Bispectrum in Single-Field Inflation Beyond Slow-Roll

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Outline

• Planck results for features in the power spectrum.

• Generalized Slow Roll: power spectrum.

• Generalized Slow Roll: bispectrum.
Fast power spectrum oscillations increase the agreement of both WMAP and Planck data.

It can be specified by it’s frequency, amplitude and damping scale.

What kind of inflationary models can predict such oscillatory behavior?

Dvorkin&Hu arXiv:0910.2237
Adshead et al arXiv::1110.3050
Planck arXiv:1303.5082
Can slow roll approximation produce those oscillations? (Hint: Betteridge's law of headlines !)

1. It assumes that slow roll parameters are small and constant.

\[ \epsilon_H = -\frac{d \ln H}{dN} \ll 1 \quad \text{and} \quad \sigma_1 = \frac{d \ln c_s}{dN} \ll 1 \]

2. Then, we get a space-time solution with time translation symmetry which in turn generates a scale invariant power spectrum (no oscillations).

3. To generate oscillations in the power spectrum, we need to set up a new time scale in the problem (breaking time translation symmetry).

Questions

1. What kind of models can give such oscillatory behavior? (ex: sharp steps)

2. How can we evaluate the power spectrum and the bispectrum in models that strongly violate the slow roll assumptions?

3. What are the predictions for the bispectrum? Can we use them as a way to distinguish sharp features models?

Generalized Slow Roll: properties

1. It is valid for single field inflation, where the sound speed and the Hubble parameter are general functions of time.

2. It provides fast and precise power spectrum and bispectrum evaluation, even when the slow roll parameters are large.

3. It implements a systematic approach to obtain precise analytical solutions (application: data analysis).

As an application of the formalism, we’ve analyzed sharp transitions in DBI inflation

\[
\mathcal{L} = \left[1 - \sqrt{1 - \frac{2X}{T(\phi)}}\right] T(\phi) - V(\phi)
\]

\[
F(\phi) = F_0(\phi) \left[1 + b \left( \tanh \left( \frac{\phi - \phi_s}{d} \right) - 1 \right) \right]
\]

General sharp step

\[
F(\phi) = T(\phi), \text{ Warp Feature}
\]

\[
F(\phi) = V(\phi), \text{ Potential Feature}
\]

Can both models reproduce Planck best fit?

Can we distinguish them?
Potential Features

Best fit feature: Planck.

GSR provides percent level precision.

We’ve found new solutions that can fit Planck power spectrum at any sound speed value.

Ex: in this plot $c_s = 0.15$
Warp Features

Best fit feature: Planck. Warp features lower the power at lower scale. In principle, this could be used to distinguish them.

We also can fit Planck power spectrum at any sound speed value. Ex: in this plot

\[ c_s = 0.15 \]
Can we distinguish warp against potential sharp transitions? (Cosmic variance assuring Breaking of Betteridge’s law holds !)

In principle: YES!

For the Planck best fit data, however, cosmic variance does not allow such distinction from power spectrum alone.

Only hope: bispectrum!
How can we evaluate the bispectrum beyond slow roll?

What are the predictions for the bispectrum in the presence of sharp features?

Can we use the bispectrum to distinguish different sharp transition models?

Implementing GSR Bispectrum: key points

1. Rewrite the action in the following form.

\[
S_3 = \int d^3 x \left[ \frac{a^3 \epsilon_H}{c_s^2} \frac{d}{dt} \left( 2\epsilon_H - \eta_H + \frac{\sigma_1}{2} \right) \mathcal{R}^2 \frac{d}{dt} \left( \frac{a^3 \epsilon_H}{c_s^2} \left( 2\epsilon_H - \eta_H + \frac{\sigma_1}{2} \right) \mathcal{R}^2 \frac{d}{dt} \right) \right]

+ \left( \epsilon_H + \sigma_1 \right) \mathcal{R} \left( \mathcal{H}_2 + 2\mathcal{L}_2 \right) + \left( 1 - \frac{1}{c_s^2} \right) \frac{\dot{\mathcal{R}}}{H} \mathcal{L}_2 + \frac{a^3 \epsilon_H}{c_s^2} \mathcal{R} \left| \frac{\dot{\mathcal{R}}}{H} \right|^2 - 2 \frac{a \epsilon_H}{c_s^2} \frac{\dot{\mathcal{R}}}{H} \partial_i \mathcal{R} \partial_i \left( \partial^2 \frac{a^2 \epsilon_H}{c_s^2} \frac{\dot{\mathcal{R}}}{H} \right)
\]

First line shows the operators that are responsible for feature like behavior in canonical inflation. They are not dominant for arbitrary sound speed. (Adshead et al 1102.3435)

We’ve developed GSR for all those terms.
Implementing GSR Bispectrum: key points

Bonus: the action new form makes the proof of the consistency relation considerably easier, especially in the slow roll approximation.

\[
\frac{12}{5} f_{NL} = \lim_{k_s \to 0} \frac{\mathcal{B}_{\mathcal{R}}(k_s, k_L, k_L)}{P_{\mathcal{R}}(k_s)P_{\mathcal{R}}(k_L)} = 1 - n_s
\]

2. Expand the mode function in first order in slow for the operator below (because it does not multiply a slow roll parameter) and zero order for the remaining ones.

\[
\left(1 - \frac{1}{c_s^2}\right) \frac{\dot{\mathcal{R}}}{H} \mathcal{L}_2
\]
3. Express the final result in the form.

\[
 f_{NL} \sim \frac{G}{k_1 k_2 k_3} \sim \sum_{ij} T_{ij}(k_1, k_2, k_3) I_{ij}(K)
\]

\[
 I_{ij}(K) = S_{ij} W_{ij} + \int ds S'_{ij} W_{ij}
\]

Kernels \( S_{ij} \) are functions of slow roll parameters (which can be quite large). \( W_{ij} \) are windows.

Integrals only depend on the triangle perimeter.

\[
 K = \sum_{i} k_i
\]
Bispectrum – low sound speed

Best fit Planck

Planck hasn’t tested such higher frequency.

Limit of validity of perturbation theory

GSR works with ~10-30% precision
Bispectrum – high sound speed

Best fit Planck

Smaller oscillatory $f_{nl}$ favors features in canonical inflation in comparison to features in low sound speed models.

GSR has few percent level precision at high sound speed values.
Does consistency relation holds in the presence of sharp steps? (Finally a breaking of Betteridge's law of headlines!)

c_s=0.15
b=-0.2

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5(1-n_s)/12

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f_{NL}

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k_Ls_s
Bispectrum – Squeeze Limit

GSR also describes well the exact solution in the squeeze limit.

GSR approximation does obey the consistency relation.
Second order - Slow Roll

GSR provides an simple framework to calculate analytically higher order slow roll corrections.

We have results for arbitrary shape.

Showing only equilateral for simplicity.
Conclusions

Models of inflation with sharp step produce fast oscillations that fit both by WMAP and Planck data.

We’ve developed a general formalism that provides fast and accurate calculation of the power spectrum and the bispectrum in single field inflation even when slow roll assumptions are violated.

Our formalism provides a systematic approach to calculate accurate analytical solutions that can be used in data analysis.

Bispectrum can be used to distinguish models (low vs high sound speed). In future work we will study if at a given sound speed, warp and potential can be distinguished by observing the bispectrum.
Thank You