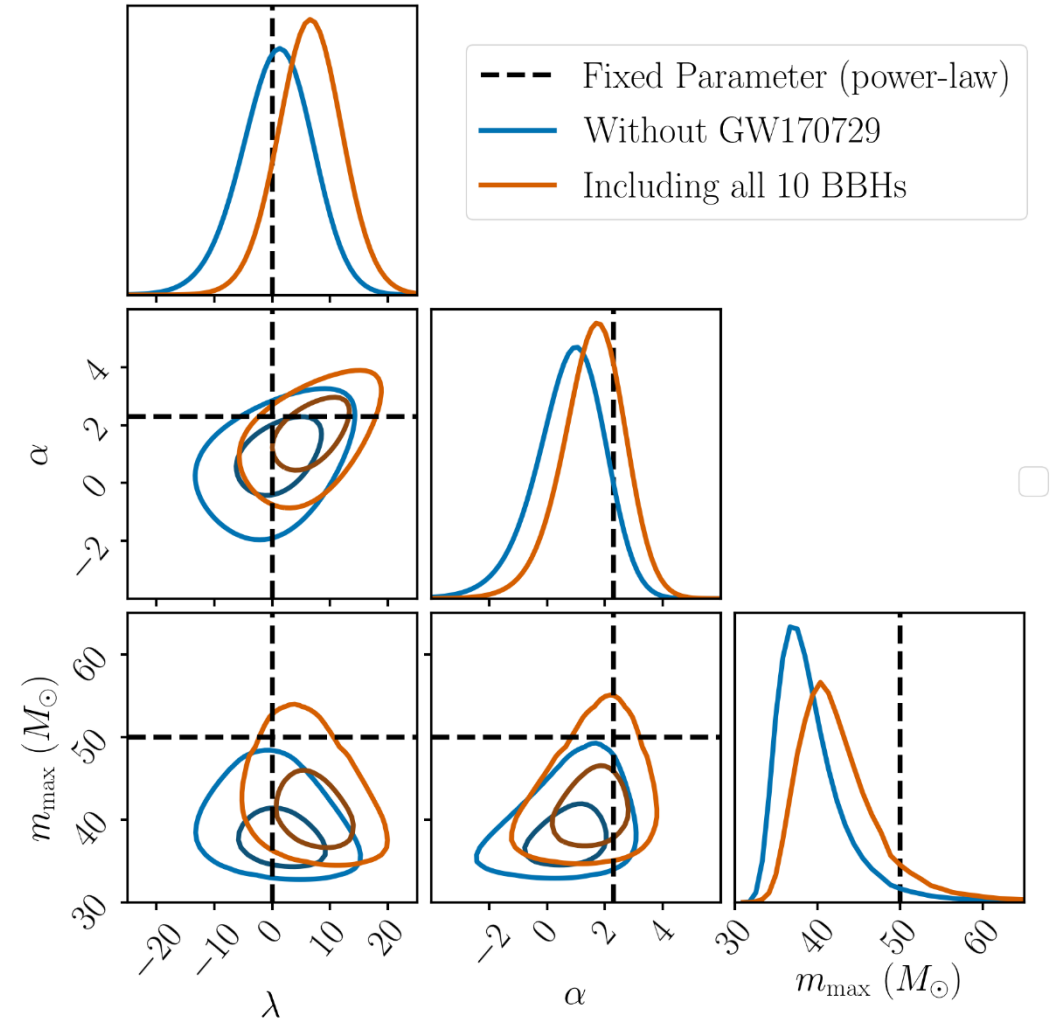
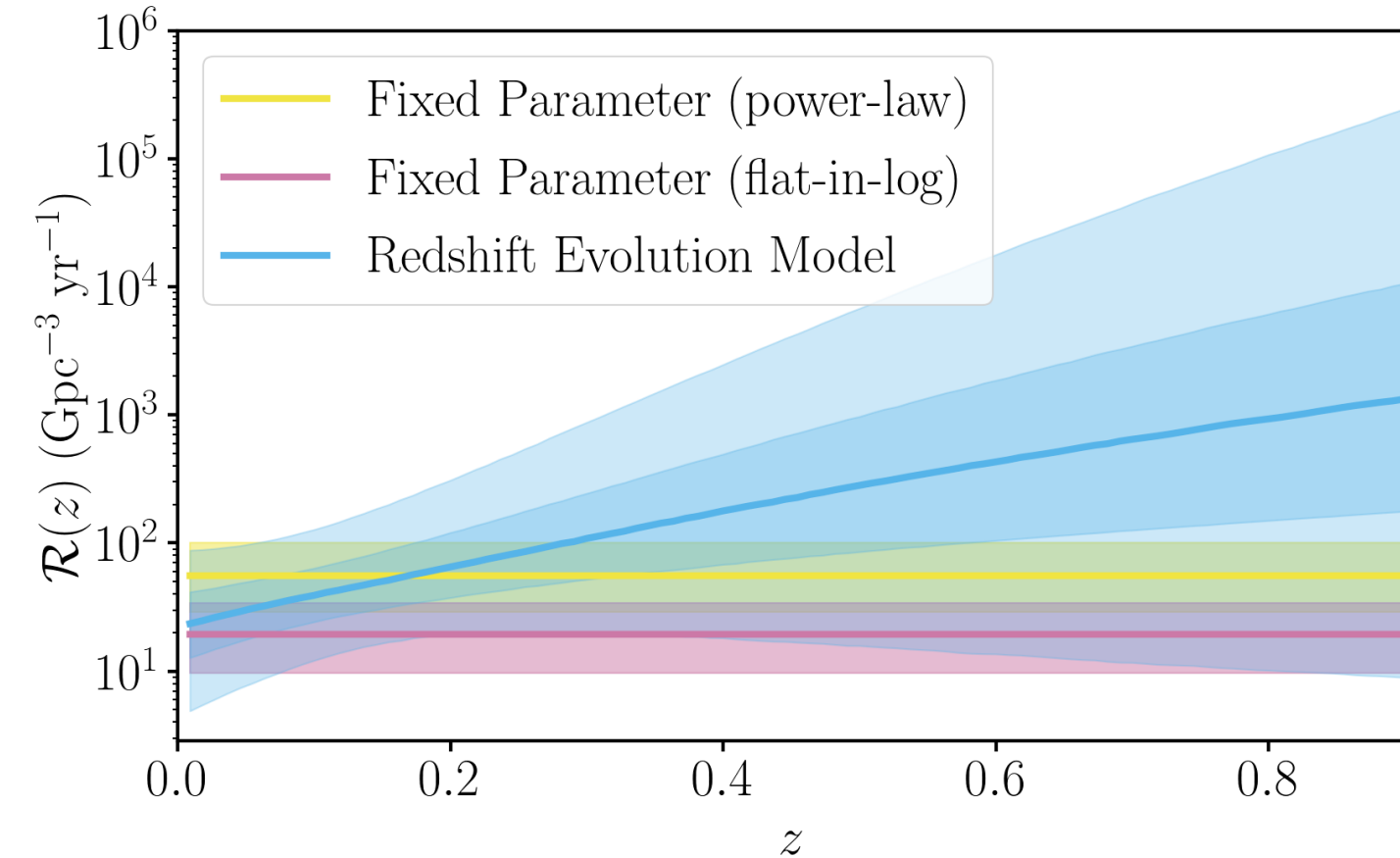
## O2 Catalog



