Astronomy 313000
Spring Quarter 2007

Take-home Final Exam

Due: Friday, June 8 at 5 pm in my box in AAC (no exceptions)

Please do not consult each other. You may consult Binney and Merrifield. If you have any questions, please contact me *before* the due date by e-mail: frieman@fnal.gov.

1. A very simple picture of spiral galaxy formation may explain why spiral disks have approximately exponential radial profiles as well as their characteristic disk scale lengths. Assume a spiral galaxy begins as a spherically symmetric, uniform density region of radius R_i, baryonic mass M_b, dark matter mass M_d, rotating as a solid body at a constant angular speed Ω (radians per sec) about some axis, say the z axis. The region collapses gravitationally; in the process, dark matter particles conserve energy, but the baryonic matter (assuming it's all gaseous) can heat up and radiate its energy away. The collapse process ends with the baryonic matter in a thin disk, with all particles orbiting on circular orbits about the z axis, embedded in a spherical dark matter halo. Assume that the baryons do not exchange angular momentum during the collapse, so that the initial component of angular momentum about the z axis is preserved for each baryon. Also assume that the final total mass distribution (which is dominated by dark matter) is such that the rotation curve is flat, i.e., the baryons in the disk orbit with circular speed v_c, independent of radius r.

   (a) Derive an expresssion for the final surface mass density $\Sigma(r)$ (mass per unit area) of the baryonic disk as a function of radius r, in terms of v_c, Ω, R_i, and the inital baryon mass density $\bar{\rho}_r = 3M_b/4\pi R_i^3$. Plot $\log(\Sigma)$ vs. r and show that there is a large range in r where it has an approximately exponential shape. For the plot, you'll want to use a dimensionless measure of the radius, i.e., define a dimensionless variable $x = r/R_d$, where the characteristic disk scale length R_d is given in terms of the above quantities.

   (b) A dimensionless measure of the initial angular momentum of the system is given by the quantity

   $$\lambda = \frac{JE^{1/2}}{GM^{5/2}}$$

   where $M = M_d + M_b$ is the total mass of the system, J is the total angular momentum, and E is the binding energy of the system. Assuming that the final circular speed in the disk, v_c, is equal to $(GM/R_i)^{1/2}$, find a relation between R_d, R_i, and λ. It appears that proto-galaxies acquire their angular momentum by tidal torquing from neighboring proto-galaxies, with typical value $\lambda = 0.05$. If $R_i = 100h^{-1}$ kpc, what is the predicted disk scale length R_d?

2. As we noted in class, violent relaxation tends to result in the particles in a system having similar speeds rather than similar kinetic energies. On the other hand, there is some evidence that, in the inner parts of clusters, more luminous (and presumably more massive) galaxies have lower velocity dispersions than their less luminous siblings. One possible culprit is dynamical friction.

   (a) Consider a galaxy of mass M moving at speed V through a cluster with local mass density ρ. Assuming the 'scatterers' in the cluster (e.g., dark matter particles or stars in other galaxies) are much less massive than M, write down an expression for the friction force on the galaxy in terms of the above three quantities and the 'Coulomb' logarithm $\ln \Lambda$. For this expression, assume for simplicity that all the scatterers have velocities smaller than V.

   (b) In terms of the same quantities, derive an expression for the characteristic timescale for the galaxy to slow down due to dynamical friction.

   (c) Now assume the cluster density profile is that of a singular isothermal sphere of velocity dispersion σ_v; also assume the galaxy is initially orbiting at radius r. Derive a value for the slow-down time in Gigayears in terms of (σ_v/1000 km/sec), (V/1000 km/sec), (r/100 kpc), and ($M/10^{12} M_\odot$).

3. This problem has to do with the effects of relativistic beaming in radio galaxies. Suppose that all radio jets emit radiation with frequency spectrum $S_\nu \propto \nu^{-\alpha}$ and that they all move with the same relativistic speed $\beta = v/c$ relative to the core. Let $F_\nu(0)$ be the flux emitted by the jet if it moves perpendicular to the line of sight. For simplicity, consider only jets that have positive semi-definite velocity component toward the observer (since those moving away from the observer tend to be suppressed by beaming).

   (a) For a single jet, write down the expression for the observed flux $F_\nu(\cos\phi)$, where ϕ is the angle between the direction of the jet and the line of sight from AGN to the observer. Assuming $\alpha = 0.7$ and $\beta = 0.95$, what is the maximum value of the observed flux in terms of $F_\nu(0)$?

   (b) Averaging over all lines of sight, derive the normalized probability distribution $P(F_\nu)$ in terms of $F_\nu(0)$, β, and α. Sketch or plot this pdf between the minimum and maximum values of F_ν. Defining $A = F_\nu/F_\nu(0)$, derive an expression for the mean value of A and evaluate it for the values of α, β above.

   (c) Now assume that AGN have a power-law distribution of 'intrinsic' flux, $dN/dF_\nu(0) \propto F_\nu(0)^{-\delta}$. Derive an expression for the distribution of observed flux, dN/dF_ν. For large positive δ, is the number of sources above some flux limit enhanced or decreased compared to $dN/dF_\nu(0)$? Is the shape of the observed flux distribution changed compared to the 'intrinsic' distribution?

4. An inventory of the baryonic mass in galactic stars, in galactic gas and dust, and in the intracluster medium shows that it falls below the cosmic baryon density, $\Omega_b = 0.04$, inferred from both big bang nucleosynthesis and the cosmic microwave background (CMB) anisotropy. One possibility for these 'missing baryons' is that they are in a hot intergalactic medium (IGM) uniformly distributed throughout the Universe. This idea originally had some appeal, since a sufficiently hot IGM could possibly give rise to the part of the X-ray background that is not accounted for by resolved sources. However, this idea is severely constrained by the fact that COBE observed the CMB to have an essentially perfect black body spectrum.

   (a) Assume the IGM has uniform temperature T and is composed of ionized hydrogen with mass density $\rho_{IGM} = \Omega_{IGM}\rho_{crit}$, where the mean density of an Einstein-de Sitter Universe is $\rho_{crit} = 3H_0^2/8\pi G$ and H_0 is the Hubble parameter. Assume that photons from the CMB have to traverse a distance of c/H_0 to reach us. Derive an expression for the Compton y parameter in terms of H_0, Ω_{IGM}, T and fundamental constants (assuming particle masses are fundamental constants).

   (b) Defining $h = H_0/100$ km/sec/Mpc, evaluate the above expression, i.e., give an expression for y in terms of Ω_{IGM}, h, and the scaled temperature ($kT/10$ keV).

   (c) The COBE constraint on departure from the Planck spectrum corresponds to the upper bound $y < 1.5 \times 10^{-5}$. The X-ray background suggests emission from hot gas with $kT = 30$ keV. Using $h = 0.7$, what is the upper bound on Ω_{IGM} if it has this temperature? Can an IGM this hot contain an appreciable fraction of the cosmic baryon budget? How does your answer change if the IGM is much cooler, say $kT = 1$ keV?