

The background of the slide is a night sky filled with stars, with the Milky Way galaxy clearly visible as a bright, diagonal band of light. In the foreground, three large, white, dome-shaped astronomical observatories are visible, each with a corrugated metal base. The observatories are arranged in a row, with the central one being the tallest and most prominent.

Astronomy 182: Origin and Evolution of the Universe

Prof. Josh Frieman

Lecture 7
Oct. 30, 2015

Today

- Relativistic Cosmology
- Dark Side of the Universe I: Dark Matter

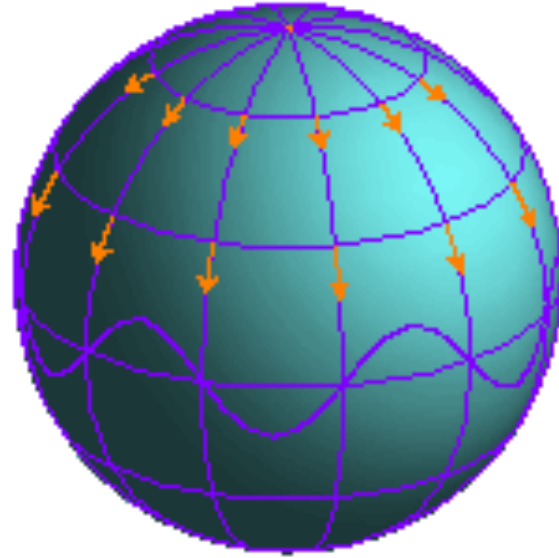
Assignments

- **This week:** read Hawley and Holcomb, Chapters 7-9, 13.
- **Today:** Lab 2 write-up due in class.
- **Next Fri., Nov. 6:** Essay due on HH, Chapter 8.

The Expanding Universe

Distance between galaxies given by universal scale factor:

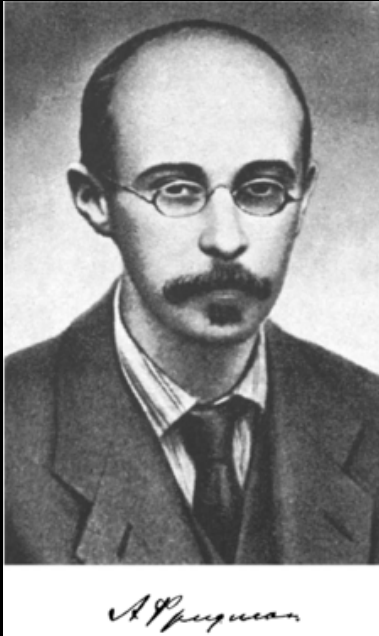
$$d(t) = d_0 a(t)$$



Dynamics of Cosmic Expansion

- Separation between any pair of galaxies is increasing due to expansion: $d(t)=d_0a(t)$, where $a(t)$ is a universal function. This is consistent with Hubble's law and observed homogeneity and isotropy.
- What determines the time-dependence of the cosmic scale factor $a(t)$?
- On large scales, gravity is the only force that matters. Use General Relativity to determine evolution of $a(t)$.

Friedmann–Lemaitre–Robertson–Walker model



Alexander Friedmann
Russian
1922-24 derivations
(died in 1925)



George Lemaitre
Belgian priest
1927 derivations



Howard Percy Robertson
American
+



Arthur Geoffrey Walker
English

1935 – proof that FLRW expression for spacetime interval is the only one for a universe that is both homogeneous and isotropic

Space vs Spacetime Curvature

Curvature of 3-dimensional Space vs. Curvature of 4-dimensional Spacetime:

General Relativity: implies that Spacetime is generally curved.

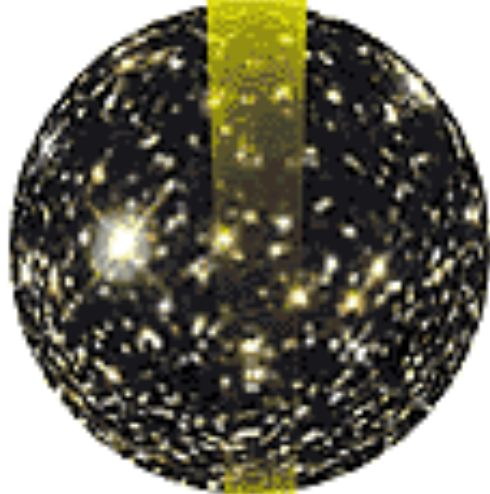
Cosmology: mainly concerned with the curvature of 3-dimensional space (K) (i.e., of a 'slice' through spacetime at a fixed time) since it is related to the density and fate of the Universe.

Global Curvature of Space

Positive curvature

$$C/r < 2\pi$$

Σ angles > 180 deg

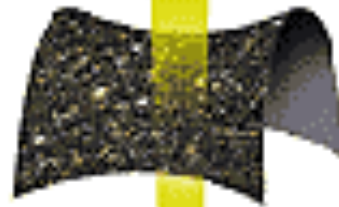
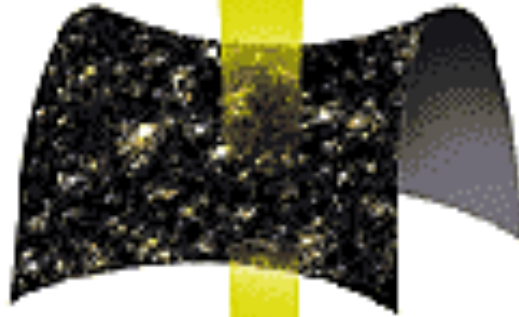


$$K > 0$$

Negative curvature

$$C/r > 2\pi$$

Σ angles < 180 deg

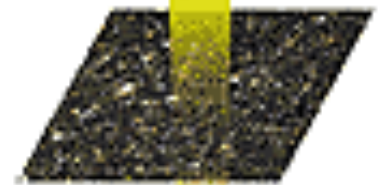
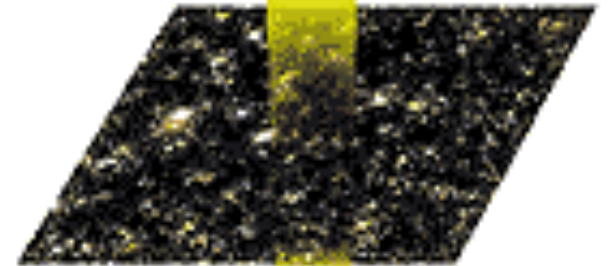


$$K < 0$$

Flat (Euclidean)

$$C/r = 2\pi$$

Σ angles $= 180$ deg



$$K = 0$$

Dynamics of Cosmic Expansion

- **GR**: curvature of spacetime determined by mass-energy (Einstein equations).
- Apply this to the homogeneous and isotropic Universe: **Friedmann equation**:

$$\left(\frac{1}{a} \frac{\Delta a}{\Delta t}\right)^2 + \frac{k}{a^2} = \frac{8\pi G\rho}{3}$$

Expansion rate Spatial Curvature Density of Matter

Dynamics of Cosmic Expansion

- **Example 1: $\rho=0, k=-1$:** empty, negatively curved universe. In this case, $(\Delta a/\Delta t)=1$, so $a(t)\sim t$. Hence, $d(t)\sim t$, and galaxies recede at fixed speeds.

$$\left(\frac{1}{a} \frac{\Delta a}{\Delta t}\right)^2 + \frac{k}{a^2} = \frac{8\pi G\rho}{3}$$

Expansion rate

Spatial Curvature

Density of Matter

Dynamics of Cosmic Expansion

- **Example 2: $k=0$:** flat universe. For ordinary (and dark) matter, $\rho \sim 1/a^3$, so $(\Delta a/\Delta t) \sim 1/a^{1/2}$, and $a(t) \sim t^{2/3}$. Hence, $d(t) \sim t^{2/3}$, and galaxies slow down over time due to gravity.

$$\left(\frac{1}{a} \frac{\Delta a}{\Delta t} \right)^2 + \frac{k}{a^2} = \frac{8\pi G \rho}{3}$$

Expansion rate Spatial Curvature Density of Matter

Dynamics of Cosmic Expansion

- **GR:** curvature of spacetime determined by mass-energy (Einstein equations).
- Apply this to the homogeneous and isotropic Universe: Friedmann equation:

$$H^2 = \left(\frac{1}{a} \frac{\Delta a}{\Delta t} \right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2}$$