## O2 Populations

# Rate Evolution

Probability rate increases with  $z$ : 0.88



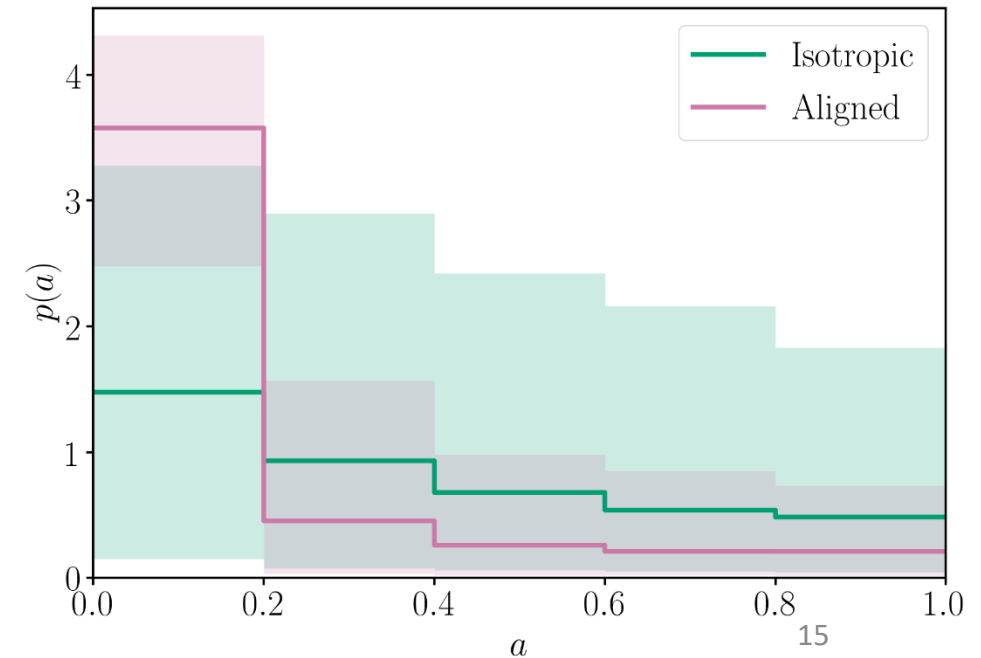
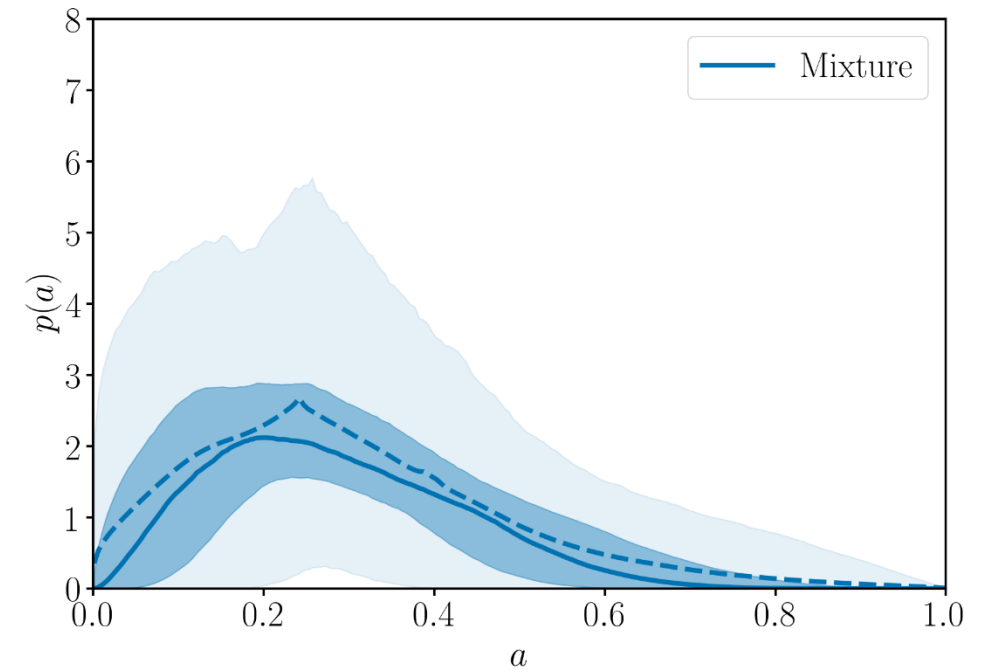
$$\frac{d\mathcal{R}}{dm_1 dm_2}(z) = \mathcal{R}_0 p(m_1, m_2 | \alpha, m_{\max})(1+z)^\lambda$$

