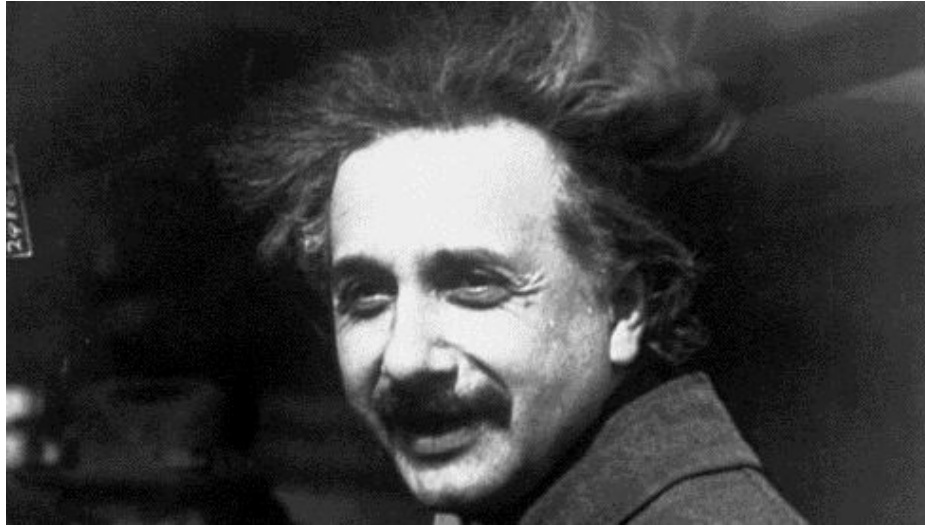


MISC

1. Final exam is scheduled for **Monday, March 17, 1:30-3:30PM here in KPTC 106.**
2. Second midterm is next Wednesday, usual class time (will be proctored by Dr. Brant Robertson and TAs).
3. Class grading curve (tentative, can be somewhat adjusted later this month)
 - > 85% = A
 - 70-85% = B
 - 55-70% = C
 - 40-55% = D
 - < 40% = F
4. Today, we are covering material of **S 22-2, 26-2, 26-3**

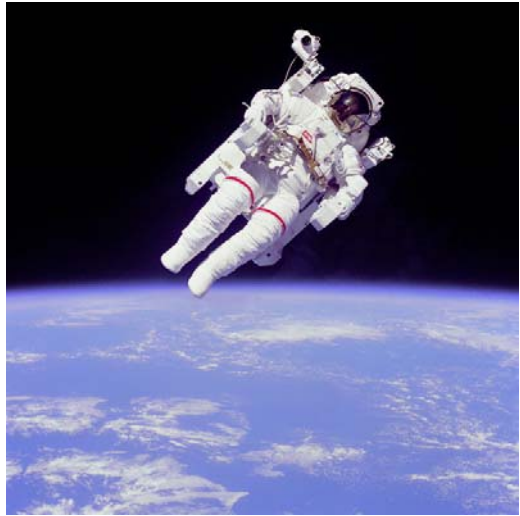
A new General Theory of Relativity & Gravity



In 1907-1915, in a series of papers, Einstein developed a new theory of gravity based on *two key observations*:

1. *Inertial mass appeared to be equivalent to the gravitational mass*
2. *Effect of gravity is mimicked by acceleration*

Weightlessness in free-fall gravity-like feeling in acceleration



NASA's 20-g centrifuge

Theory of General Relativity

“The theory appeared to me then, and still does, the greatest feat of human thinking about nature, the most amazing combination of philosophical penetration, physical intuition, and mathematical skill. But its connections with experience were slender. It appealed to me like a great work of art, to be enjoyed and admired from a distance. – Max Born



The famous equation of Einstein's general relativity can be written simply as:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

GEOMETRY = MATTER + ENERGY

Where geometry refers to a measure of how curved the space is

Unfortunately, in practice the mathematical description of curvature and of matter content and energy is quite involved...

In mathematical form they can be written as...

$$\begin{aligned}
 & \frac{1}{2}g^{rs} \left(-\frac{\partial^2 g_{ij}}{\partial x^r \partial x^s} + \frac{\partial^2 g_{is}}{\partial x^r \partial x^j} + \frac{\partial^2 g_{rj}}{\partial x^i \partial x^s} - \frac{\partial^2 g_{rs}}{\partial x^i \partial x^j} \right) + \\
 & \frac{1}{4}g^{qp} \left(-\frac{\partial g_{is}}{\partial x^p} + \frac{\partial g_{pi}}{\partial x^s} + \frac{\partial g_{ps}}{\partial x^i} \right) \left(\frac{\partial g_{qj}}{\partial x^r} + \frac{\partial g_{qr}}{\partial x^j} - \frac{\partial g_{rj}}{\partial x^q} \right) - \\
 & \frac{1}{4}g^{qp} \left(-\frac{\partial g_{ij}}{\partial x^p} + \frac{\partial g_{pi}}{\partial x^j} + \frac{\partial g_{pj}}{\partial x^i} \right) \left(\frac{\partial g_{qr}}{\partial x^s} + \frac{\partial g_{qs}}{\partial x^r} - \frac{\partial g_{rs}}{\partial x^q} \right) - \\
 & \frac{1}{4}g_{ij}g^{rs}g^{uv} \left(-\frac{\partial^2 g_{rs}}{\partial x^u \partial x^v} + \frac{\partial^2 g_{rv}}{\partial x^u \partial x^s} + \frac{\partial^2 g_{us}}{\partial x^r \partial x^v} - \right. \\
 & \quad \left. \frac{\partial^2 g_{uv}}{\partial x^r \partial x^s} \right) + \frac{1}{8}g_{ij}g^{rs}g^{uv}g^{qp} \left(\frac{\partial g_{qr}}{\partial x^v} + \frac{\partial g_{qv}}{\partial x^r} - \frac{\partial g_{rv}}{\partial x^q} \right) \times \\
 & \left(\frac{\partial g_{ps}}{\partial x^u} + \frac{\partial g_{pu}}{\partial x^s} - \frac{\partial g_{us}}{\partial x^p} \right) - \frac{1}{8}g_{ij}g^{rs}g^{uv}g^{qp} \left(\frac{\partial g_{qr}}{\partial x^s} + \right. \\
 & \quad \left. \frac{\partial g_{qs}}{\partial x^r} - \frac{\partial g_{rs}}{\partial x^q} \right) \left(\frac{\partial g_{pu}}{\partial x^v} + \frac{\partial g_{pv}}{\partial x^u} - \frac{\partial g_{uv}}{\partial x^p} \right) = \frac{8\pi G}{c^4} T_{ij}.
 \end{aligned}$$