Expansion rate

Matter Density

Spatial Curvature

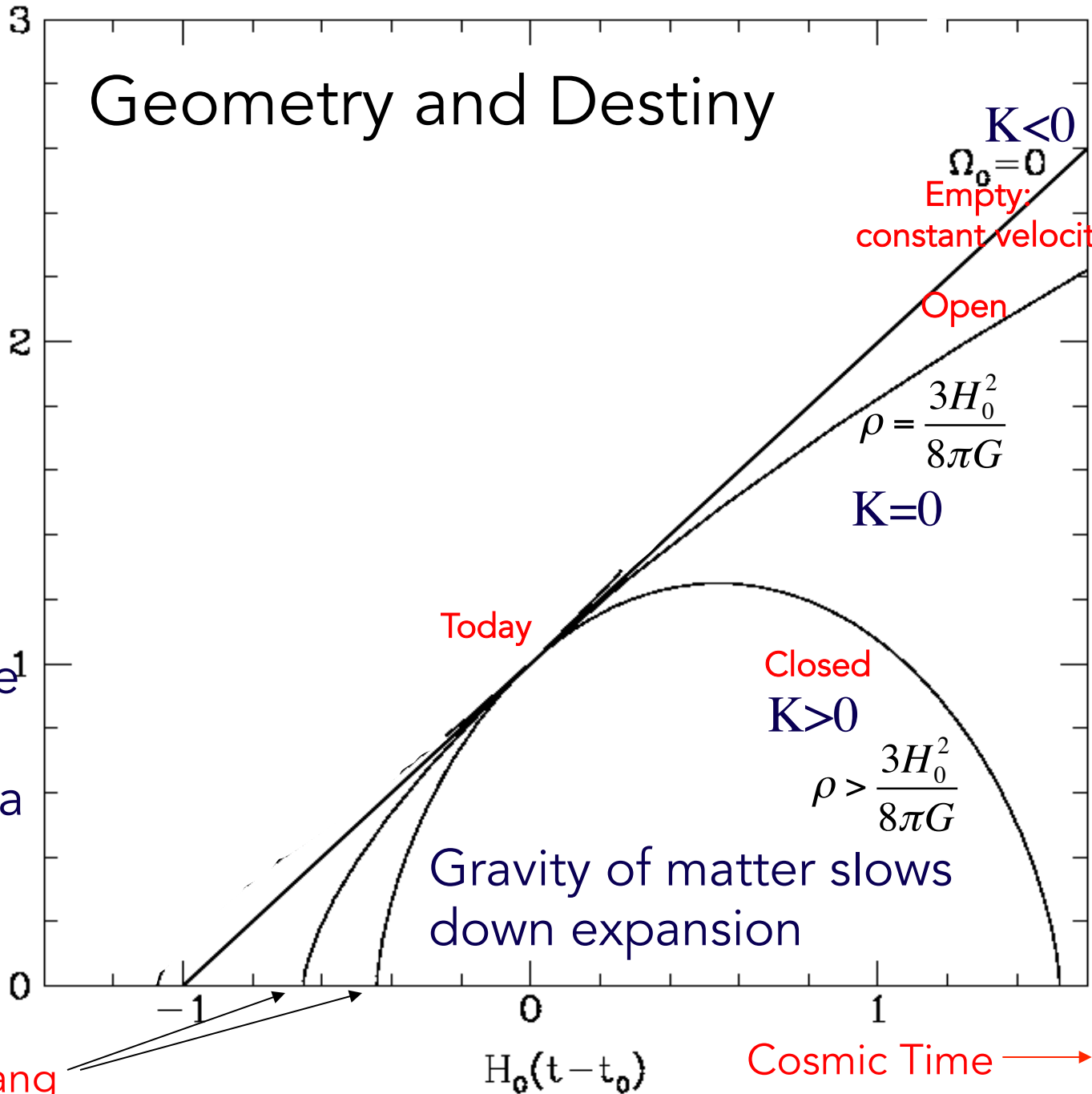
Geometry and Destiny

Size of the Universe



$$a(t)/a(t_0)$$

Will the Universe Expand forever or recollapse in a Big Crunch?



Big Bang

$$H_0(t-t_0)$$

Cosmic Time

Fate of the Universe

- Will the Universe expand forever or recontract to a Big Crunch?
- Is the mass density of the Universe smaller or larger than the **critical value**,

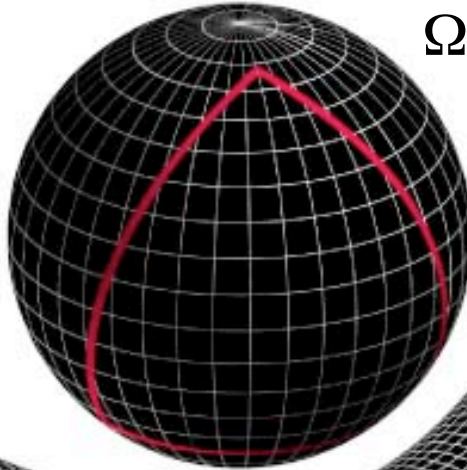
$$\rho_{crit} = \frac{3H_0^2}{8\pi G} = 2 \times 10^{-29} \text{ gm/cm}^3$$

- Density of atoms is well below this. But how much unseen—dark---matter is there? This question dominated discourse through the 1980's.

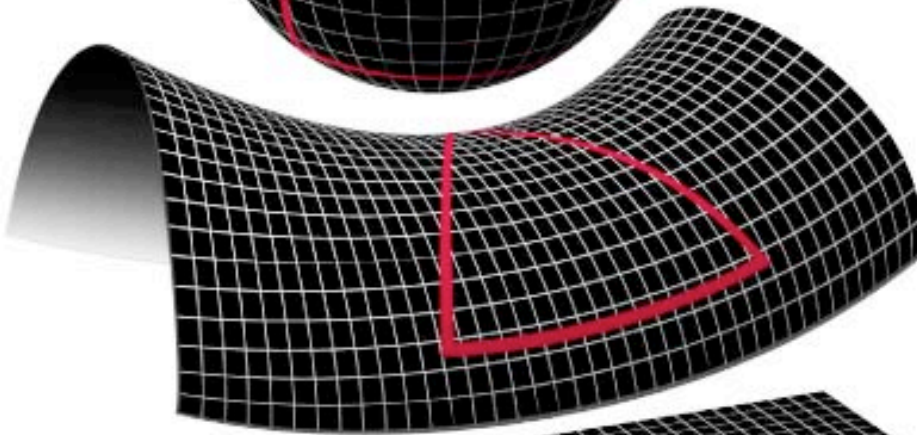
Geometry and Density

$$\Omega = \frac{\rho}{\rho_{crit}}, \text{ where } \rho_{crit} = \frac{3H_0^2}{8\pi G} = 2 \times 10^{-29} \text{ gm/cm}^3$$

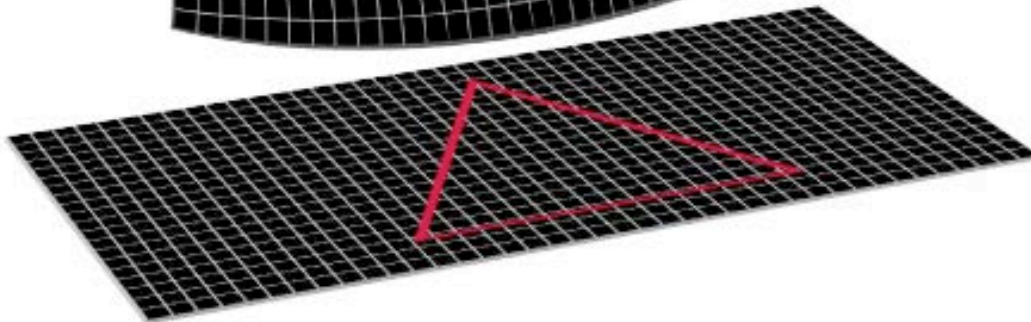
$\Omega_0 > 1$



$\Omega_0 < 1$



$\Omega_0 = 1$



Fate of the Universe

- Will the Universe expand forever or recontract to a Big Crunch?
- Is the mass density of the Universe smaller or larger than the **critical value**,

$$\rho_{crit} = \frac{3H_0^2}{8\pi G} = 2 \times 10^{-29} \text{ gm/cm}^3$$

- Density of atoms is well below this. But how much unseen—dark---matter is there? This question dominated discourse through the 1980's.
- Discovery of **cosmic acceleration** in 1998 changed the dynamics from this picture.

Dynamics of Cosmic Expansion

- **GR:** curvature of spacetime determined by mass-energy (Einstein equations).
- Apply this to the homogeneous and isotropic Universe: Friedmann equation:

$$H^2 = \left(\frac{1}{a} \frac{\Delta a}{\Delta t} \right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3}$$

Expansion rate

Matter Density

Spatial Curvature

Cosmological Constant

Dynamics of Cosmic Expansion

- **Example 3: $k=\rho=0$:** flat, empty universe with cosmological constant. In this case, $a(t) \sim \exp[(\Lambda/3)^{1/2}t]$. Universe grows exponentially: cosmic acceleration. Even for non-zero k and ρ , this is the asymptotic solution in the future.