# Spin Magnitude / Tilt

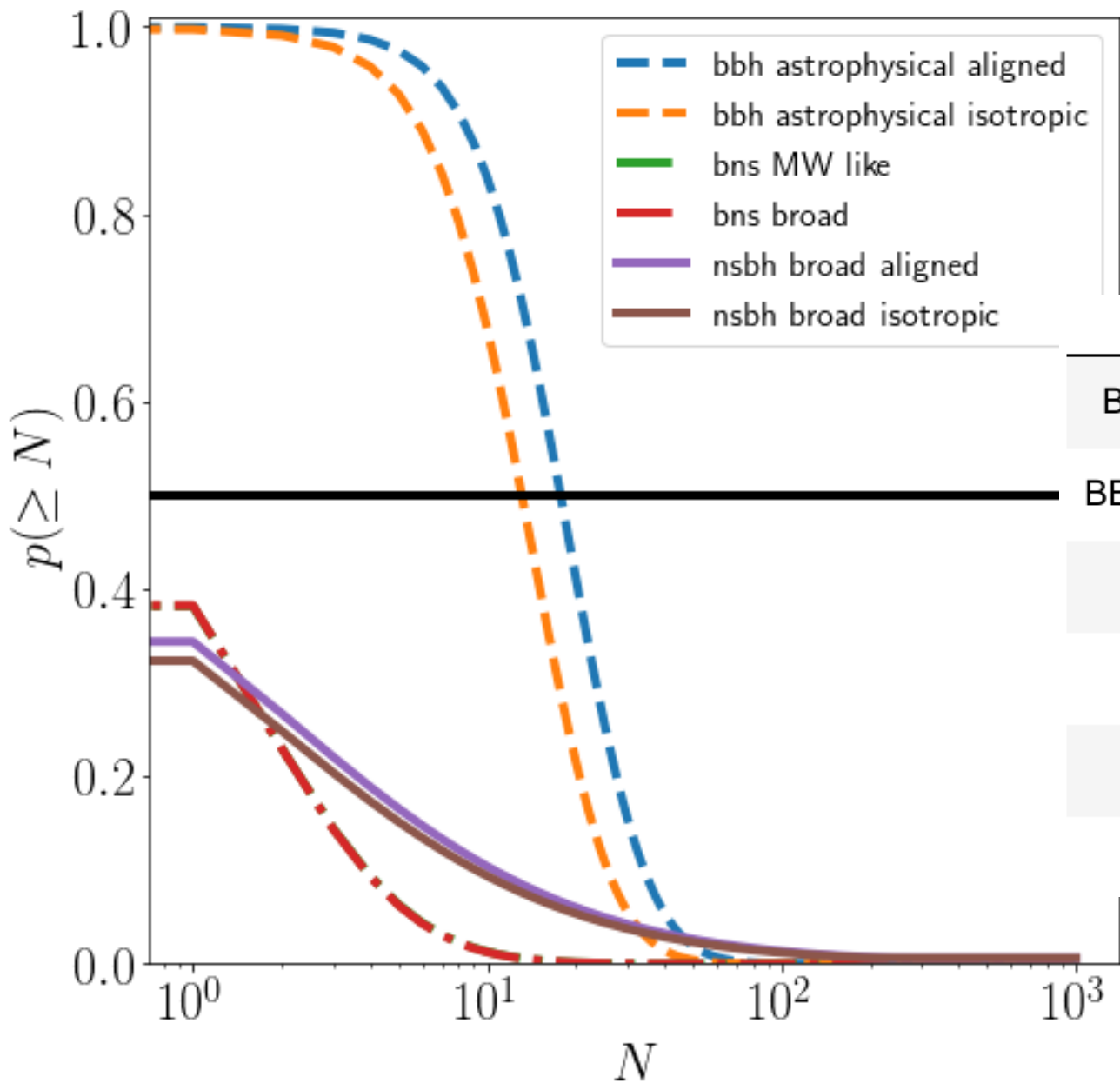
- Parametric (top) distribution, marginalizing over all mass and spin tilt / mixture parameters
  - Some preference for spins which decline away from zero

$$p(a_i | \alpha_a, \beta_a) = \frac{a_i^{\alpha_a - 1} (1 - a_i)^{\beta_a - 1}}{B(\alpha_a, \beta_a)}$$

- Non-parametric (bottom), 5 bin analysis, fix tilts to isotropic or *exactly* aligned
  - Aligned distribution favors lower spins
  - Isotropic spins mostly flat



# O3 Predictions



source category	VT (1 yr, 0.5 DC)	$N_d$
BBH / bbh_astrophysical_aligned	$3.5 \times 10^{-1} \text{ Mpc}^3 \text{ yr}$	$18_{-11}^{+23}$
BBH / bbh_astrophysical_isotropic	$2.6 \times 10^{-1} \text{ Mpc}^3 \text{ yr}$	$14_{-9}^{+17}$
BNS / bns_mw_like	$1.7 \times 10^{-3} \text{ Mpc}^3 \text{ yr}$	$1_{-1}^{+5}$
BNS / bns_broad	$2.3 \times 10^{-3} \text{ Mpc}^3 \text{ yr}$	$1_{-1}^{+5}$
NSBH / nsbh_broad_aligned	$2.4 \times 10^{-2} \text{ Mpc}^3 \text{ yr}$	$0_{-0}^{+25}$
NSBH / nsbh_broad_isotropic	$2.1 \times 10^{-2} \text{ Mpc}^3 \text{ yr}$	$0_{-0}^{+22}$