Theory of General Relativity

“The theory appeared to me then, and still does, the greatest feat of human thinking about nature, the most amazing combination of philosophical penetration, physical intuition, and mathematical skill. But its connections with experience were slender. It appealed to me like a great work of art, to be enjoyed and admired from a distance. – Max Born



The famous equation of Einstein's general relativity can be written simply as:

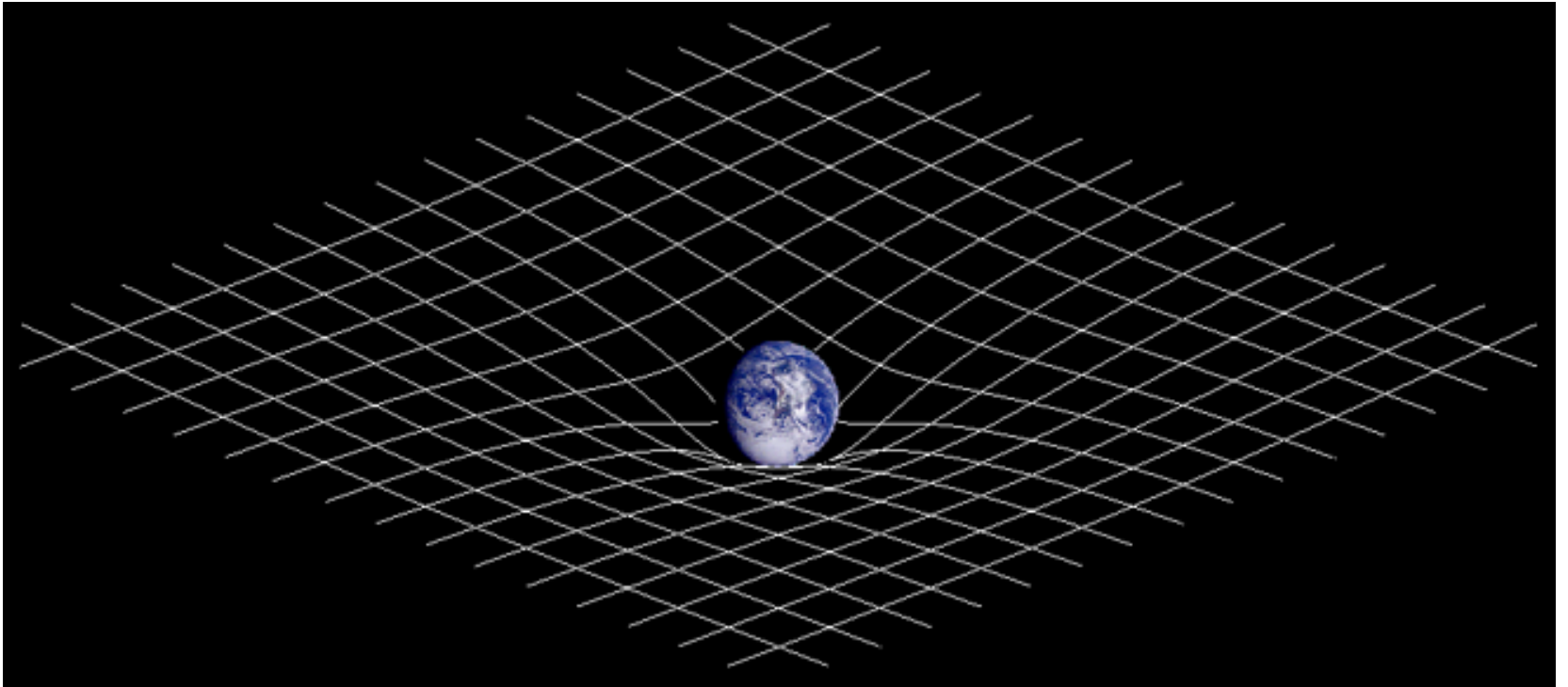
$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

GEOMETRY = MATTER + ENERGY

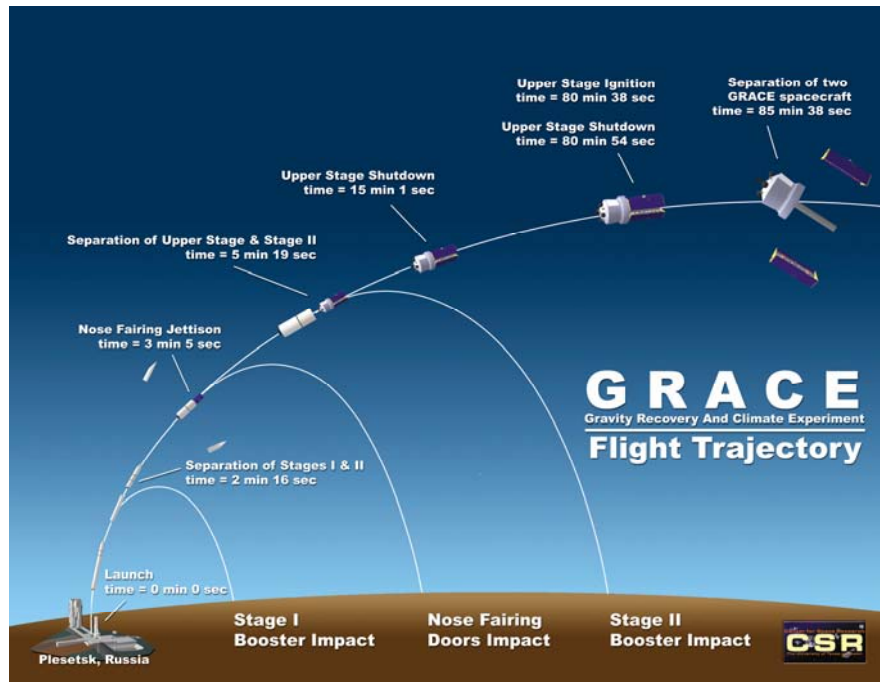
Where geometry refers to a measure of how curved the space is

