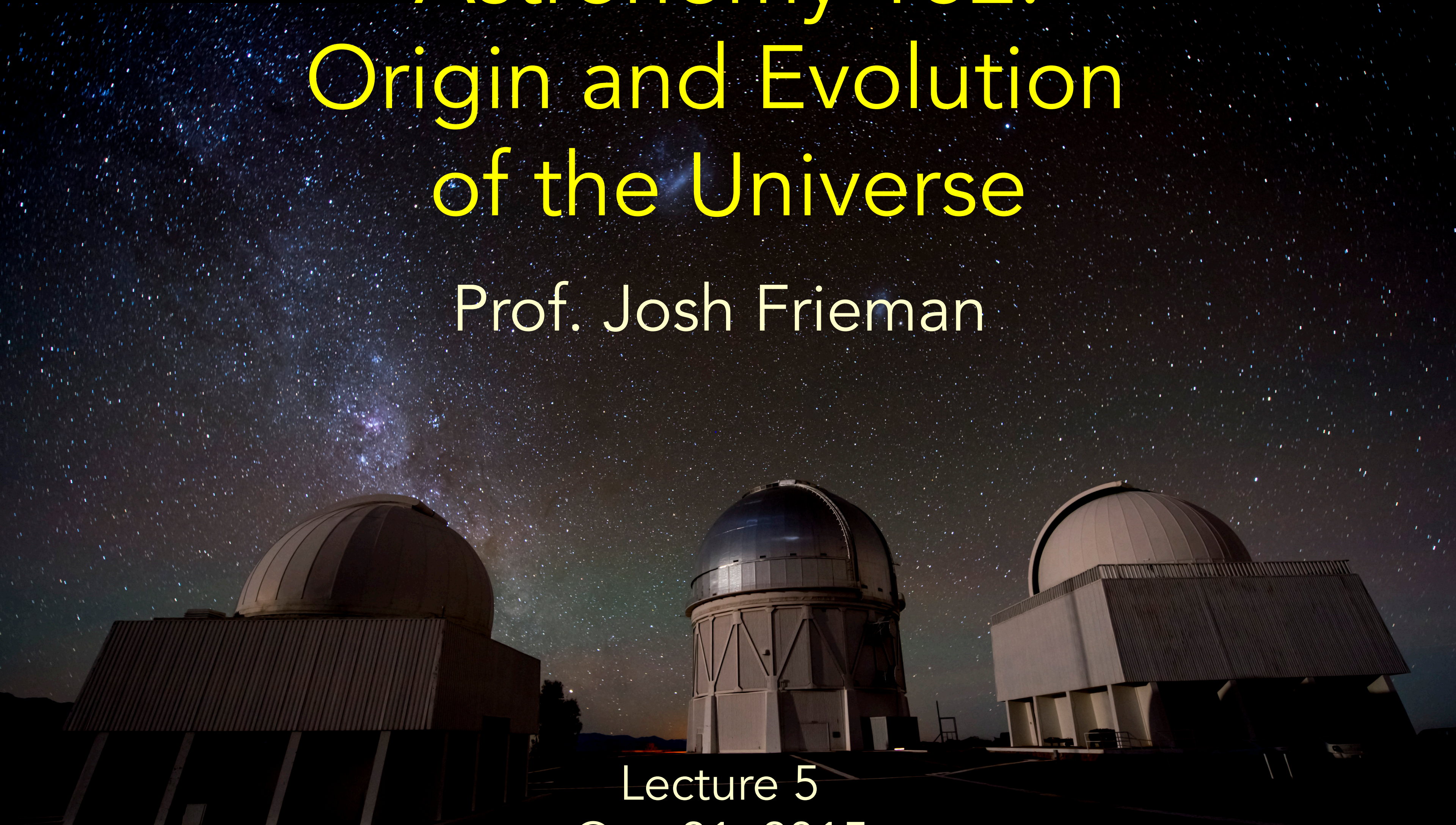
Astronomy 182: Origin and Evolution of the Universe

Prof. Josh Frieman



Lecture 5
Oct. 21, 2015

Today

- Wrap up of Einstein's Special Relativity
- General Relativity
  - Principle of Equivalence
  - Curved Spacetime
  - Black Holes