# Conclusions

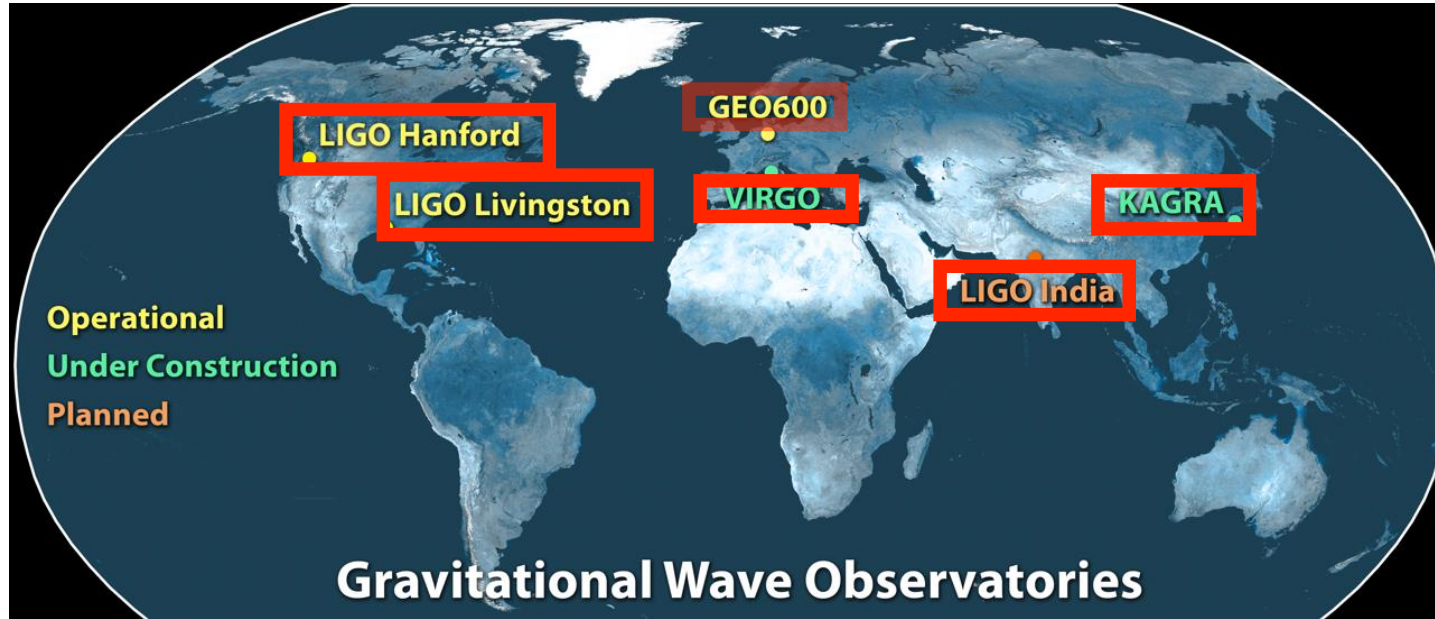
- **4 new BBH:** No new significant NSBH or BNS
- **More detections:** 10s more in O3 will tighten rate and model constraints as time goes on
- Power law slopes: **shallower** than O1, low mass binaries **more frequent** than high mass
  - **Lower mass gap:** not enough sensitivity below  $5 M_{\odot}$
  - **Heavy BH constraints:** most BH  $< 45 M_{\odot}$
  - **Mass ratio:** power index on  $q$  bounded above 0 at 90%: B [0.8, 11.5] / C [0.1, 11.4]
  - **Mild evidence** for second population component ( $\lambda_m$ : [0.1, 0.7],  $\ln\text{BF} \sim 2$ )

# Conclusions (cont.)

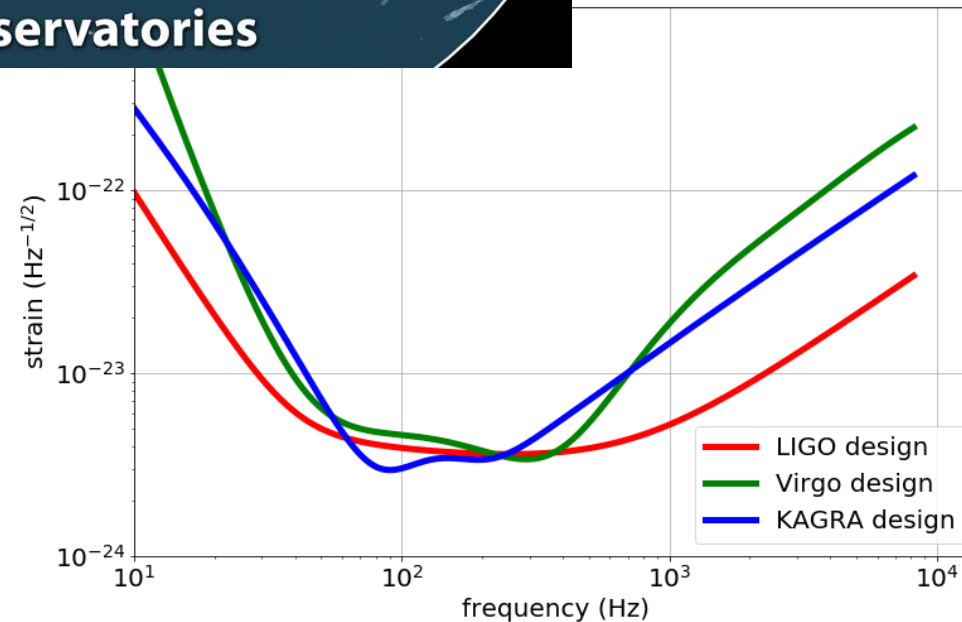
- Spin distribution **disfavors extremely high spins** under **aligned** scenario, **isotropic spins less constrained**
  - **Spin magnitude**: most extreme spins excluded
  - **Spin tilt / mixtures**: no conclusive statements
  - **Specific magnitude models** explored and **consistent** with above
- **Rate evolution with redshift**: upward sloping and uniform in comoving are favored
  - **Local rate distribution is consistent** even when varying over mass model parameters
- Models allowing for **self-consistent integration** with detection significance also being developed
- Public paper and data release: <https://dcc.ligo.org/LIGO-P1800324/public>