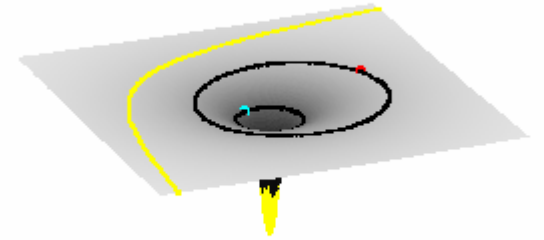
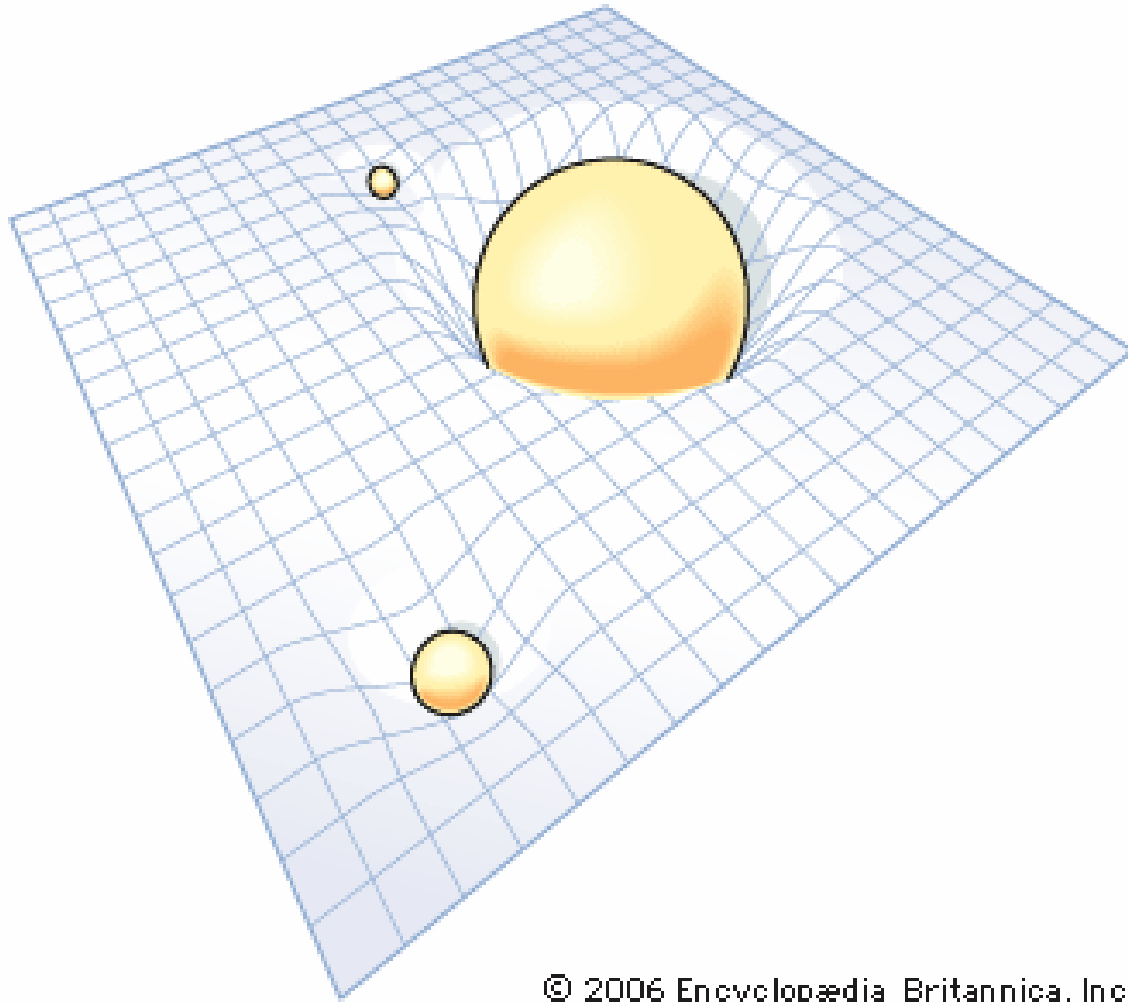
Unfortunately, in practice the mathematical description of curvature and of matter content and energy is quite involved...

An analogy...



“Spacetime grips mass, telling it how to move, and mass grips spacetime, telling it how to curve” – John Archibald Wheeler.

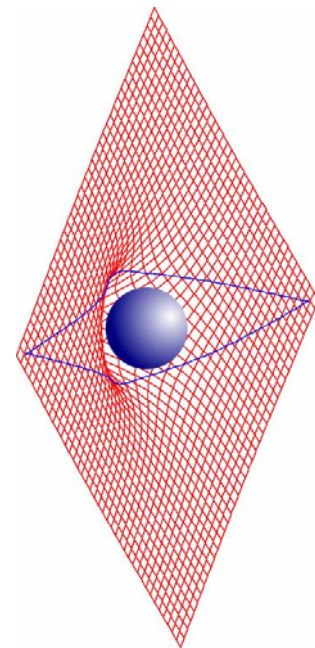
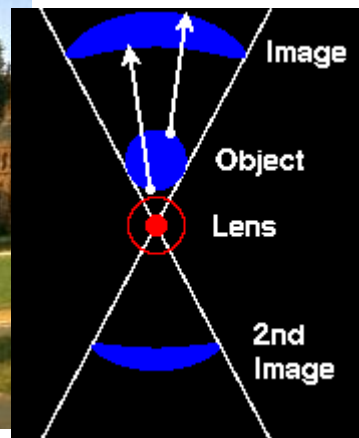
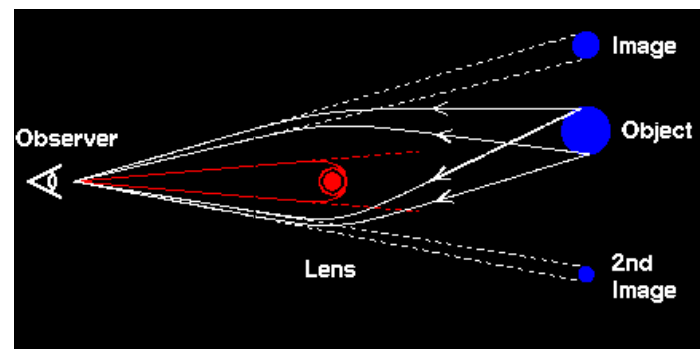