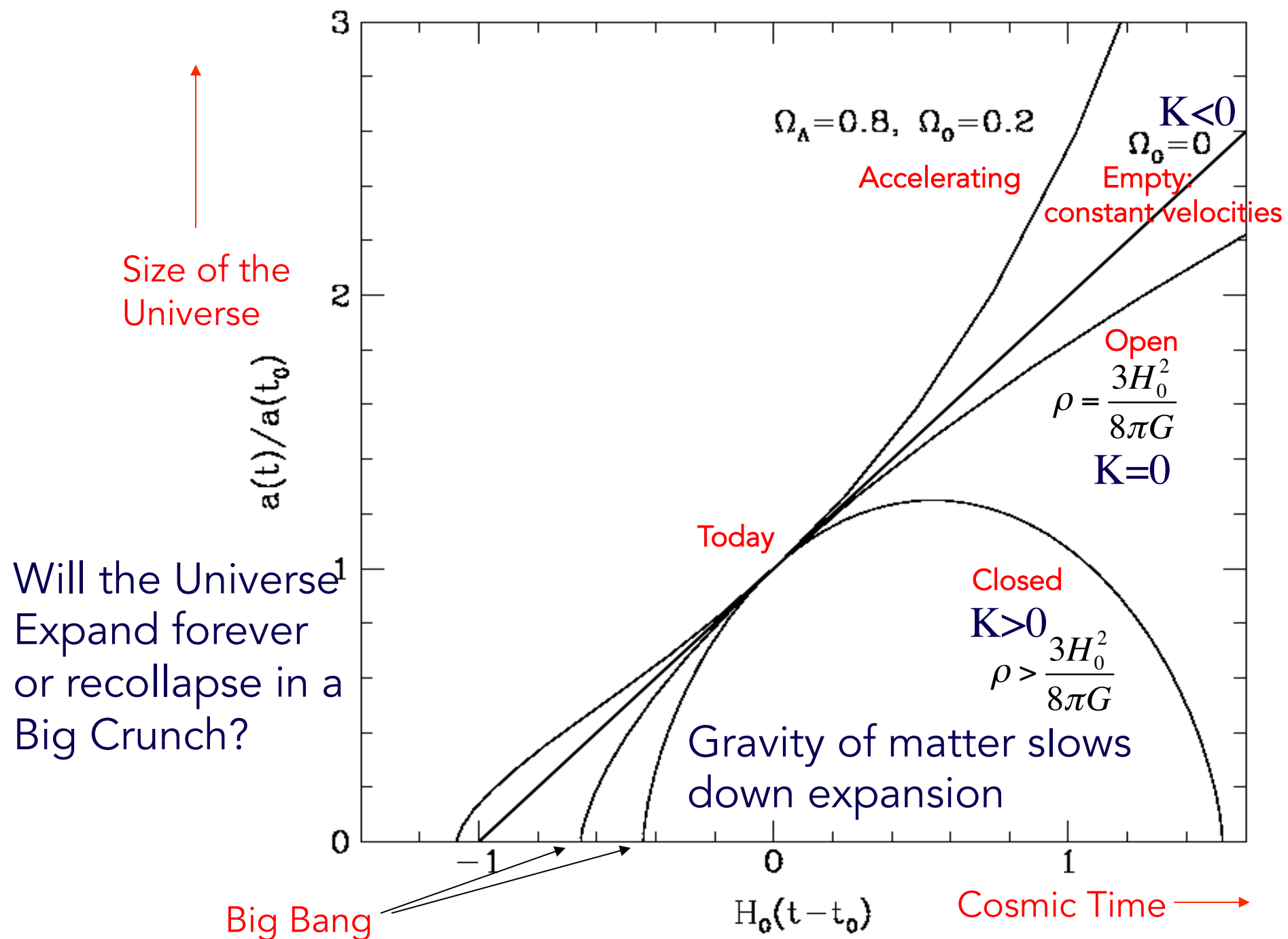
$$H^2 = \left(\frac{1}{a} \frac{\Delta a}{\Delta t} \right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3}$$

Expansion rate

Matter Density

Spatial Curvature

Cosmological Constant

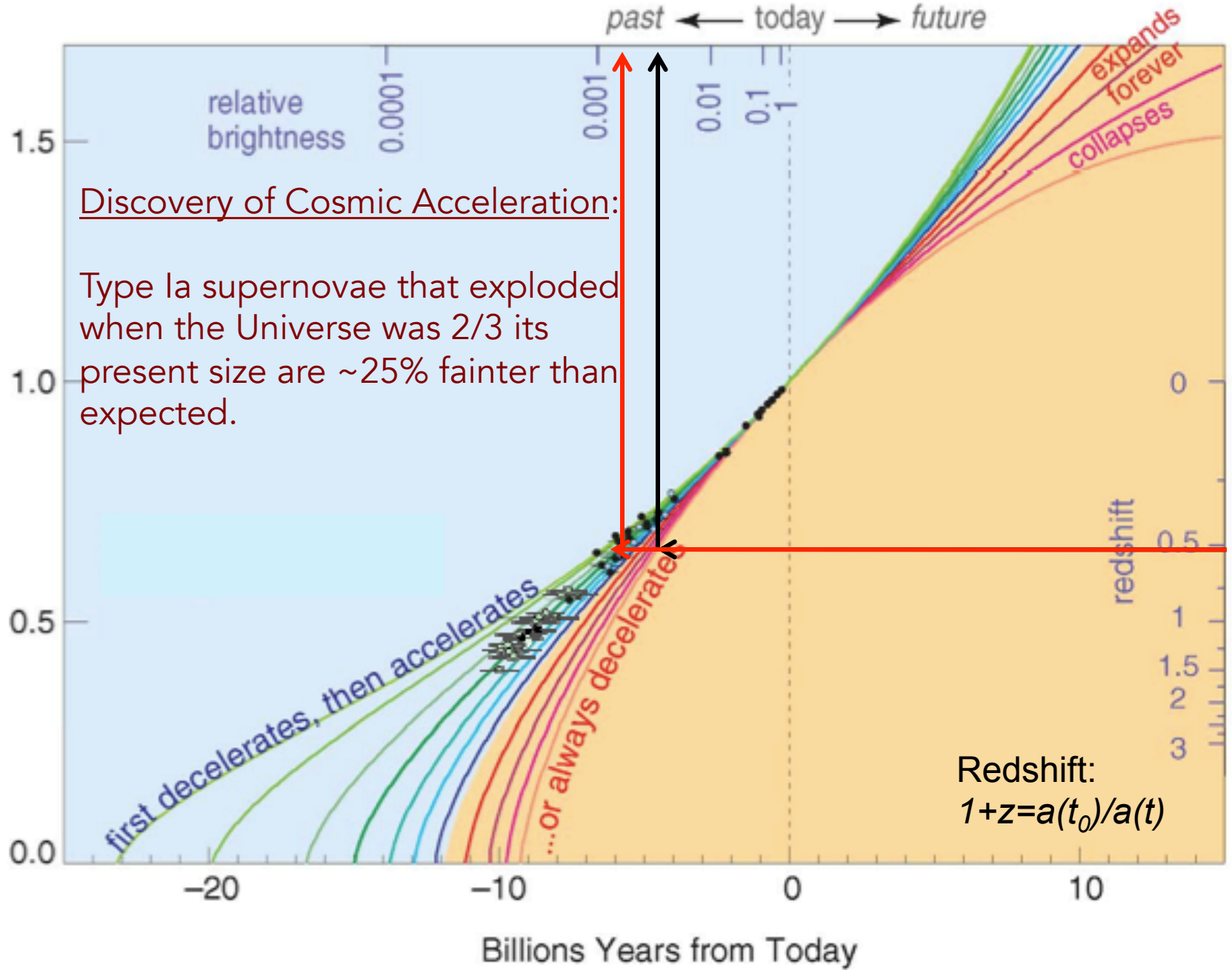


Expansion History of the Universe

Supernova Data (1998)

$$\frac{a(t)}{a(t_0)}$$

Average Distance Between Galaxies
Relative to Today's Average



Discovery of Cosmic Acceleration:

Type Ia supernovae that exploded when the Universe was 2/3 its present size are ~25% fainter than expected.

Redshift:
 $1+z = a(t_0)/a(t)$

Cosmic Acceleration

- Throughout the 20th Century, cosmologists attempted to measure the deceleration (slowing) of the expansion due to gravity, but the measurements were inconclusive.
- **1998**: Two teams of astronomers, using type Ia supernovae as standard candles, found that instead the expansion is *speeding up* (Nobel Prize 2011).
- To explain this, we either need to modify our understanding of how gravity works on cosmological scales or else invoke a dominant, new component of the Universe---**dark energy**---that has very different properties from ordinary matter.

Historical Note: Einstein Static Universe

- Friedmann equations:

$$H^2 = \left(\frac{1}{a} \frac{\Delta a}{\Delta t} \right)^2 = \frac{8\pi G \rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3}$$