Assignments

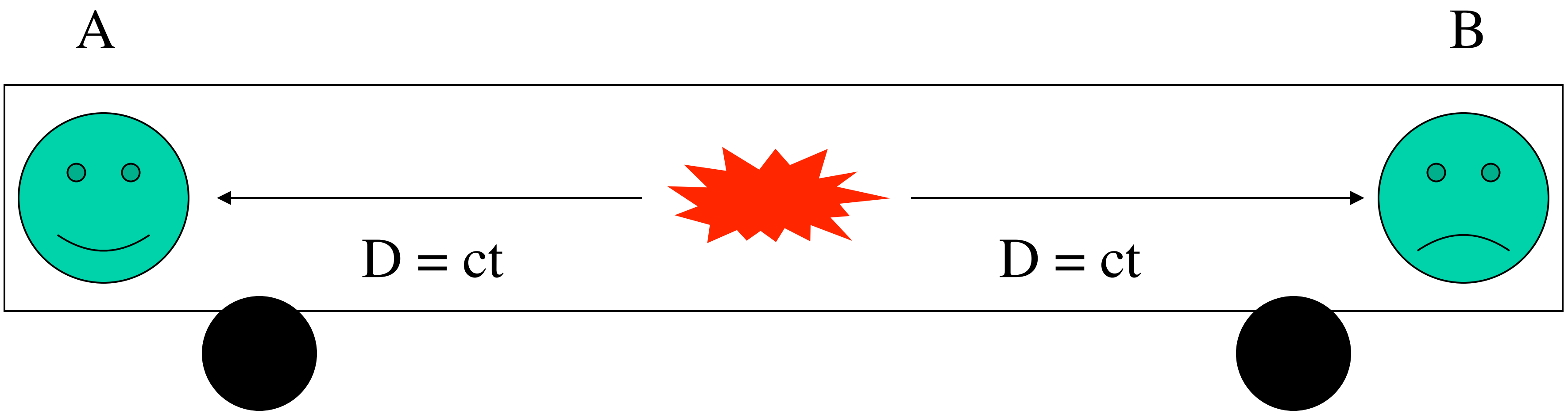
- Today: Essay due on Hawley and Holcomb, Chapter 10.
- This week and next: read Hawley and Holcomb, Chapters 7-9.
- This Friday: Lab 2 in class with Jason.
- Next Fri., Oct. 30: Lab 2 write-up due in class.

Special Relativity (1905)