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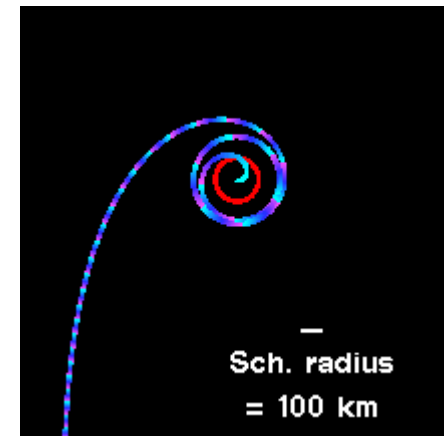
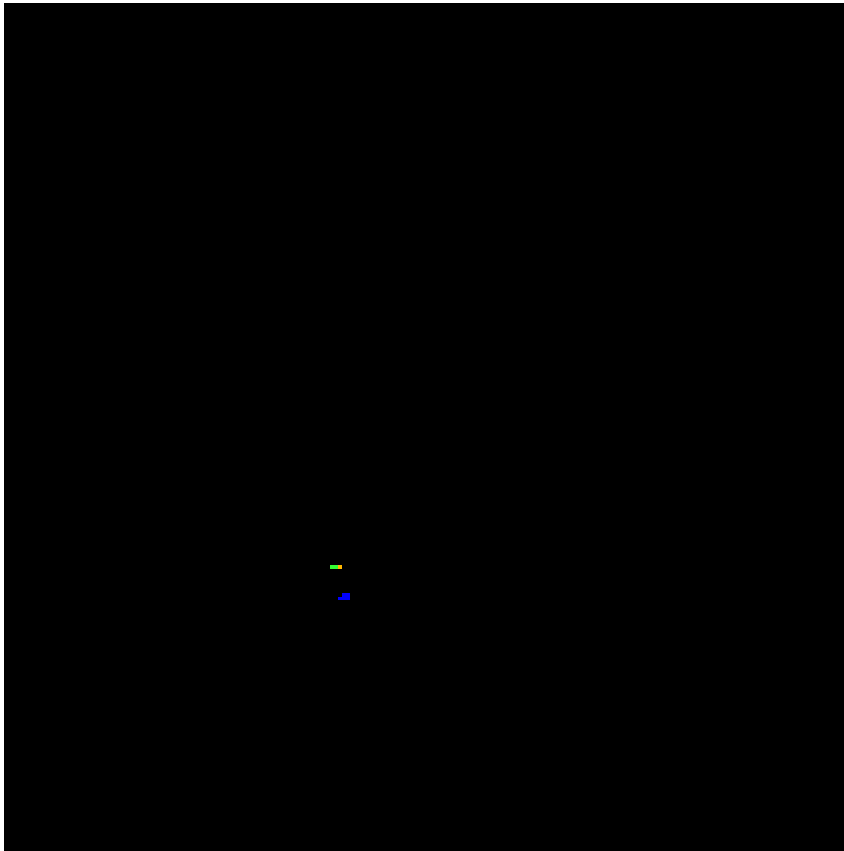
Strongly curved space around black hole And gravitational lensing

<http://cfa-www.harvard.edu/~bmcleod/castle.html>



Lensing of the Castle (Mall, Washington, DC) by a black hole with the mass of Saturn

A simulation of a flight towards a black hole



trajectory

Credit: Andrew Hamilton (Colorado University)
<http://casa.colorado.edu/~ajsh/approach.html>