# Bonus Slides

# The Next ~5 Years



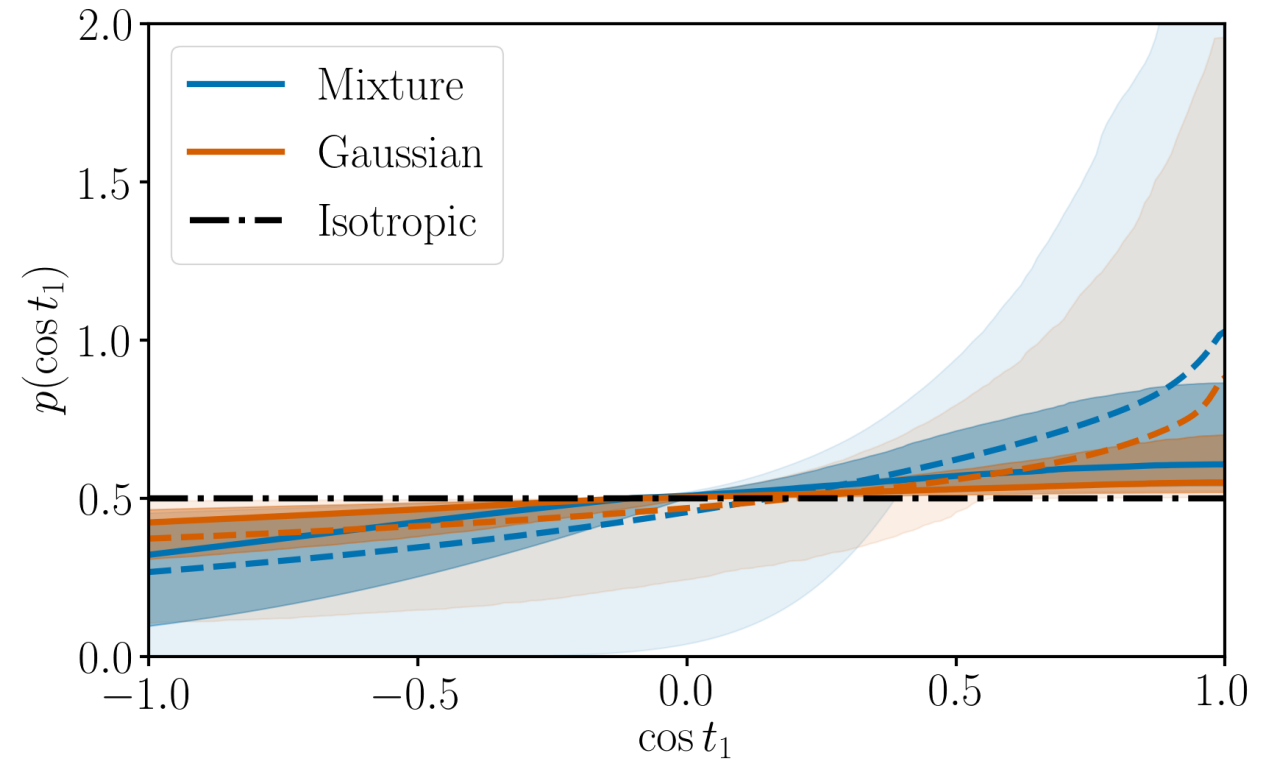
- Kagra likely to join O3
- 202X LIGO to meet design sensitivity (blue curve)
- 4 and 5 detector networks by mid 2020s
- LIGO India tentatively for 2024



# Spin Magnitude / Tilt

- **Gaussian** model ( $\zeta = 1$ ) and **Mixture** model ( $0 < \zeta < 1$ )
- **Mixture** fraction ( $\zeta$ ) uninformative
  - both model constraints return similar distributions
- **Gaussian** model: large *allowed* misalignment
  - wide tilt distributions which resemble isotropy

$$p(\cos t_1, \cos t_2 | \sigma_1, \sigma_2, \zeta) = \frac{(1 - \zeta)}{4} + \frac{2\zeta}{\pi} \prod_{i \in \{1,2\}} \frac{\exp(-(1 - \cos t_i)^2 / (2\sigma_i^2))}{\sigma_i \operatorname{erf}(\sqrt{2}/\sigma_i)}.$$



**Isotropic**  
**(uniform in  $\cos t_1$ )**

**Gaussian**

# New in O2

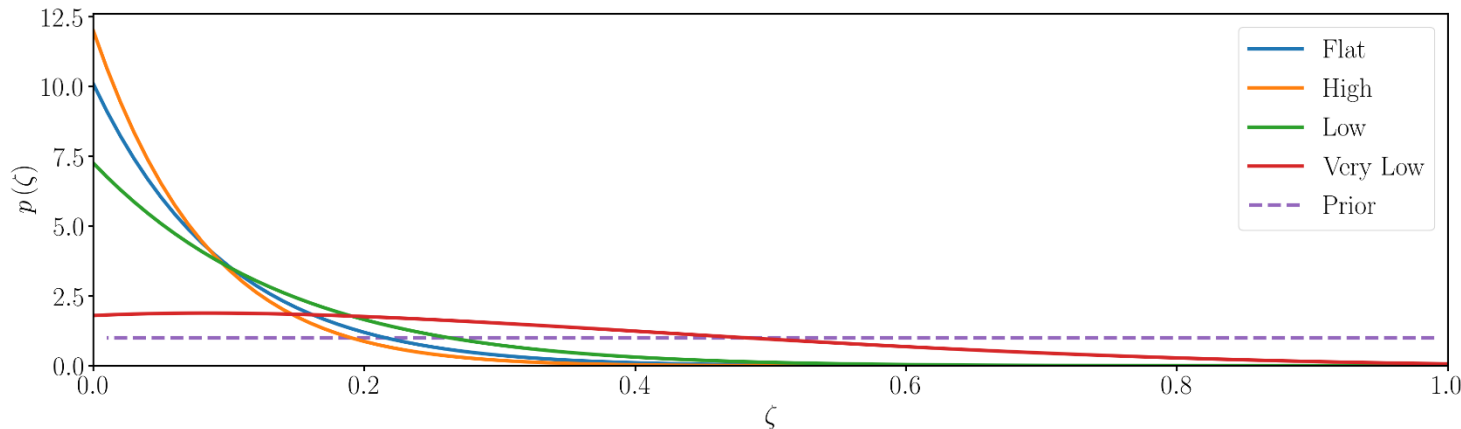
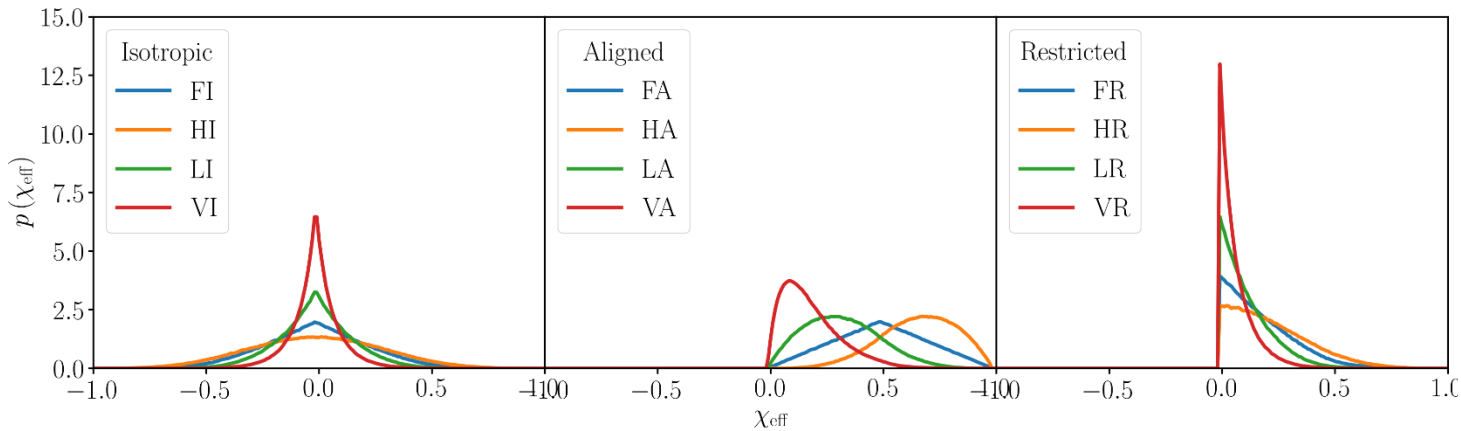
- Found four new binary black hole merger events: **GW170729**, **GW170809**, **GW170818**, **GW170823**
- **151012** designated as a GW event
  - higher significance because of improved detection pipelines and better determined rate estimates (...and personal intervention by Chase Kimball)
- Not all events found with all searches
  - All “GW” monikered events have  $p_{\text{noise}} < 0.05$