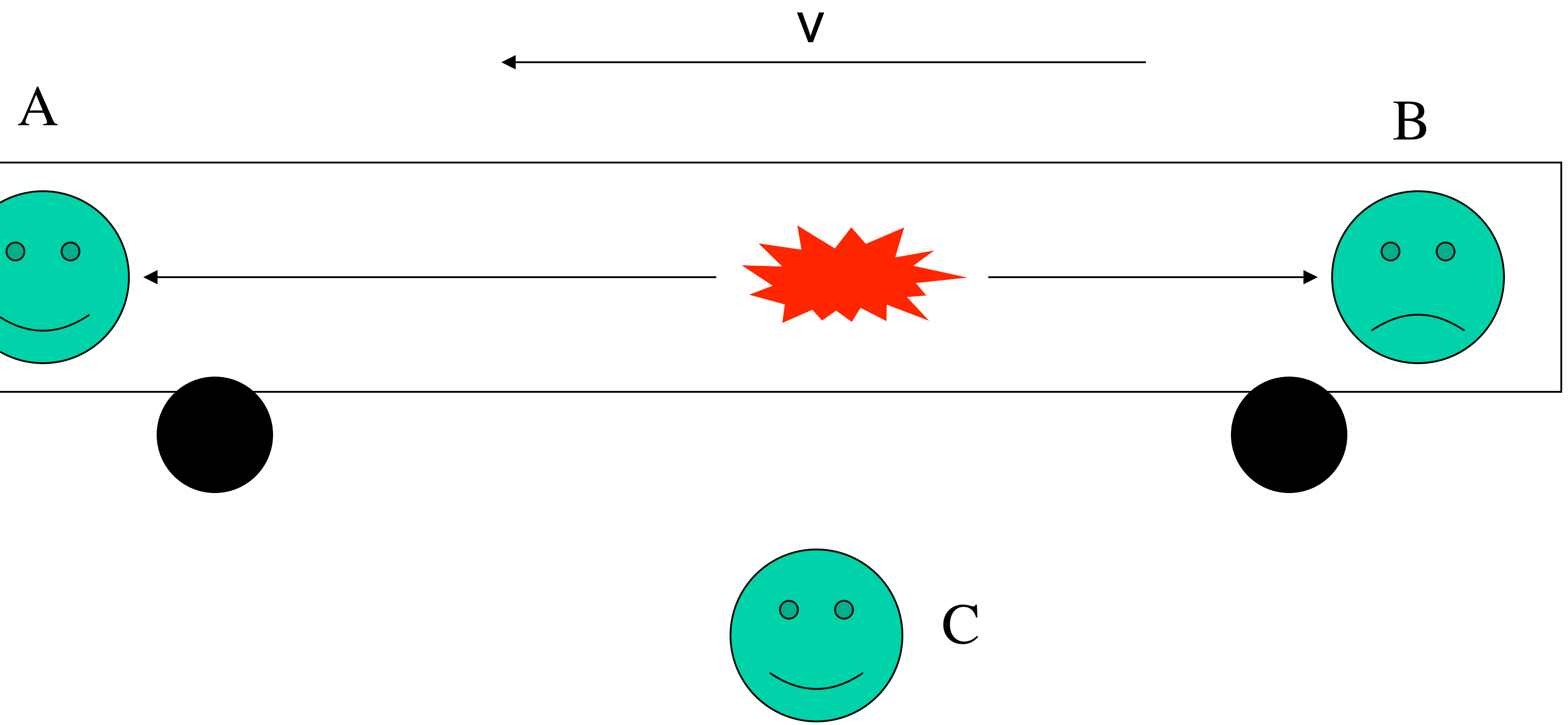
- Grew out of Einstein's aim to resolve Maxwell's Theory of Electromagnetism, and constancy of the speed of light, with Newtonian physics.

- Special Relativity Postulates:
  - Principle of Relativity: constant-velocity, aka inertial, motion is relative. The Laws of physics must be identical for all inertial observers.
  - The speed of light is constant for all inertial observers.

As seen by observers on the train, light takes same amount of time to reach each end of the car.



As seen by observer C at rest on the platform, light takes more time to reach A than B, since it travels a longer distance

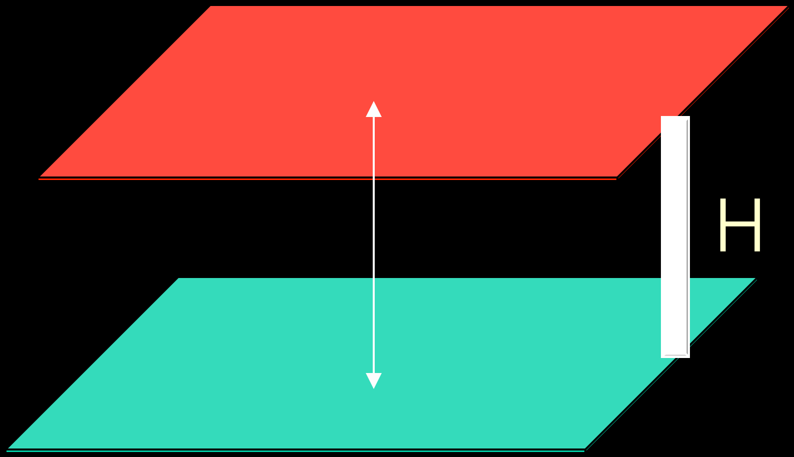


Time: relativity of simultaneity

- Who's right, those on the train or on the ground?
- According to the Principle of Relativity, they both are:

- Simultaneity is relative. Observers in relative motion do not agree on which events occur at the same time.

- Einstein: this means their conceptions of time must differ.
- How do we measure time?