gravitational lensing as a test of GR theory



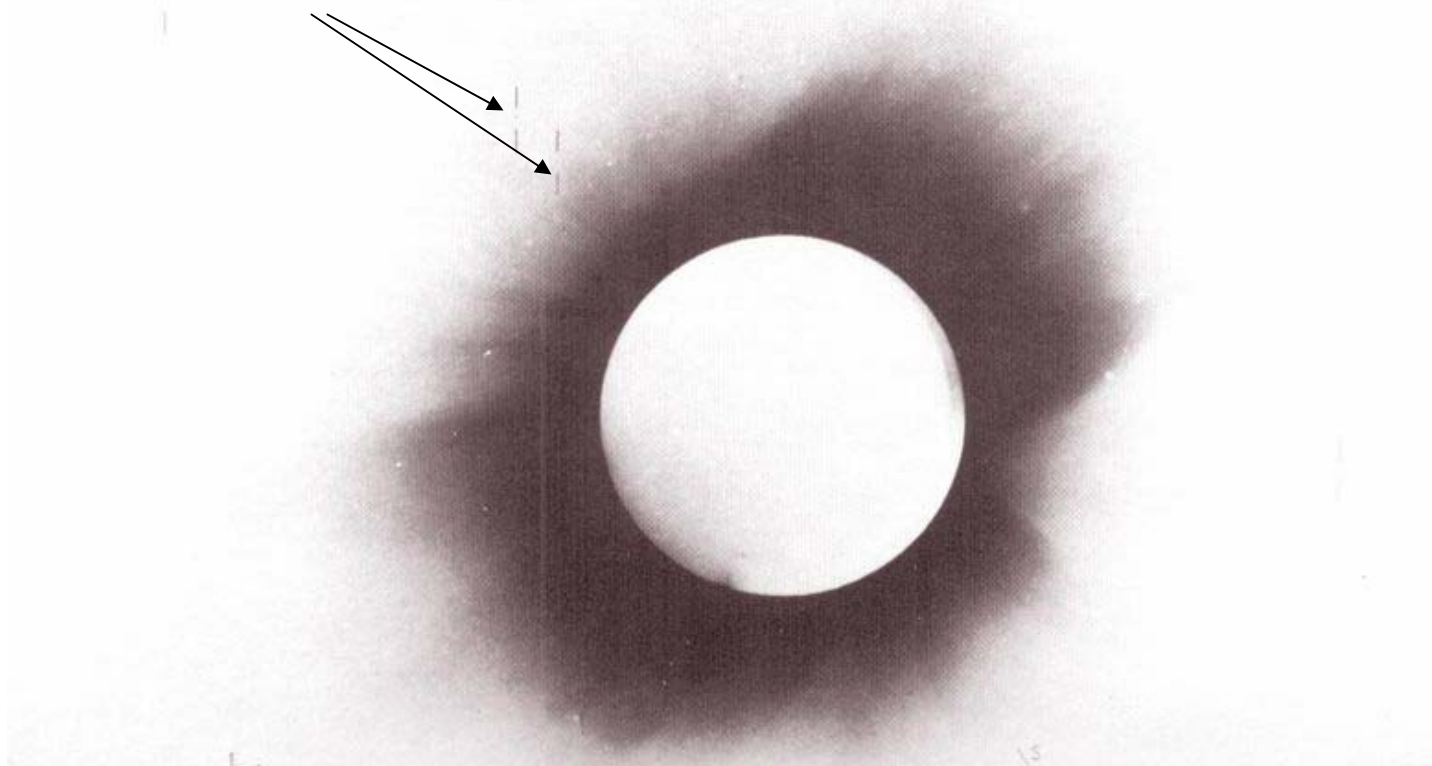
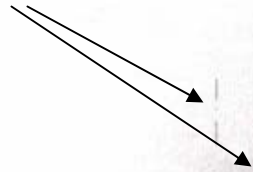
**Sir Arthur
Stanley
Eddington**

- ❑ In 1919, Eddington went into expedition to the island of Principe to observe the total solar eclipse
- ❑ during the eclipse he observed stars in the stellar cluster called Hyades and he measured their relative positions to other stars
- ❑ Eddington measured that when Hyades stars were close to the Sun they were deflected by $\sim 1.6 \pm 0.3$ arc seconds, which was consistent with the deflection Angle of 1.75 arc seconds predicted by Einstein

This is considered as one of the first experimental confirmations of the GR, but the real story is more complicated

(inverted) photograph of solar eclipse in 1919

***some of the stars
used to measure
gravitational
deflection of light***



Tests of General Relativity

All tests of GR can be separated into two types: **weak field limit** (i.e. testing GR when deviations from Newton's gravity are weak) and **strong field limit** (deviations from the Newton's law are large)

Weak field limit tests are numerous:

bending of light by the Sun's gravity field
precession of planetary orbits (in particular Mercury's)
time delay due to the Earth gravity
gravitational redshift from white dwarfs

Jury is still out on the validity of the GR at the weakest fields (largest scales, evolution of the Universe).

Strong field limit tests are much more difficult to perform, but only they can convincingly confirm (or reject!) the GR:

gravitational radiation from binary pulsars orbiting each other at relatively small separation (Nobel Prize in physics in 1993)

$$\text{GEOMETRY} = \text{MATTER} + \text{ENERGY}$$

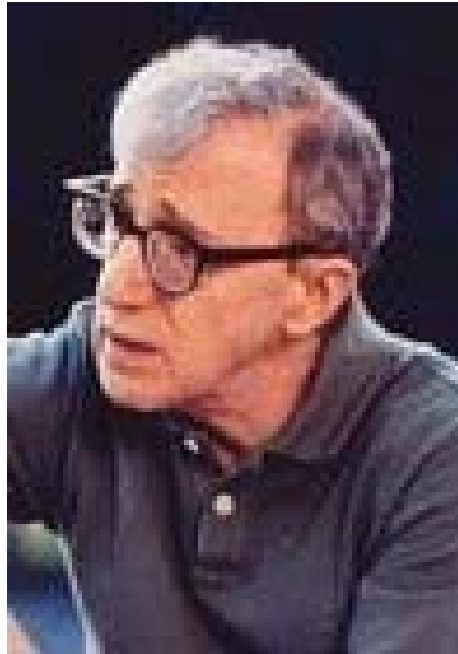


Einstein's equations can be applied to the Universe as a whole, if one makes assumptions about what stuff contributes to the matter and energy density (r.h.s. of the equations) in the Universe *and* assumes that universe is isotropic and homogeneous (on large scales) – aka the cosmological principle.

Contents of the Universe, defining its global curvature, are all existing components (protons, neutrons, hypothetical dark matter and dark energy) can contribute to gravity and can influence the rate with which the Universe expands