Courtesy: Michael Purrer

Event	UTC Time	FAR [ $y^{-1}$ ]			Network SNR		
		PyCBC	GstLAL	cWB	PyCBC	GstLAL	cWB
GW150914	09:50:45.4	$< 1.53 \times 10^{-5}$	$< 1.00 \times 10^{-7}$	$< 1.63 \times 10^{-4}$	23.6	24.4	25.2
GW151012	09:54:43.4	0.17	$7.92 \times 10^{-3}$	–	9.5	10.0	–
GW151226	03:38:53.6	$< 1.69 \times 10^{-5}$	$< 1.00 \times 10^{-7}$	0.02	13.1	13.1	11.9
GW170104	10:11:58.6	$< 1.37 \times 10^{-5}$	$< 1.00 \times 10^{-7}$	$2.91 \times 10^{-4}$	13.0	13.0	13.0
GW170608	02:01:16.5	$< 3.09 \times 10^{-4}$	$< 1.00 \times 10^{-7}$	$1.44 \times 10^{-4}$	15.4	14.9	14.1
GW170729	18:56:29.3	1.36	0.18	0.02	9.8	10.8	10.2
GW170809	08:28:21.8	$1.45 \times 10^{-4}$	$< 1.00 \times 10^{-7}$	–	12.2	12.4	–
GW170814	10:30:43.5	$< 1.25 \times 10^{-5}$	$< 1.00 \times 10^{-7}$	$< 2.08 \times 10^{-4}$	16.3	15.9	17.2
GW170817	12:41:04.4	$< 1.25 \times 10^{-5}$	$< 1.00 \times 10^{-7}$	–	30.9	33.0	–
GW170818	02:25:09.1	–	$4.20 \times 10^{-5}$	–	–	11.3	–
GW170823	13:13:58.5	$< 3.29 \times 10^{-5}$	$< 1.00 \times 10^{-7}$	$2.14 \times 10^{-3}$	11.1	11.5	10.8

# Fixed Mag. Distr. Model Selection

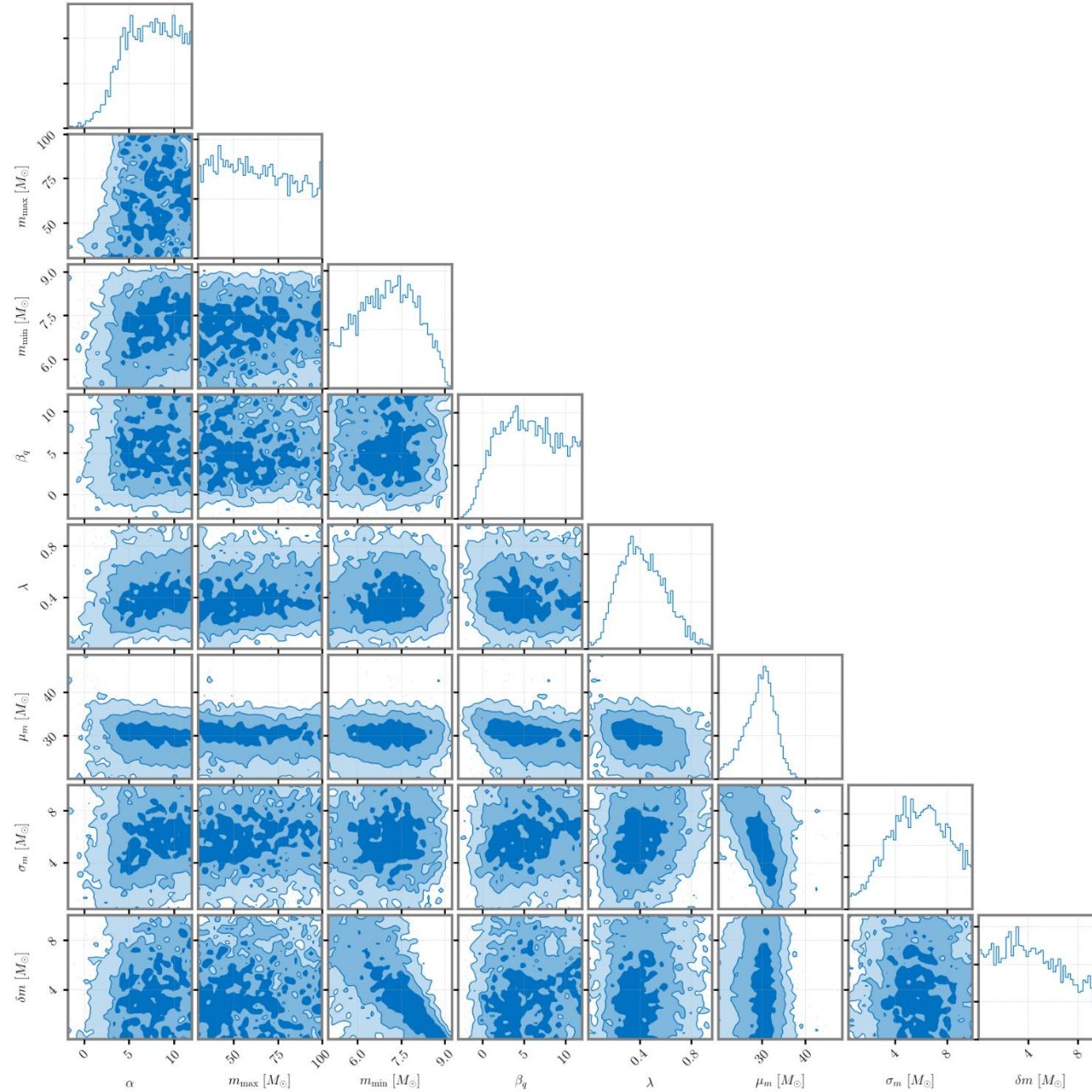
**Bottom** : Magnitude models, mixture posterior  
**Right**: log Bayes Factors for variety of fixed magnitude / tilt distributions