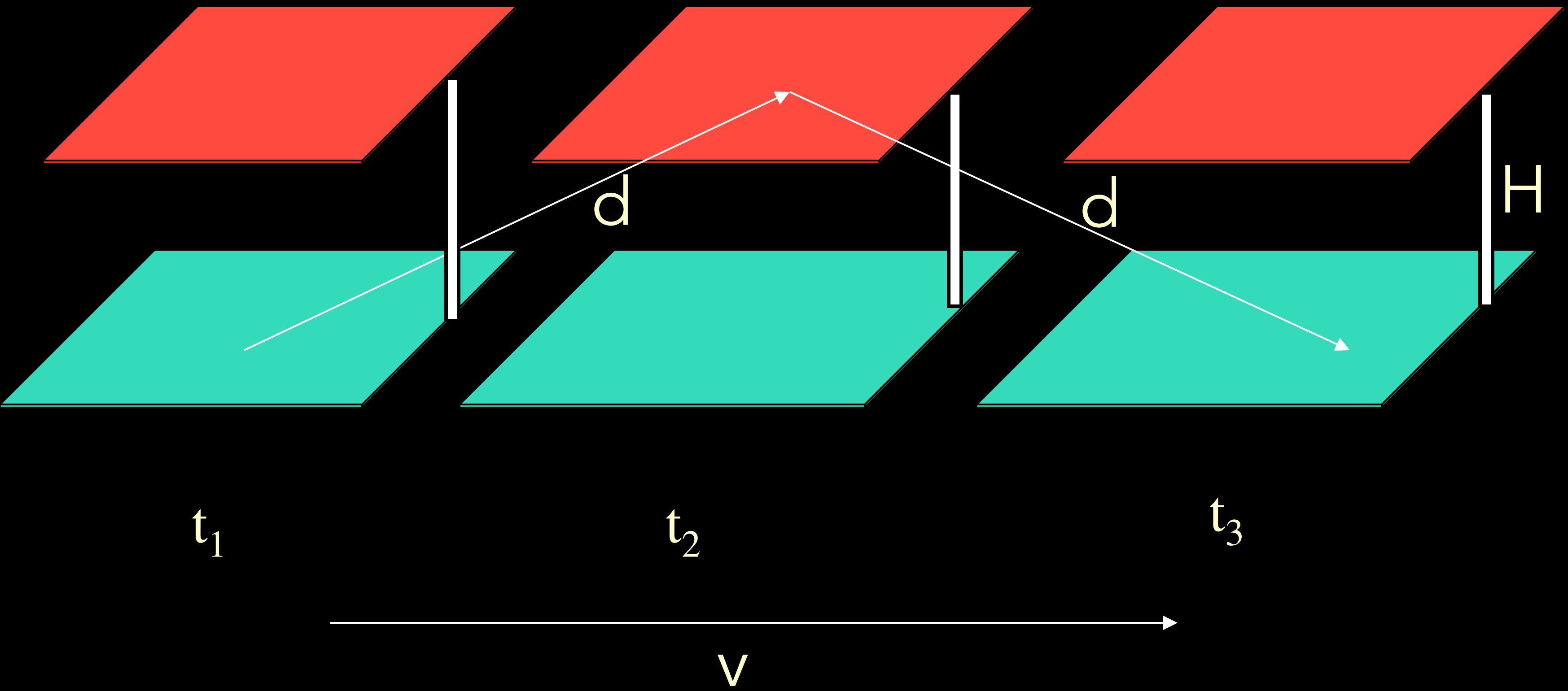


Light Clock: ticks every time a light particle (photon) bouncing between the fixed walls makes one complete vertical circuit. It ticks when $ct=2H, 4H, 6H, \ldots$, so the time between ticks is $\Delta t=2H/c$

Let's assume observers A, B, and C have such clocks.

Now consider the light clock held by observer A on the train, moving past stationary observer C: according to C, the photon in the moving clock travels a longer round-trip path. Since speed of light is constant, it takes longer between ticks:

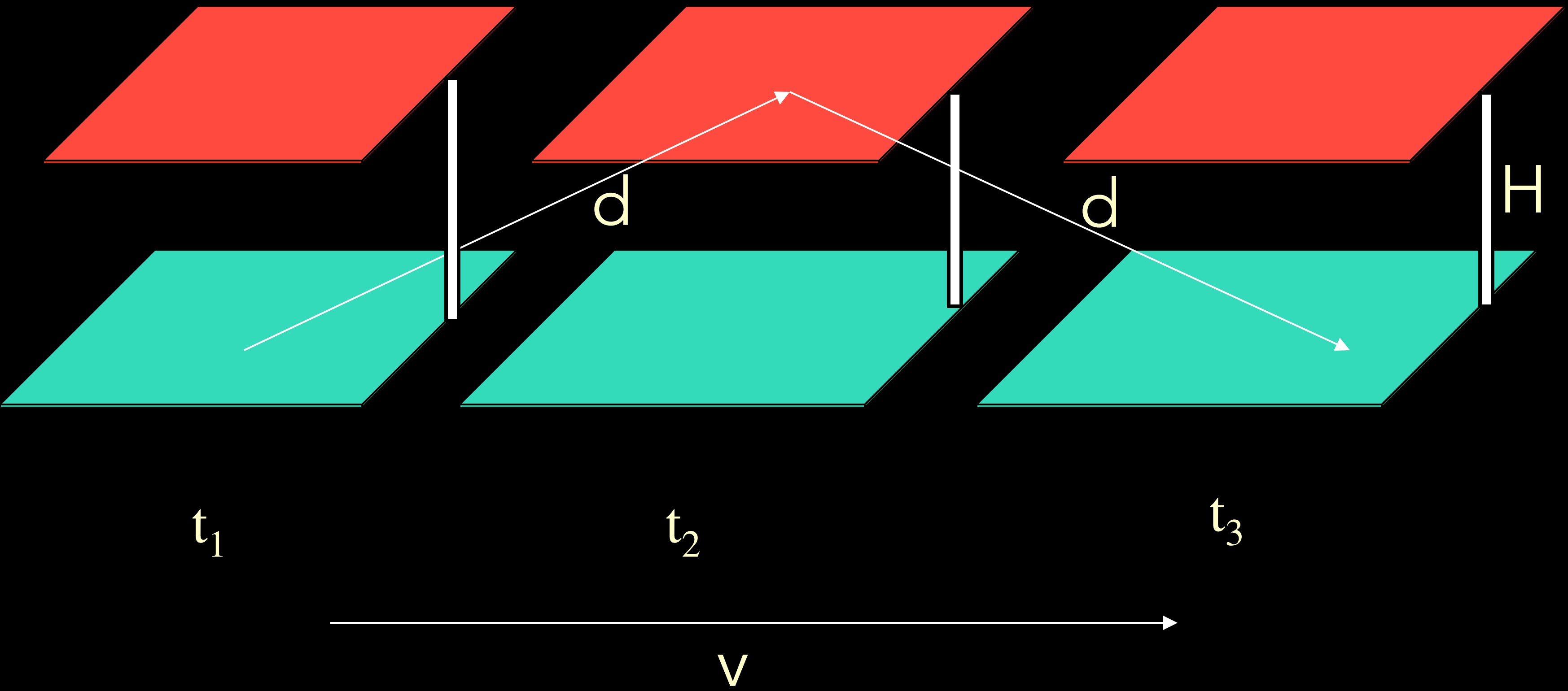
<u>the moving clock slows down according to the stationary observer (and vice versa according to Principle of Relativity)</u>



According to Observer C on the platform, the time between ticks of A's clock has increased to