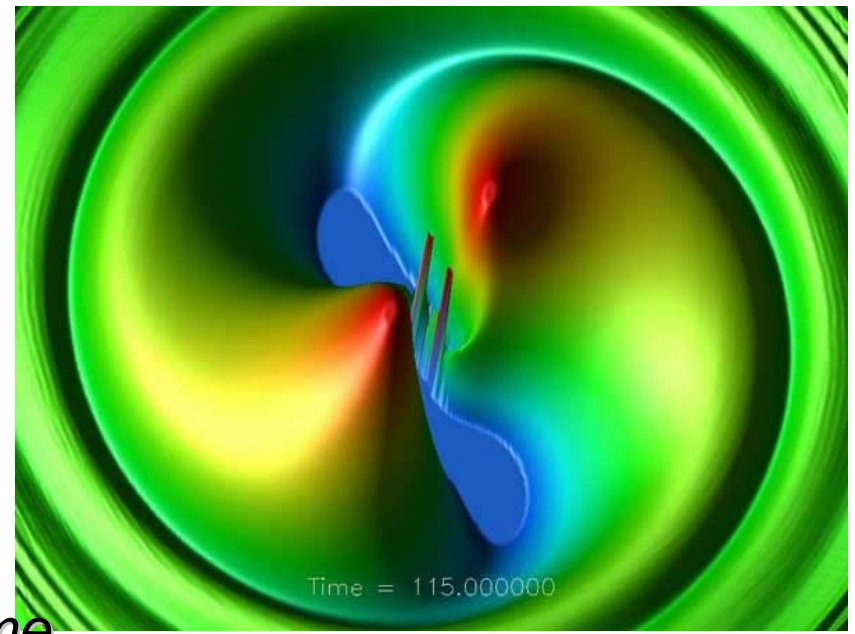
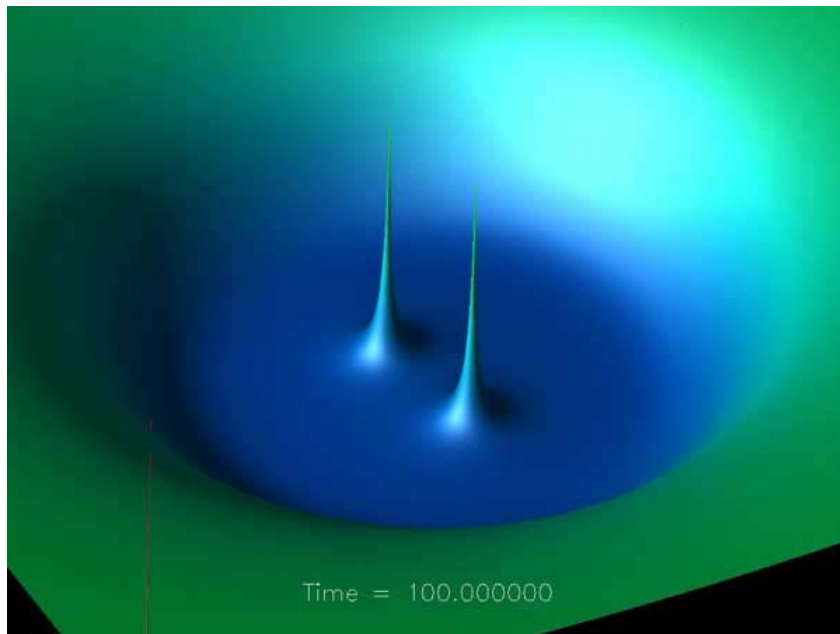
"I'm astounded by people who want to 'know' the universe when it's hard enough to find your way around Chinatown."

- Woody Allen



Solution of Einstein's equations of gravity is very complicated in general

Merger of two black holes was numerically solved only three years ago due to complicated warping of space accompanying such an event.



time



"Computers are incredibly fast, accurate, and stupid; humans are incredibly slow, inaccurate and brilliant; together they are powerful beyond imagination." --Einstein

The Copernícan príncíple

The Earth, the solar system, the Milky Way Galaxy, our cosmological neighborhood are *not* special locations

An extension is the Cosmological principle

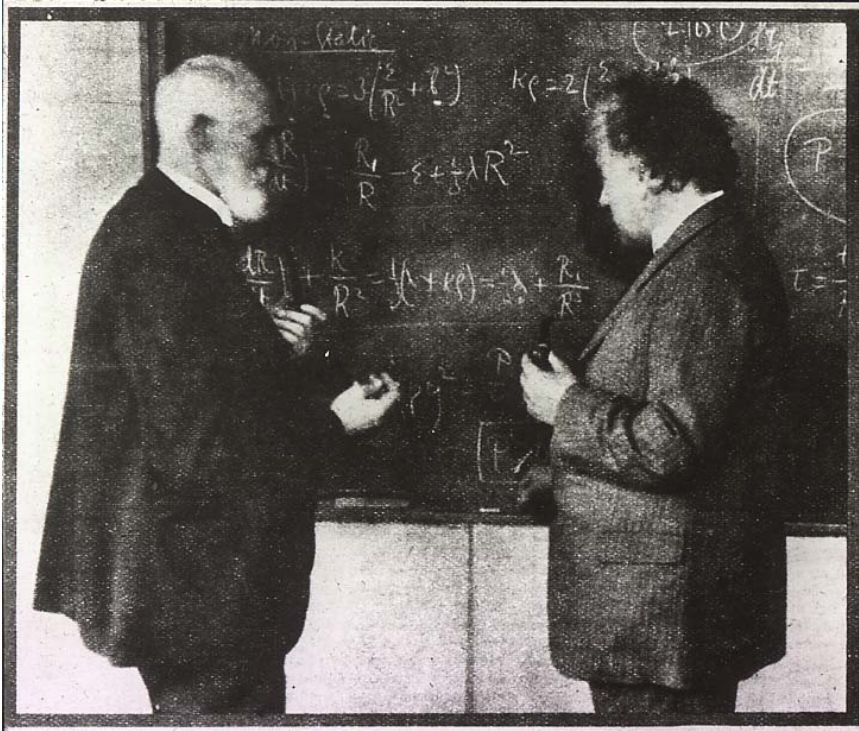
The Universe, *on average*, is homogeneous (equal in density and in other properties in different locations) and isotropic (it looks the same in all directions)



Note that *isotropy + Copernican principle = homogeneity*

The cosmological principle and equations of the general relativity are key ingredients of the standard Big Bang model

Einstein's "biggest blunder"



Willem de Sitter and Einstein discuss non-static solution to the Einstein's equation of gravity

1917. Einstein recognized the dynamic nature of his equations. The Milky Way did not appear to expand or contract, so he introduced a repulsive term, *the cosmological constant*, that was weak on scales of the solar system, but could counteract gravity on large scales

1917. Willem de Sitter solved Einstein's equation for the ideal case of a universe with repulsive cosmological constant, but no matter. He showed that Einstein's equations implied that in such a universe space would expand exponentially with different points of space receding from each other with velocities proportional to the distance between them...