$q = 1$	Very low	Low	Flat	High
Isotropic	1.10	0.0	-0.93	-2.07
Restricted	3.39	3.26	1.31	0.11
Aligned	1.58	-4.12	-12.92	-32.37
$q = 0.5$	Very low	Low	Flat	High
Isotropic	1.14	0.0	-1.03	-2.41
Restricted	3.45	3.26	1.23	-0.25
Aligned	1.69	-3.71	-12.22	-30.73
fixed param.	Very low	Low	Flat	High
Isotropic	1.40	0.0	-2.63	-4.61
Restricted	1.76	0.11	-2.78	-4.88
Aligned	-3.78	-14.45	-24.28	-48.00

$$\chi_{\text{eff}} = \frac{(\chi_1 + q\chi_2) \cdot \hat{L}}{1 + q}$$

# Model C (PL + Gaussian)





# Population Modelling Basics

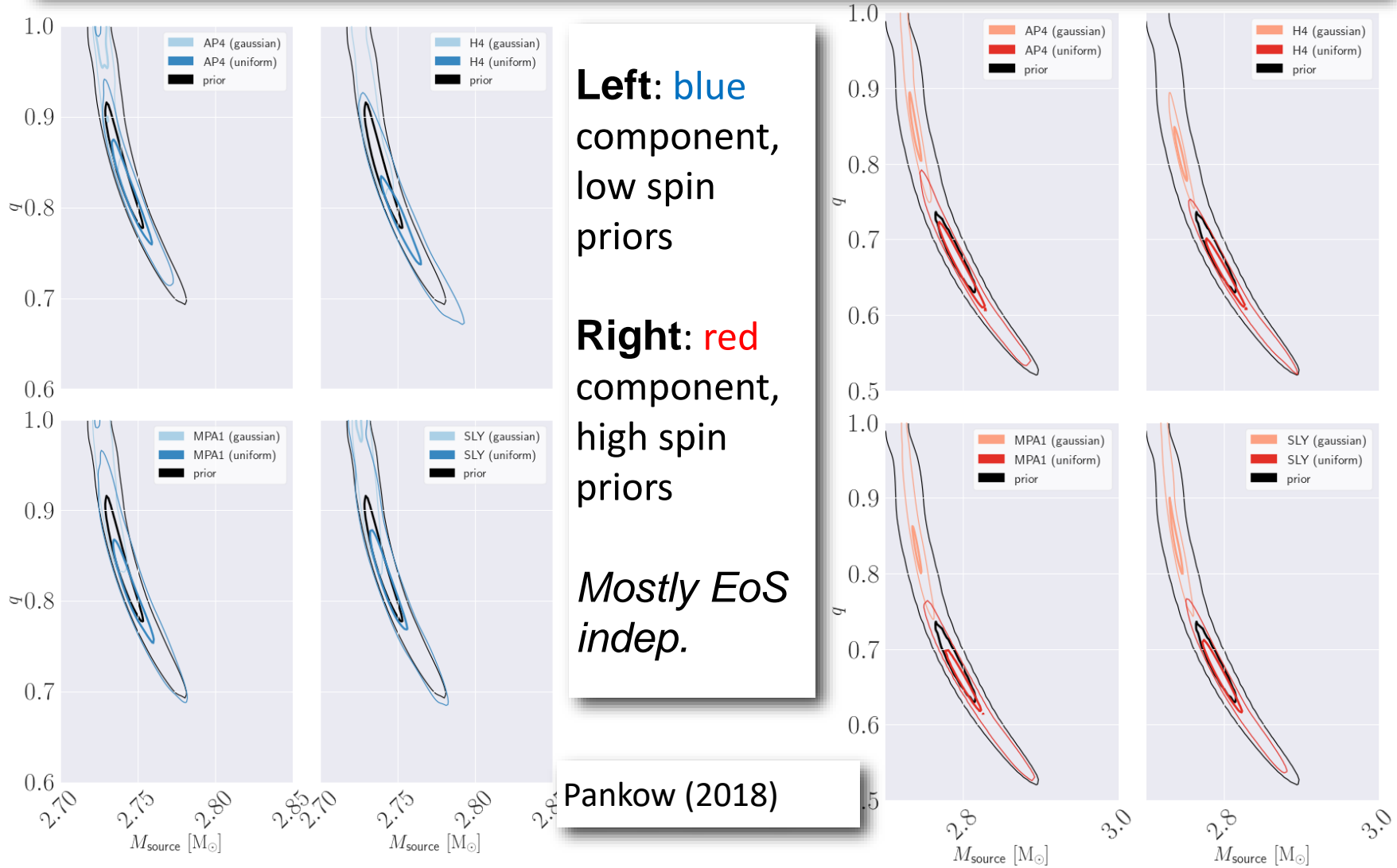
- **The goal:** determine the particulars of binary evolution physics through a parameterized ( $\lambda$ ) model
- **How?:** Determine the imprint of those processes on the **GW binary physical parameter distribution ( $\theta$ )**

$$p(\{x_i\}|\vec{\lambda}) = \prod_{i=1}^N p(x_i|\vec{\lambda}) = \prod_{i=1}^N \int d\vec{\theta} p(x_i|\vec{\theta}) p(\vec{\theta}|\vec{\lambda})$$