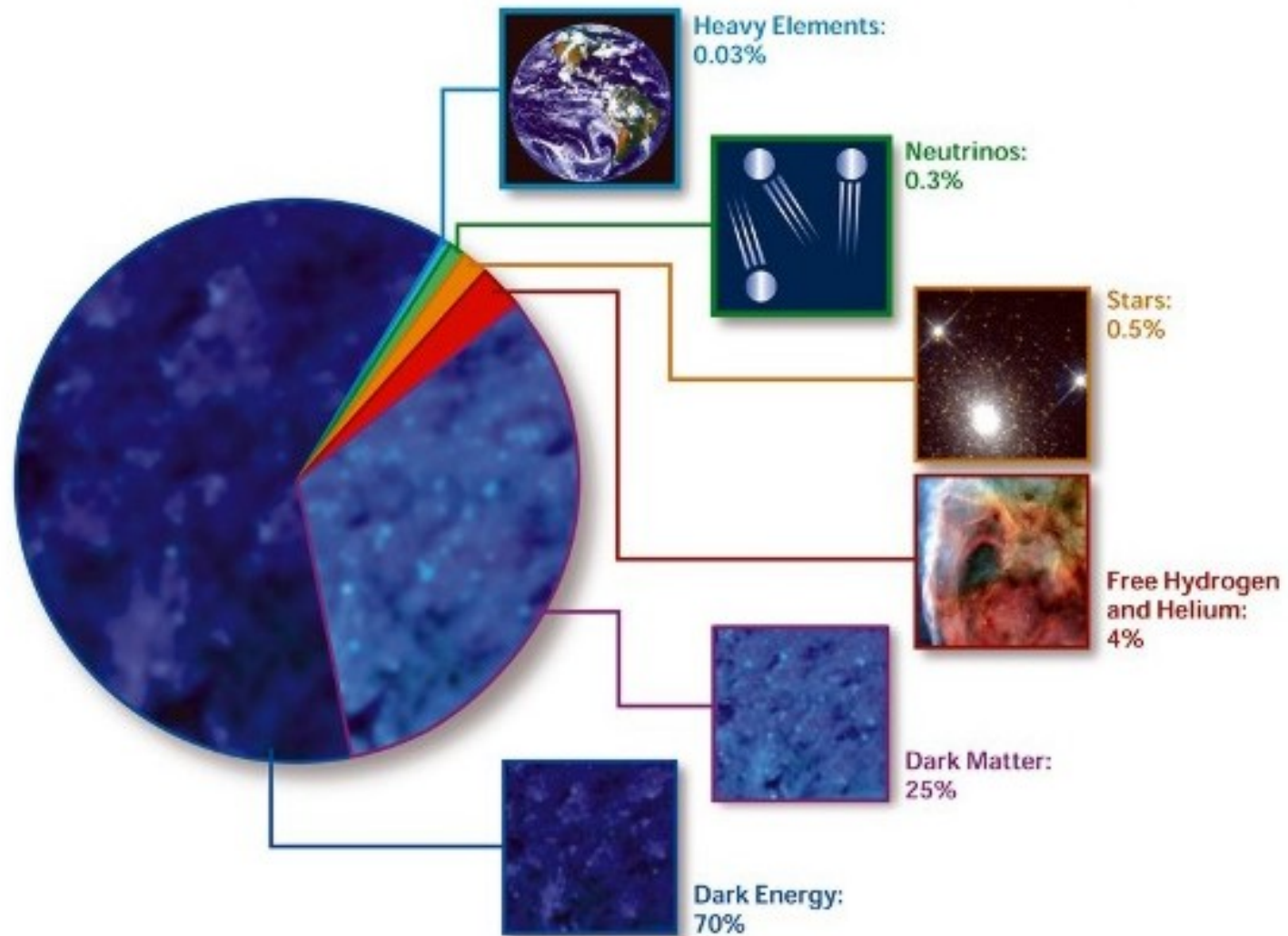
$$\frac{1}{a} \frac{\Delta}{\Delta t} \left(\frac{\Delta a}{\Delta t} \right) = -\frac{4\pi G}{3} \rho + \frac{\Lambda}{3}$$

- For static solution: $\rho = \Lambda / 4\pi G = 1 / 4\pi G a_0^2 = \text{constant}$
- Problem: this solution is unstable and will eventually expand or contract.

Contents of the Universe

- To determine the evolution of the Universe, we need to know **what forms** of matter and energy there are and **how much** there is of each:
 - Ordinary matter (stars, gas, planets,..., made of atoms)
 - Neutrinos
 - Electromagnetic Radiation (Cosmic Microwave Background)
 - Dark Matter
 - Dark Energy
 - ...

Contents of the Universe



Non-relativistic Matter

- Includes ordinary matter (stars, gas, planets, ..., anything made of atoms), dark matter, massive neutrinos: anything that moves slowly compared to the speed of light c

- Recall Einstein's famous equation:

$$E=mc^2=m_0c^2/(1-v^2/c^2)^{1/2}$$

where m_0 is the 'rest mass'

- For a slowly moving particle, $v \ll c$, and $E \approx m_0c^2$.
- In this case, the energy per unit volume is

$$\rho = nE = nm_0c^2$$

where n is the number of particles per unit volume

Non-relativistic Matter

- Now assume that particles are not created or destroyed. Then the total number of particles in a volume V is fixed:

$$N = nV = \text{constant}$$

- Thus the number density of particles obeys

$$n \sim 1/V$$

- In the expanding Universe, the volume $V \sim a^3$ so that

$$n \sim 1/a^3$$

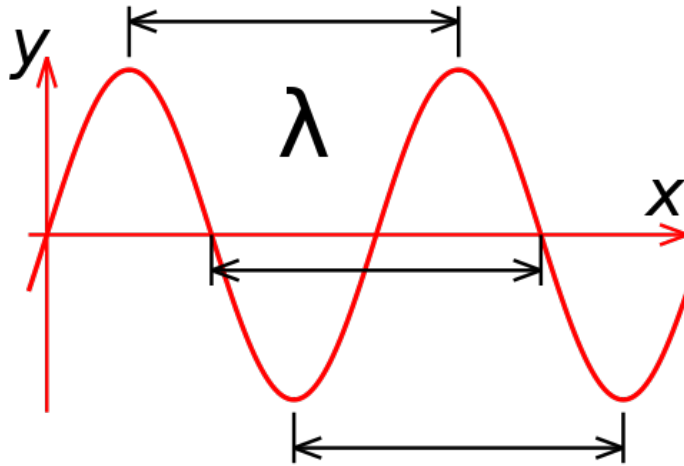
and therefore for non-relativistic matter:

$$\rho = nm_0c^2 \sim 1/a^3$$

Radiation (Relativistic Particles)

- Includes electromagnetic radiation (light) and any other stuff that moves at or very near the speed of light (e.g., gravity waves or extremely light particles).

Light is an Electromagnetic Wave



λ – wavelength –
distance between wavecrests

$\lambda \nu = c$
(speed of light)

ν – frequency
 $\nu = 1/\Delta t =$
1/period (time
between wavecrests)
Unit: Hz = 1/second

Energy per photon:
 $E_\gamma = h\nu = hc/\lambda$ (quantum)
Since expansion stretches
wavelength, $\lambda \sim a(t)$ (redshift), it
follows that $E_\gamma \sim 1/a(t)$

Radiation

- Now assume that radiation (e.g., photons) is not created or destroyed. Then the total number of particles in a volume V is fixed:

$$N = nV = \text{constant}$$

- Thus the number density of particles obeys

$$n \sim 1/V$$

- In the expanding Universe, the volume $V \sim a^3$ so that

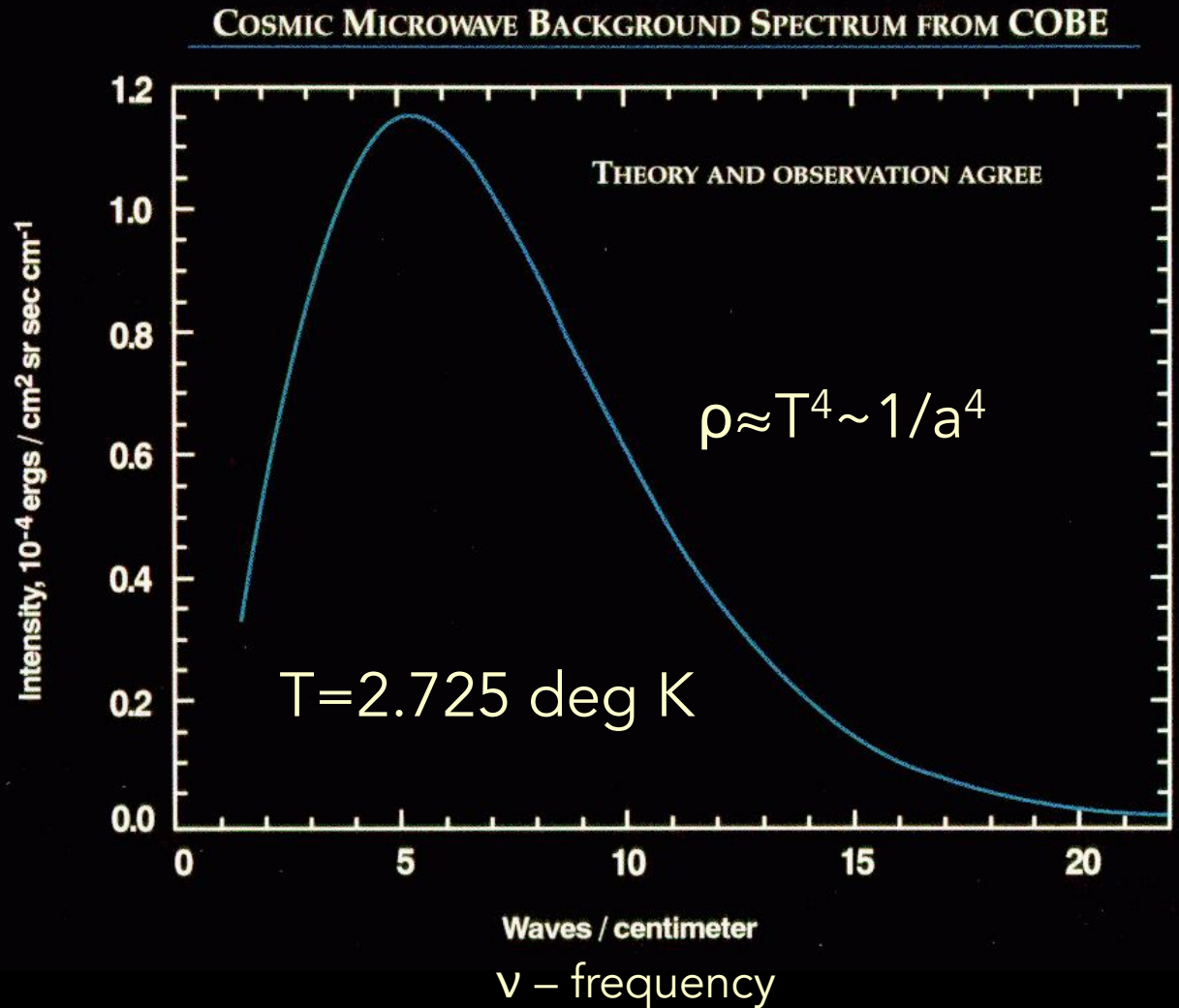
$$n \sim 1/a^3$$

and therefore for radiation:

$$\rho = nE_\gamma \sim (1/a^3)(1/a) \sim 1/a^4$$

Cosmic Microwave Background

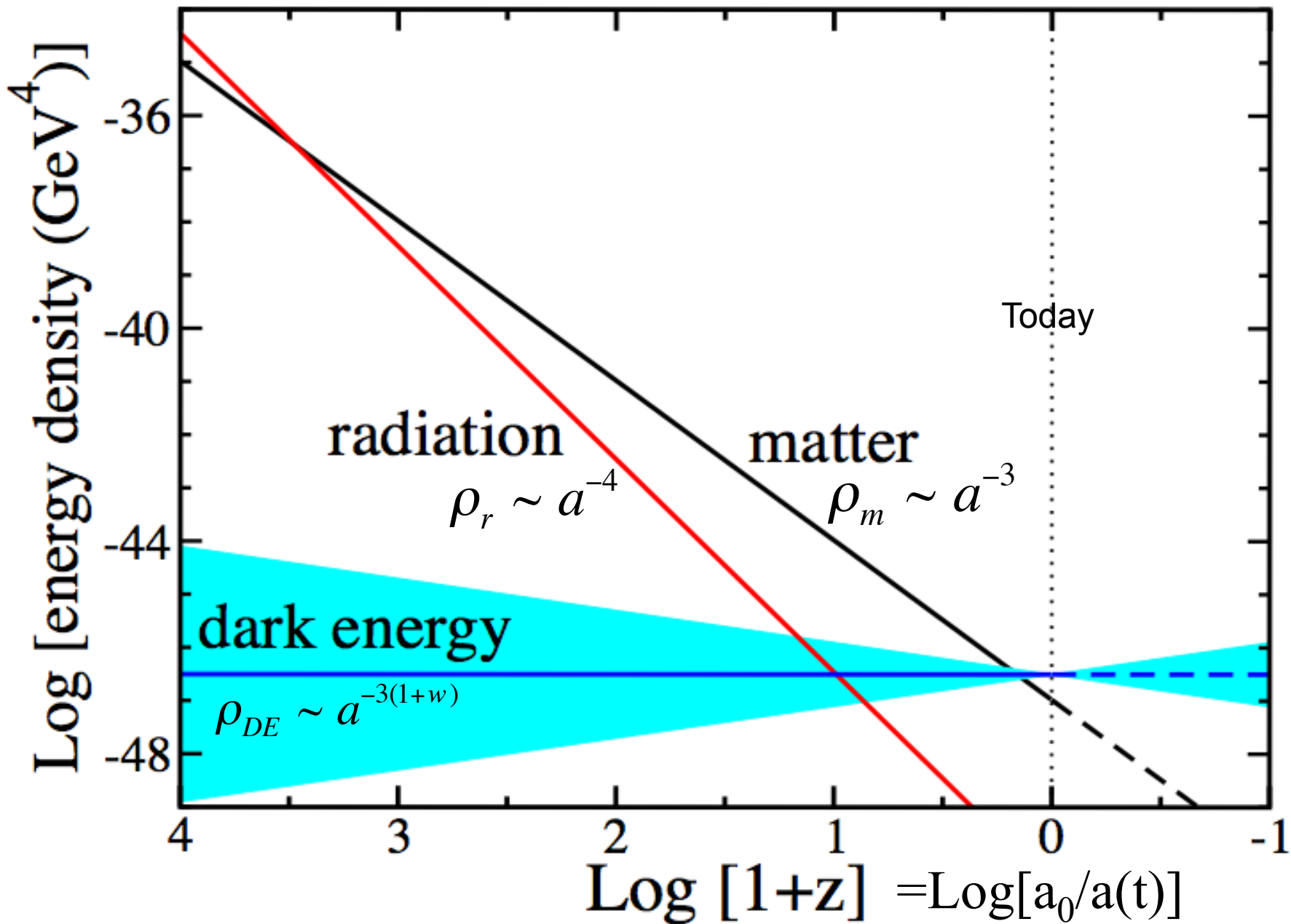
- Universe is filled with thermal electromagnetic radiation: the Cosmic Microwave Background (CMB) radiation, remnant from the hot early Universe.



Radiation dominated

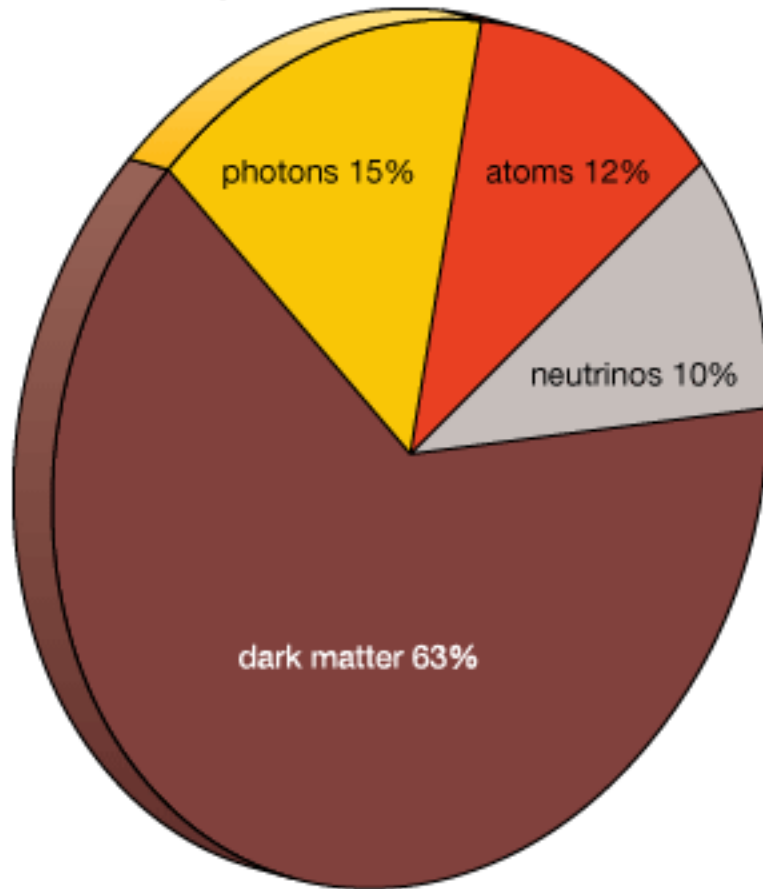
Matter dominated

Dark Energy dominated

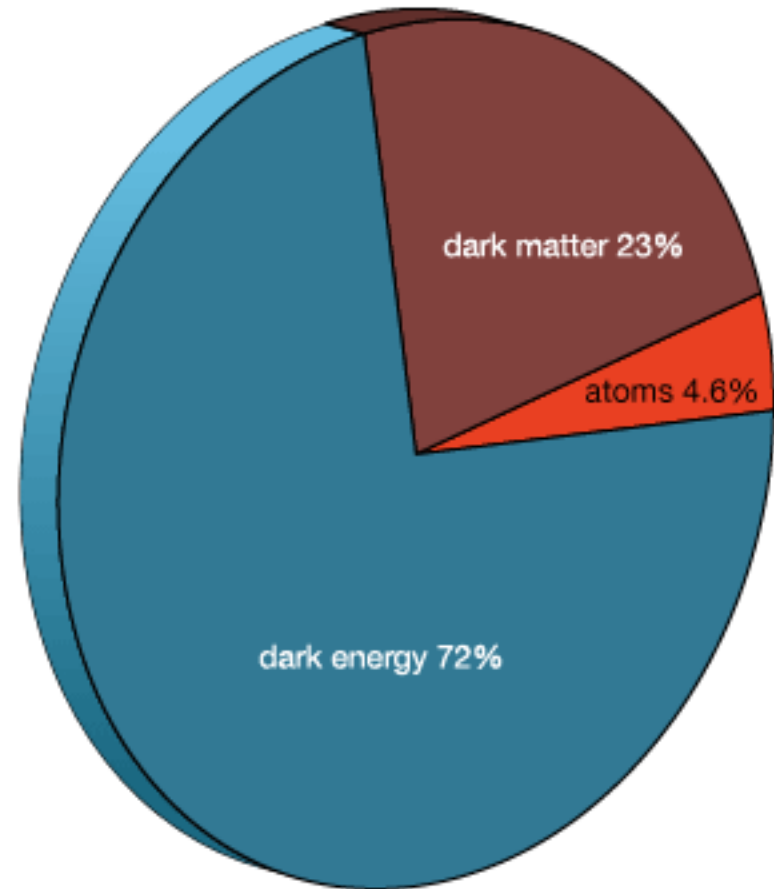


Components Then and Now

Matter-energy content of the universe



13.7 billion years ago
(universe 380,000 years old)



today

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Dark Matter

- A component that does not interact with (emit or absorb) light but whose presence is inferred from its gravitational effect on luminous matter or light.
- 1930's: initial evidence for dark matter (clusters)
- 1970's-80's: mounting evidence for dark matter (spiral galaxy rotation curves)
- 1990's-2000's: confirmation via gravitational lensing and cosmological measurements