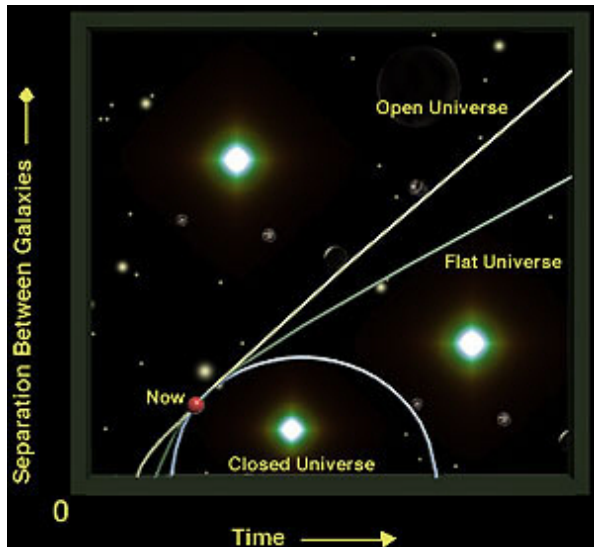
Alexander Friedmann
(1888-1925)

Friedmann's solution

1920-22. Alexander Friedmann derived a general solution to the Einstein's equations using the Cosmological principle (homogeneity and isotropy) and published it in a German journal and forwarded paper to Einstein's.

The solution showed that in general Universe is unstable and should either expand or contract, depending on the average density of matter and energy in it.

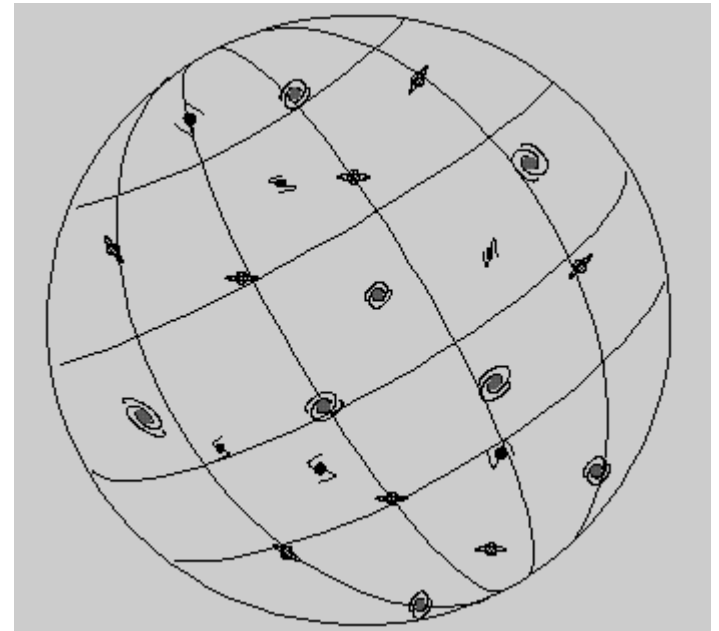
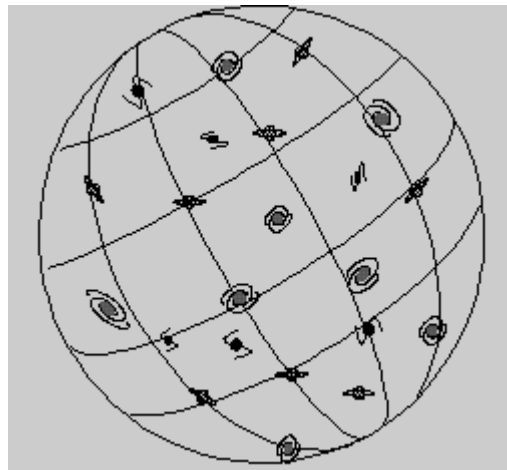
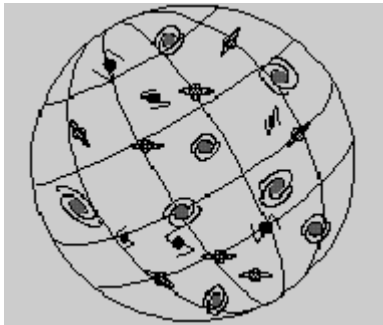
Einstein at first thought that Friedmann's solutions were incorrect, but later acknowledged their validity.



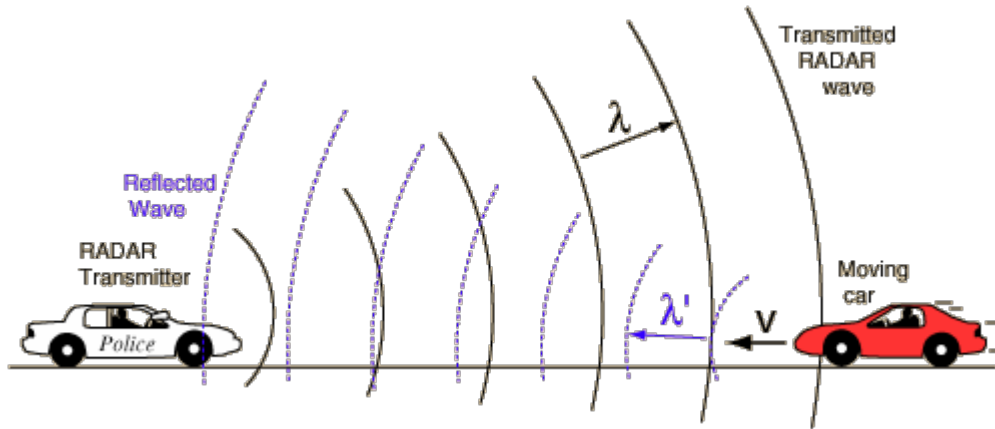
In the Friedmann's solution the space itself is expanding or contracting