- Fold **GW posterior measurements ( $\{x_i\}$ )** together with modelling to determine **most favored model parameters**, from the observations and thus the physical prescriptions

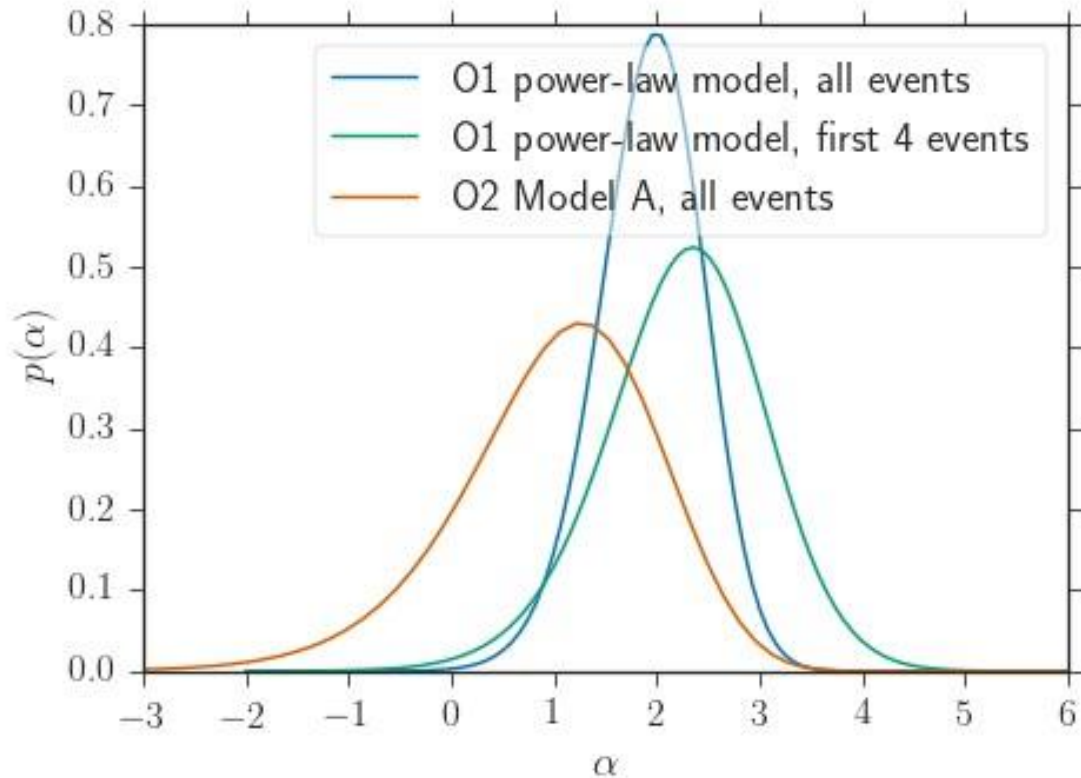
$$p(\{x_i\}|\vec{\lambda}) = \prod_{i=1}^N \frac{1}{S} \sum_{k=1}^S \frac{p(\vec{\theta}_k|\vec{\lambda})}{p(\vec{\theta}_k)}$$

Joint total mass / mass ratio posteriors with constrains from kN ejecta mass applied --- mass asymmetry has moderate tension with Galactic DNS distribution



# What if?

- ...we had carried over the O1 model to new events?



Courtesy: Maya Fishbach

Green: Power law index with **four** events

$$m_{\min} = 5, m_{\max} = 100$$

Blue: Power law index with **ten** events

$$m_{\min} = 5, m_{\max} = 100$$

Orange: Power law index with ten events

Model A

# Mass Distributions

**Model A/B/C: power law (primary mass)**

$\alpha$  : power law slope

$m_{\min}$  : minimum power law cutoff

$m_{\max}$  : maximum power law cutoff

**Model B/C: power law (mass ratio)**

$\beta_q$  : Power law slope

$$p(m_1, m_2 | m_{\min}, m_{\max}, \alpha, \beta_q) = \begin{cases} C(m_1) m_1^{-\alpha} q^{\beta_q} & \text{if } m_{\min} \leq m_2 \leq m_1 \leq m_{\max} \\ 0 & \text{otherwise.} \end{cases}$$

**Power Law**

$\alpha, m_{\min}, m_{\max}$

**Gaussian**

$\lambda_m, \mu_m, \sigma_m$

**Low Mass  
Tapering**

$$p(m_1 | \theta) = \left[ (1 - \lambda_m) A(\theta) m_1^{-\alpha} \Theta(m_{\max} - m_1) + \lambda_m B(\theta) \exp\left(-\frac{(m_1 - \mu_m)^2}{2\sigma_m^2}\right) \right] S(m_1, m_{\min}, \delta m),$$

$$p(q | m_1, \theta) = C(m_1, \theta) q^{\beta_q} S(m_2, m_{\min}, \delta m).$$

# Spin Distributions

## O2 -- Model A/B/C: Beta distribution (spin magnitudes)

$\alpha_a \beta_a$ : Beta parameters

$\alpha > \beta$  -> inclining

$\alpha < \beta$  -> declining

$$p(a_i | \alpha_a, \beta_a) = \frac{a_i^{\alpha_a - 1} (1 - a_i)^{\beta_a - 1}}{B(\alpha_a, \beta_a)}$$

## O2 -- Model A/B/C: mixture distribution (spin tilts)

$\zeta$ : Mixture parameter

$\sigma_i$ : degree of spin alignment allowed

$$p(t_1, t_2 | \sigma_1, \sigma_2, \zeta) = \frac{(1 - \zeta)}{4} + \frac{\zeta}{2\pi} \prod_{i \in \{1, 2\}} \frac{e^{-(1 - \cos t_i)^2 / (2\sigma_i^2)}}{\sigma_i \operatorname{erf}(\sqrt{2}/\sigma_i)}.$$

Isotropic  
(uniform in  $\cos t_1$ )

Gaussian