Clusters of Galaxies: Size ~ few Million light years
Mass ~ 1 quadrillion (1000 x 1 trillion) M_{sun}
Contain ~10s to 1000s of galaxies
Evolution time scale: ~few billion years

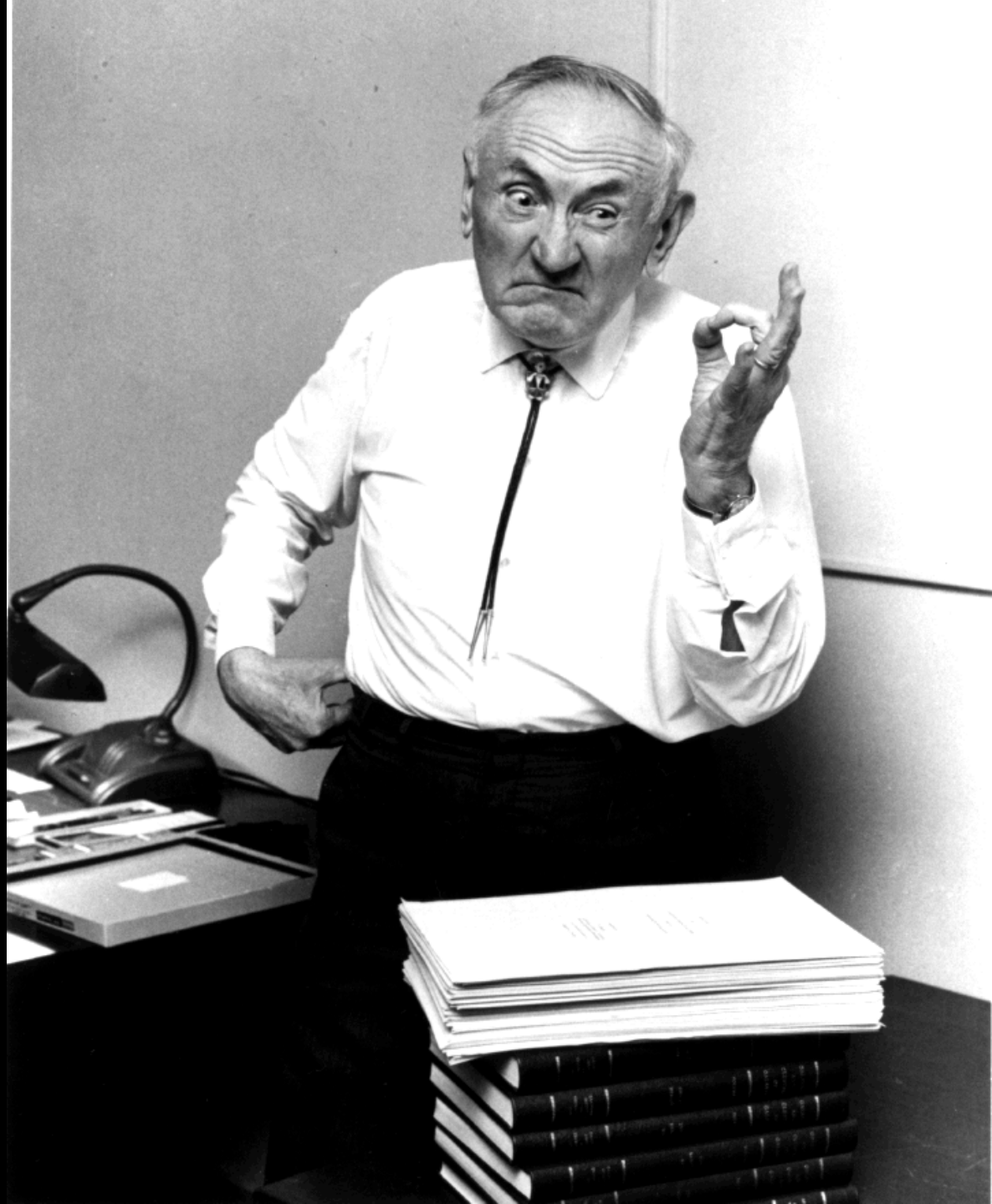




Coma Cluster of Galaxies

Fritz Zwicky (1898-1974)

1930's: studied the motions of galaxies within the Coma cluster, found they are moving too fast, ~ 1000 km/sec, to remain confined by Coma's gravitational field. Why is Coma still there?



Dark Matter (F. Zwicky)

- The galaxies in Coma cluster are moving around faster than we can explain.
- The gravity of something *that we can't see* must be keeping the galaxies from flying off into space: **Dark Matter**
- Clusters are mostly made of **dark matter**: galaxies are like sprinkles on dark matter ice cream.
- We know **dark matter** is there because it exerts gravitational pull on the galaxies we can see in clusters.

How do we determine Masses of Astrophysical Objects?

Recall Newtonian acceleration:

$$a = \frac{F}{m} = \frac{GMm}{r^2 m} = \frac{GM}{r^2}$$

For body in circular motion,

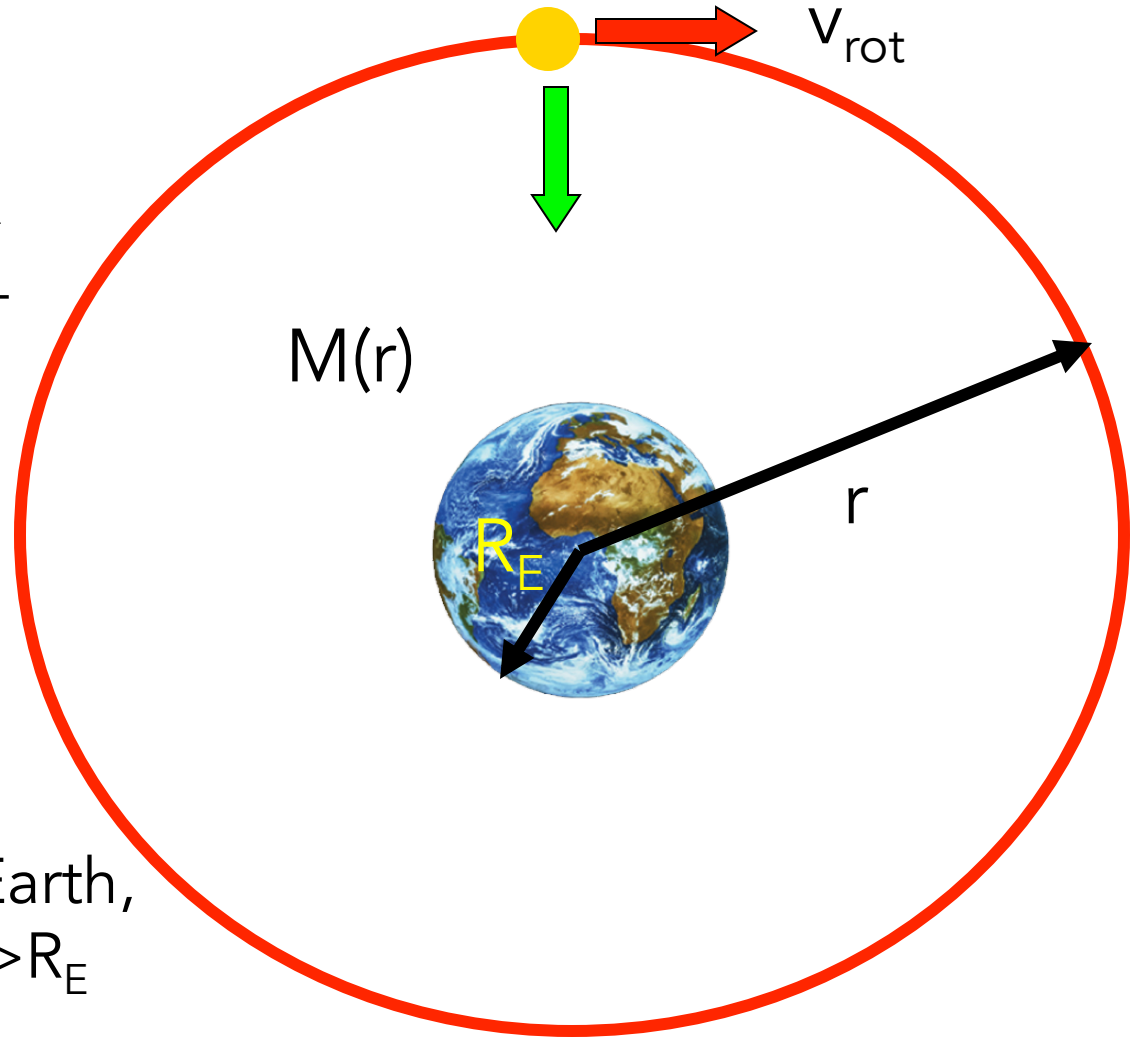
$a = v_{\text{rot}}^2 / r$, so

$$v_{\text{rot}}^2 = GM(r) / r$$

For satellite around Earth,

$M(r > R_E) = \text{const.}$ for $r > R_E$

$$v_{\text{rot}}(r > R_E) \propto r^{-1/2}$$



How do we determine Masses of Astrophysical Objects?