Two dimensional analogy of an inflating balloon

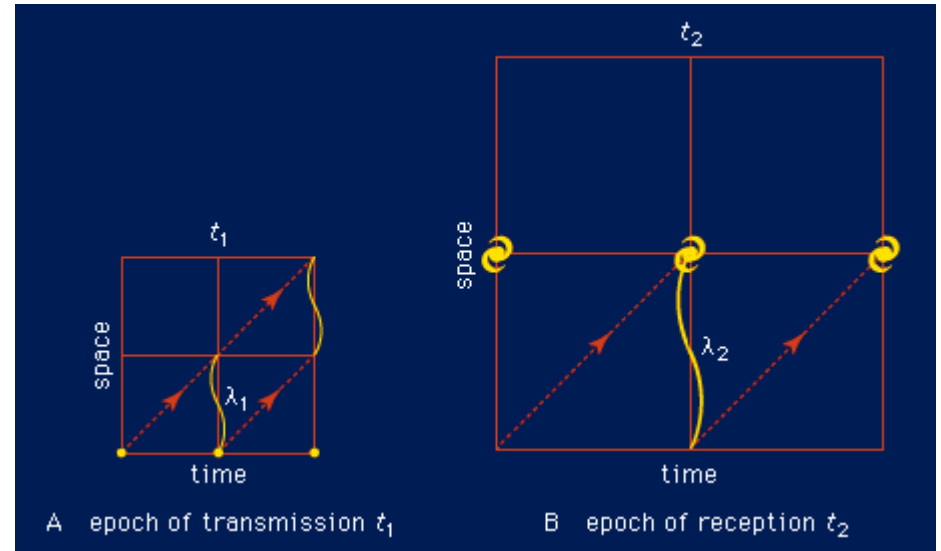
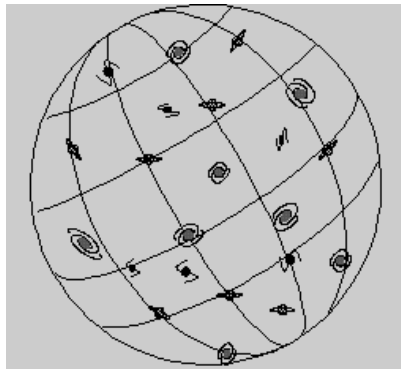
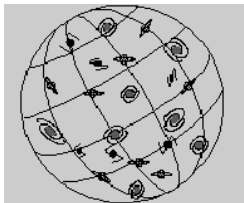
*(**caution!** This analogy is not entirely correct, as we do not know whether our Universe expands into any additional dimension. Rather the space itself is created)*



Two sources of spectral line shifts



Usual spectral red- or blue-shift due to physical motion of objects away or towards observer



Red-shift due to expansion of space itself

Cosmological redshift due to expansion of space

$$1+z = R_{\text{now}}/R_{\text{then}} = 1/R_{\text{then}}$$

Where z corresponds to a Doppler-like shift of wavelength and frequency of light as the Universe expands from the epoch when scale factor is R_{then} to the current epoch corresponding to R_{now} .

For example, $z=1$ corresponds to the epoch when the Universe was twice “smaller” than now.

appearance the spectrum is very much like spectra of the Milky Way clouds in Sagittarius and Cygnus, and is also similar to spectra of binary stars of the W Ursae Majoris type, where the widening and depth of the lines are affected by the rapid rotation of the stars involved.

The wide shallow absorption lines observed in the spectrum of N. G. C. 7619 have been noticed in the spectra of other extra-galactic nebulae, and may be due to a dispersion in velocity and a blending of the spectral types of the many stars which presumably exist in the central parts of these nebulae. The lack of depth in the absorption lines seems to be more pronounced among the smaller and fainter nebulae, and in N. G. C. 7619 the absorption is very weak.

It is hoped that velocities of more of these interesting objects will soon be available.

*A RELATION BETWEEN DISTANCE AND RADIAL VELOCITY
AMONG EXTRA-GALACTIC NEBULAE*

BY EDWIN HUBBLE

MOUNT WILSON OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

Communicated January 17, 1929

Determinations of the motion of the sun with respect to the extra-galactic nebulae have involved a K term of several hundred kilometers which appears to be variable. Explanations of this paradox have been sought in a correlation between apparent radial velocities and distances, but so far the results have not been convincing. The present paper is a re-examination of the question, based on only those nebular distances which are believed to be fairly reliable.

Distances of extra-galactic nebulae depend ultimately upon the application of absolute-luminosity criteria to involved stars whose types can be recognized. These include, among others, Cepheid variables, novae, and blue stars involved in emission nebulosity. Numerical values depend upon the zero point of the period-luminosity relation among Cepheids, the other criteria merely check the order of the distances. This method is restricted to the few nebulae which are well resolved by existing instruments. A study of these nebulae, together with those in which any stars at all can be recognized, indicates the probability of an approximately uniform upper limit to the absolute luminosity of stars, in the late-type spirals and irregular nebulae at least, of the order of M (photographic) = -6.3 .¹ The apparent luminosities of the brightest stars in such nebulae are thus criteria which, although rough and to be applied with caution,

The paper that revolutionized cosmology

1929. Hubble published his first paper showing correlation between recession velocity and distance to the galaxy

“A Relation Between Distance and Radial Velocity Among Extra-Galactic Nebulae”

6 pages, 1 table, 1 figure

The last paragraph of the Hubble's 1929 paper

The outstanding feature, however, is the possibility that the velocity-distance relation may represent the de Sitter effect, and hence that numerical data may be introduced into discussions of the general curvature of space. In the de Sitter cosmology, displacements of the spectra arise from two sources, an apparent slowing down of atomic vibrations and a general tendency of material particles to scatter. The latter involves an acceleration and hence introduces the element of time. The relative importance of these two effects should determine the form of the relation between distances and observed velocities; and in this connection it may be emphasized that the linear relation found in the present discussion is a first approximation representing a restricted range in distance.

¹ *Mt. Wilson Contr.*, No. 324; *Astroph. J.*, Chicago, Ill., 64, 1926 (321).

² *Harvard Coll. Obs. Circ.*, 294, 1926.

³ *Mon. Not. R. Astr. Soc.*, 85, 1925 (865-894).

⁴ These PROCEEDINGS, 15, 1929 (167).

Hubble constant: and the age of the universe

Inverse of the Hubble constant has units of time
and corresponds to the *age of the Universe*

If we use H to be ~ 500 km/s/Mpc, as estimated by Hubble, the age will be ~ 2 billion years.