$$\Delta t_C = 2d/c > 2H/c$$

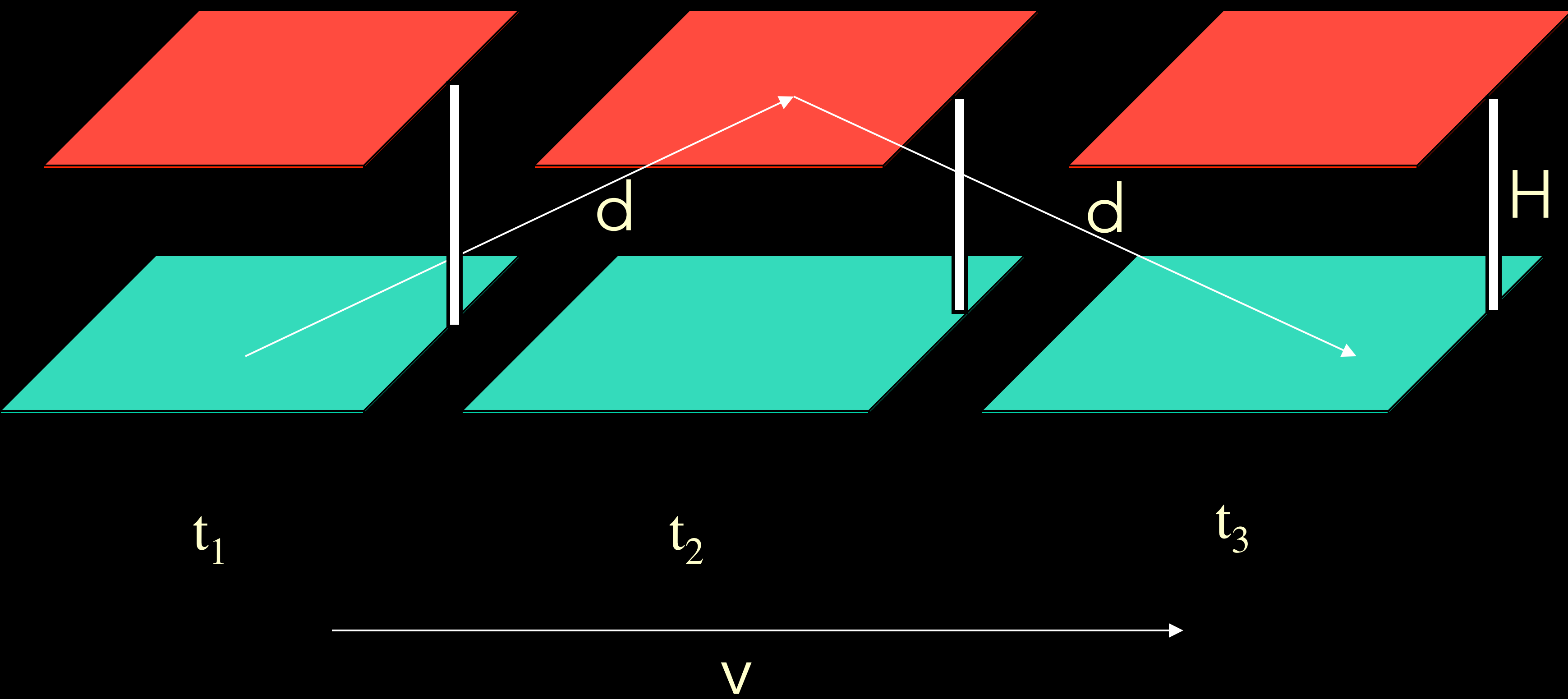
the moving clock slows down according to the stationary observer (and vice versa according to Principle of Relativity)



According to Observer C on the platform, the time between ticks of A's clock has increased to