Recall Newtonian acceleration:

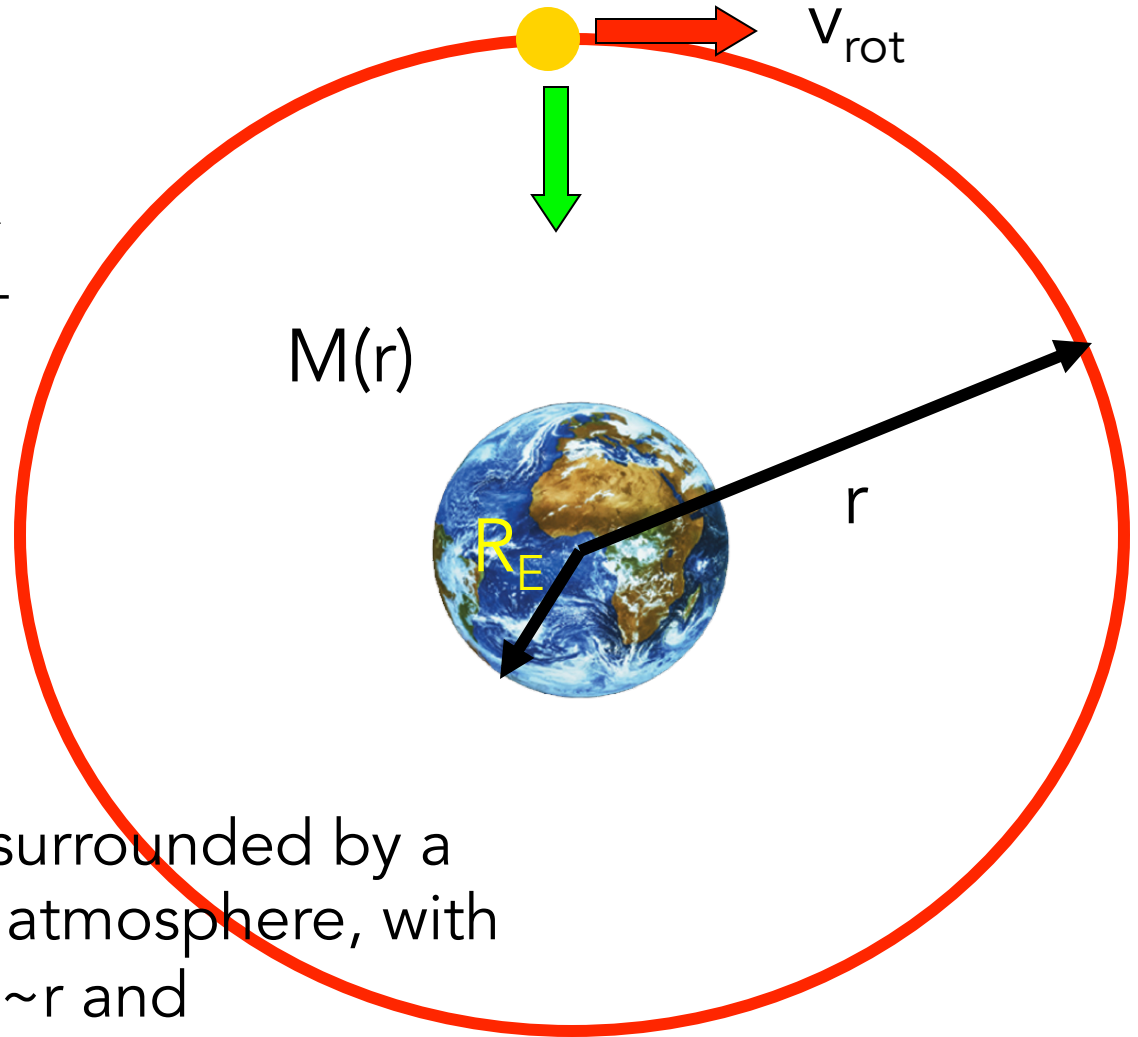
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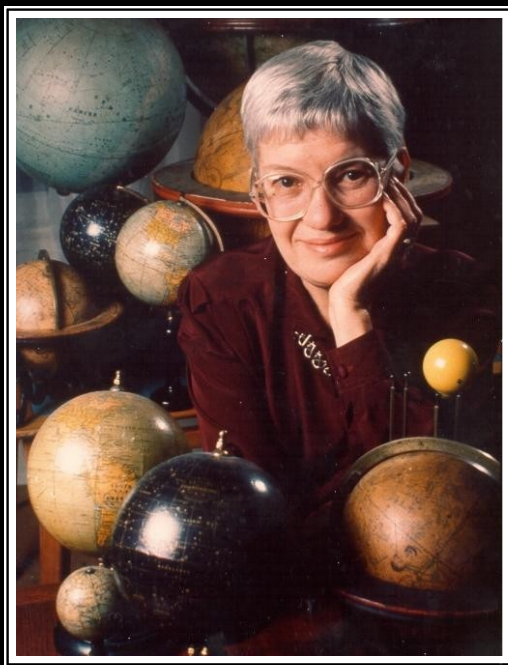
For body in circular motion,

$a = v_{\text{rot}}^2 / r$, so

$$v_{\text{rot}}^2 = GM(r) / r$$

Suppose earth were surrounded by a much thicker, denser atmosphere, with $\rho \sim 1/r^2$. Then $M(r > R_E) \sim r$ and $v_{\text{rot}}(r > R_E) \propto \text{constant}$

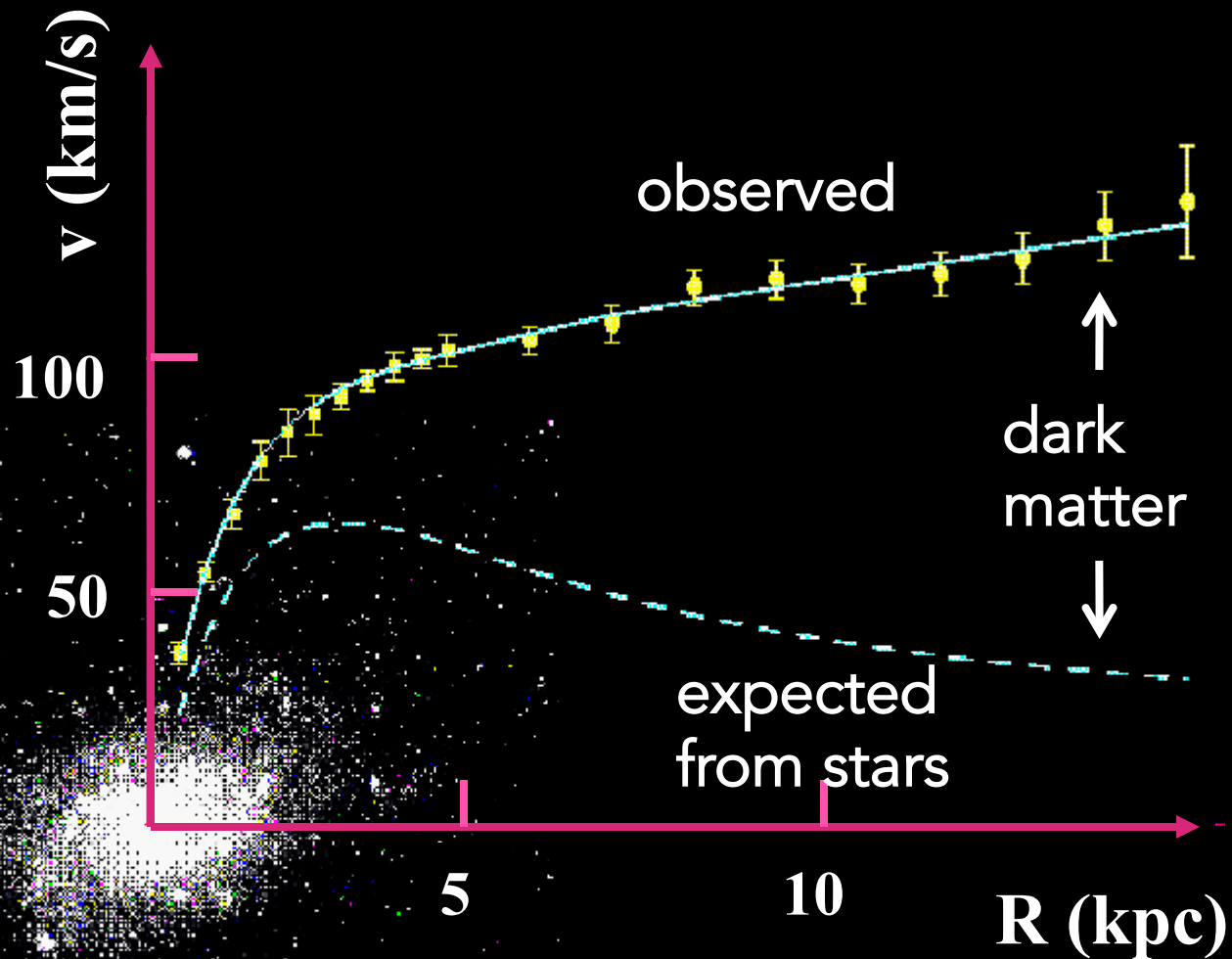




Vera Rubin
(1970's)

