How sweet it is – How many sugar cubes do you need to fill the big dome?

What do you think? Don’t worry about being wrong, just think critically and give it your best!

- How many sugar cubes do you think it would take to fill the large dome with sugar cubes?
- How would you know if you are right or wrong?
- How do astronomers develop models of how large or how far away something is if they can’t make direct measurements?

Procedure:

Write the answers to question and/or problems in your laboratory notebooks

Before we being lets work on some background.

If two objects have the same shape, they are said to be geometrically similar. By definition, the ratio of any two linear dimensions of one object will be same for any geometrically similar object. This is easiest to illustrate with simple geometric shapes:

For each pair of geometrically similar shapes above, the proportions are identical. That is, the ratio of width to height of the rectangles is 2.0, and the ratio of the height to the hypotenuse of the triangles is 0.6. Notice that all of the dimensions of one can be calculated by multiplying the dimensions of the other by a constant, in these examples 2 (or 1/2). If you make a geometrically similar
rectangle three times as high as the small one above, then its width will be three
times as big as well, along with its diagonal measurement and perimeter, i.e.
ANY linear dimension will be multiplied by the same factor.

Question

A. What are the relationships of geometrically similar shapes?

Now lets talk about some other characteristics, such as surface area and
volume?

Question

B. What is the surface area of a cube that has sides which are 1cm x 1-
cm?

C. What is the volume of a cube that is 1-cm x 1-cm x 1-cm?

D. Now that you know the volume of a single cube can you grow that
cube to double its volume? How many cubes do you need to double
the volume?

E. Now double the volume of the last cube. How many cubes do you
need, and what is the mathematical relationship that expresses this?
F. Fill in the table below with your calculations for each of the cubes:

<table>
<thead>
<tr>
<th>Length (cm)</th>
<th>Surface area (cm²)</th>
<th>Volume (cm³)</th>
<th>Surface area/volume</th>
<th>Surface area/length</th>
<th>Volume/length</th>
</tr>
</thead>
</table>

You can determine the areas and volumes intuitively by counting the exposed faces and imagining the hidden faces but how about mathematically? Devise a mathematical way to determine the surface area and volume of the cubes with some simple mathematical formula? What are the formulas?

G. What happens to the surface area to volume ratio as a cube becomes bigger?
You can probably guess that mathematics can be used to find relationships between lengths, areas, and volumes of any geometrically similar objects such as cubes. However, how about other geometric shapes like cylinders and spheres?

Look at the illustration of the cylinder below.

![Cylinder Illustration]

Let's look at how a cylinder is made. It is really nothing more that a piece of paper that is rolled and taped together. If we cut out a rectangle that is 5-cm in height and 10.3-cm in length and roll it in the shape of the tube we can make a cylinder.

![Rectangle and Cylinder]

Now let's fill the cylinder up with play sand. The sand is now occupying the volume of the cylinder, and in fact can be thought of as the volume of the cylinder. If we pour this sand into an empty film canister we will see that it fills the canister completely.

Now make a cylinder that is double the volume of the original paper cylinder. If you have doubled the volume of the cylinder you should be able to fill your film canister up twice.

Question

H. What are the dimensions of the cylinder that has double the volume?

I. Explain how you decided on the correct dimensions.
Use your knowledge of how to double volume and make a new cylinder that is double the volume of the last cylinder. Check your new cylinder by using sand to check its volume.

J. What can we say about how cylinders grow in size, when we double their size?

K. Construct a table similar to the one we used for cubes to illustrate how a cylinder grows

How can we determine the volume of one of our cylinders with a mathematical formula? This is a more difficult problem --we have to know some geometry! It turns out that all geometric objects that take the shape of a circle rely on a mathematic term called pi (π). Most of you have probably heard about pi -- the number 3.14159. Pi is a very interesting number, because it is infinite, we only rounded it off to 3.14159 but it actually goes on forever! (the closet one can approximate pi is with the fraction 355/113)To determine the volume of cylinder we need to know its height and its diameter or its radius (the radius is simply ½ the diameter). It turns out that the volume of cylinder can be found by the equation

\[ V = h \times \pi \times r^2 \]

L. Now that you know how to determine the volume of a cube and the volume of a cylinder, how many sugar cubes will fit inside the circle that I have given to you?

You can see that this is a more difficult question, because the two object (cubes and cylinders) are not geometrical similar. However, they both have volume and as such we can express the magnitude of their volume with mathematics.
Now lets look at one last geometric shape, a sphere.

A sphere is a 3-dimension object, and is commonly called a ball.

Question

M. What are the dimensions of a sphere?

This may have been a very difficult question for you, since a sphere is such a deceptively difficult object to measure and a difficult object to make. However, you should have been able to guess that it must have something to do with pi! Since we can’t easily make hollow spheres and fill them with sand to determine their volume, we will simply give you the formula for determining the volume of a sphere, and remind you that any geometric object as a mathematical relationship that describes its volume an area. It turns out that the volume of a sphere is given by the equation:

\[ \frac{4}{3} \pi r^3 \]

Question

N. How can you double the size of a sphere?

Now think of the building that houses the 40” telescope – the big dome. What is the shape of the building? Study the picture below and draw the individual geometric shapes that comprise the building that houses the large telescope.
Draw the shapes here:

Question

O. What do you have to know in order to determine the volume of the big dome?

P. How many sugar cubes would it take to fill the building and the dome? 
    Hint: think about how we determined the number of sugar cubes to completely fill our circle
**Part II**

THIS IS GIVEN TO THE STUDENTS AFTER SOME BRAINSTORMING ON MEASURING THE DOME AND THINKING ABOUT HOW TO DETERMINE THE VOLUME OF THE DOME

Mathematics of the Dome

You can think of the dome as consisting of two parts; a large cylinder and a hemisphere at the top. A cylinder has the shape of a large paper roll, and a hemisphere is just a ball, cut in half. To make a model of the dome, all we would have to do is find a paper roll, and cut a ball about in half and place the two together.

We know that the volume of a cylinder is given by the formula \( \pi \cdot r^2 \cdot h \); where \( r \) is the radius of the cylinder and \( h \) is the height of the cylinder before you reach the part of the dome that looks like a hemisphere.

The volume of the hemisphere can be found by using the formula for a geometric shape called a “spherical cap” We will call this a hemisphere.

To actually calculate the volume of the hemisphere gets a bit tricky unless you know calculus. The volume is really the summation of a bunch of really thin disks sitting on top of each other of decreasing size. It turns out that the formula is:

\[
V = \frac{1}{6} \pi h (3r^2 + h^2)
\]

where \( h \) is the height of the sphere, and \( r \) is the radius.

To find the total volume of the dome all we have to do is add the volume of the cylinder and the volume of the hemisphere together.
We are luck, because we have plenty of architectural drawings and people around who happen to know the dimensions of the dome, so we won’t have to go climbing around with meter sticks!

**Procedure:**
1. Determine the volume of one sugar cube.
2. Explain how the volume changes when you measure two cubes, three cubes, and so forth.
3. Determine the volume of the dome using the mathematics we learned today.
4. Compare the volume of the sugar cube to that of the dome and calculate the number of sugar cubes that it would take to completely fill the dome!
5. Explain how this activity has anything to do with how astronomers use mathematics, scale, modeling building, and estimate to help solve the mysteries of the universe.

**Some Considerations**
- When we find the dimensions of the dome we will have to be sure to convert all of them into the SI system.
- We have to measure the volume of the sugar cube in the SI system.