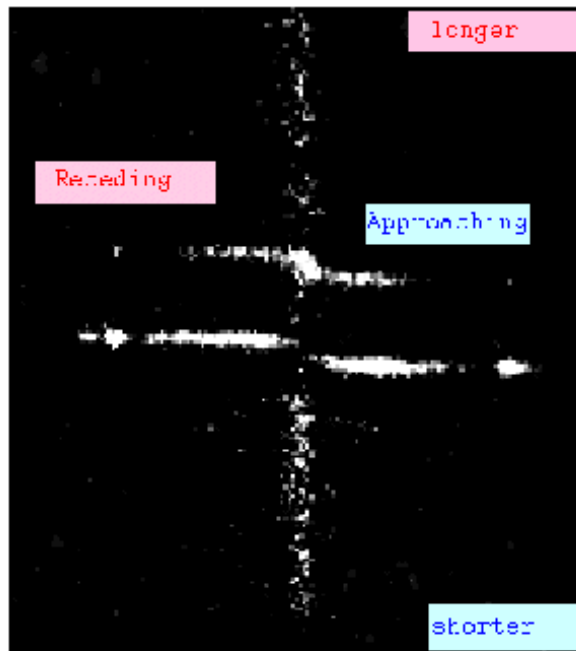
Galaxies
surrounded
by halos of
dark matter

Rotation of Stars around Galaxies



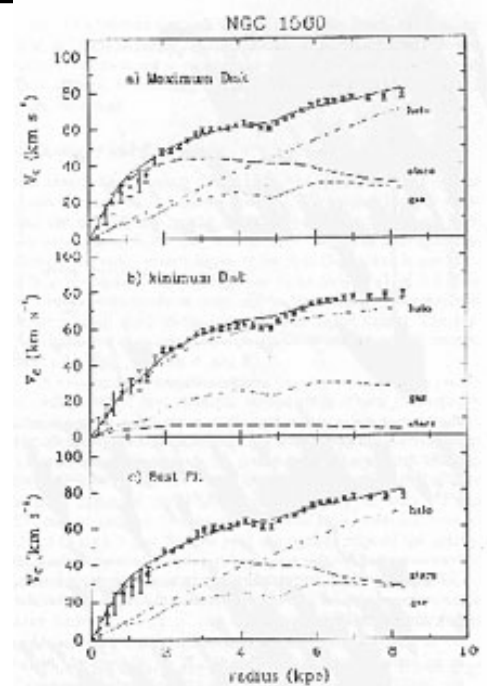
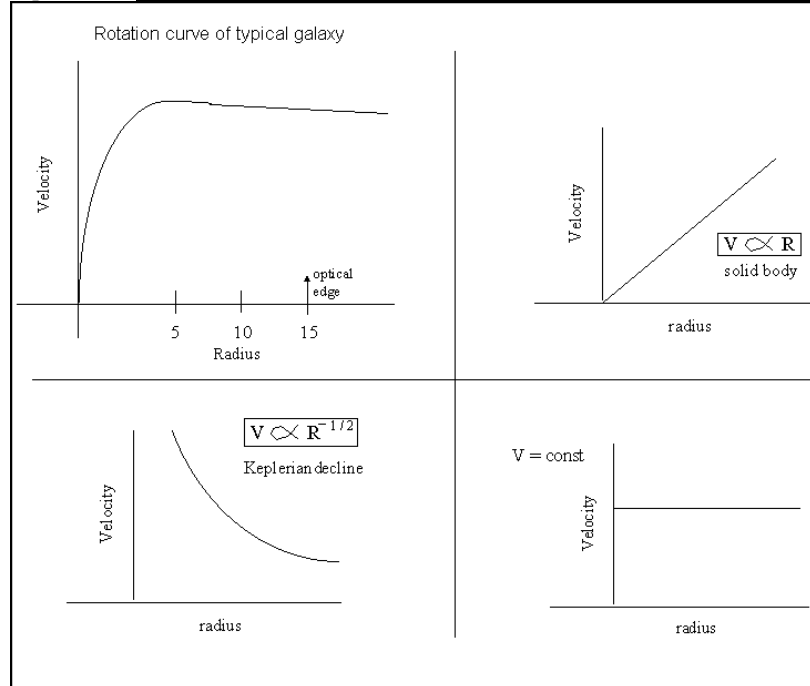
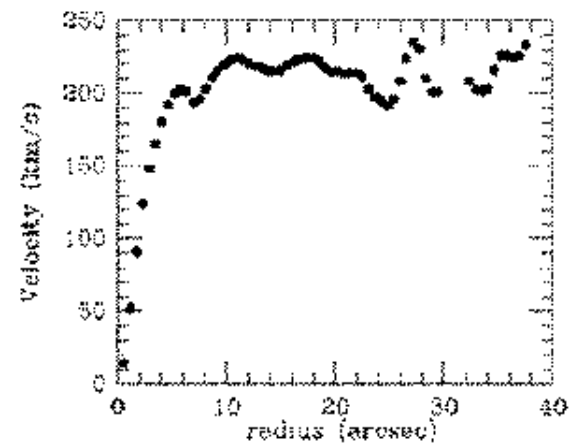
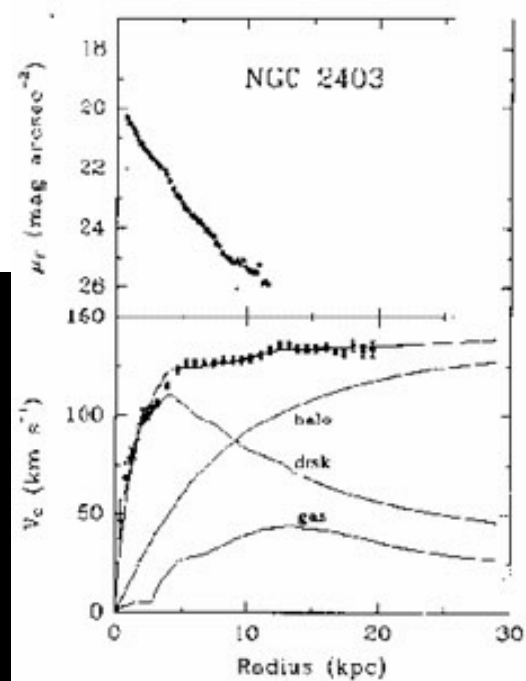
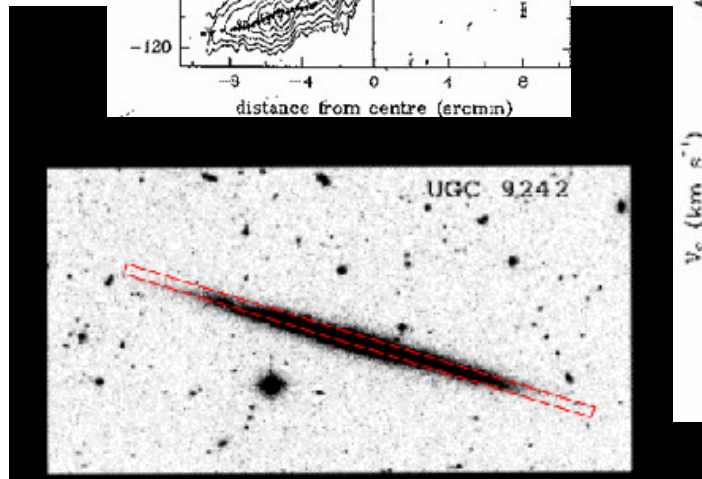
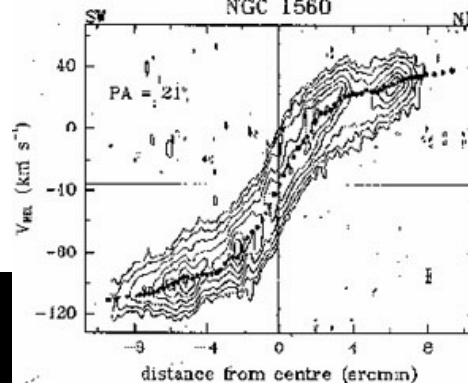
M33 rotation curve
(contrast Solar System)

Galaxy rotation curve



Distance along galaxy major axis →

↑ WAVELENGTH ↓



Dark Matter (V. Rubin)

- The stars in a galaxy are moving around faster than we can explain.
- The gravity of something *that we can't see* must be keeping the stars from flying off into space: **Dark Matter**
- Galaxies are mostly made of **dark matter**: stars are like sprinkles on dark matter ice cream.
- We know **dark matter** is there because it exerts gravitational pull on the stars we can see in galaxies.