$$\Delta t_C = 2d/c > 2H/c$$

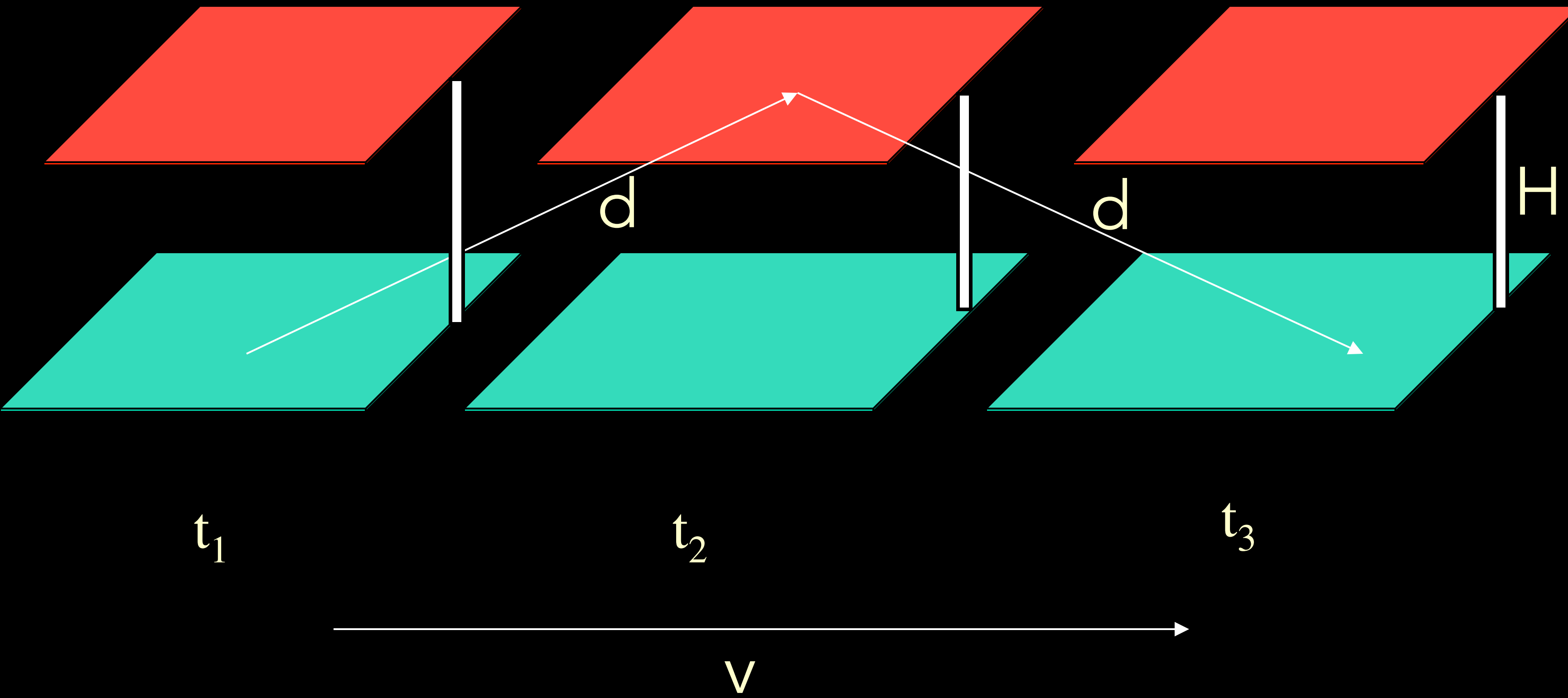
$$d^2 = H^2 + (v\Delta t_C/2)^2$$



$$\Delta t_C = 2d/c > 2H/c = \Delta t_A$$
$$d^2 = H^2 + (v\Delta t_C/2)^2$$
$$(c\Delta t_C/2)^2 = (c\Delta t_A/2)^2 + (v\Delta t_C/2)^2$$

Time between ticks of A's clock according to C:

$$\Delta t_C = \Delta t_A /(1-v^2/c^2)^{1/2}$$



Time Dilation

- For train traveling at v=0.87c (87% of the speed of light), a time interval between 2 events measured by Observer A on the train to be 30 seconds will take 60 seconds according to Observer C on the platform, and vice versa.

$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Effect of Motion on Space

Observers see moving clocks tick more slowly than their own.

Measure distance, i.e., length of a moving object such as a car, by measuring time elapsed between when its front and rear edges pass you:

$$L = v \Delta t$$

However, the driver of the car sees that your clock ticks more slowly than hers (since you are in motion relative to her), so you say less time elapses as the car passes you by than the driver does. Hence you will conclude the car is shorter than the manufacturer claims (who, like the driver, measured its length when the car was at rest).

This phenomenon is known as Length Contraction:

$$l' = l\sqrt{1 - \frac{v^2}{c^2}}$$

Observers perceive a moving object as being shortened along the direction of its motion.

Einstein: since, by the Principle of Relativity, this conclusion applies to all measurements of distance, it is a property of space itself.

Key Elements of Special Relativity

Observers in relative motion do not experience Space and Time (distances and durations) identically. Space & Time are thus not absolute, immutable, or universal (contra Newton).

Physical effects due to motion:

Length Contraction and Time Dilation

These effects of Special Relativity exist for all observers, but they are only clearly apparent if relative speeds approach the speed of light c, not a common occurrence in our experience. Our Galileian/Newtonian intuition is shaped by our slowness.

Spacetime

Time as another `coordinate' in addition to the 3 spatial coordinates needed to specify an event in `4-dimensional' <u>Spacetime:</u> meet me at position x,y,z at t o'clock.

Motion through Space and Time: when an object moves through Space relative to us, its clock slows down compared to ours. Hence, the `speed of its motion through Time' slows down.

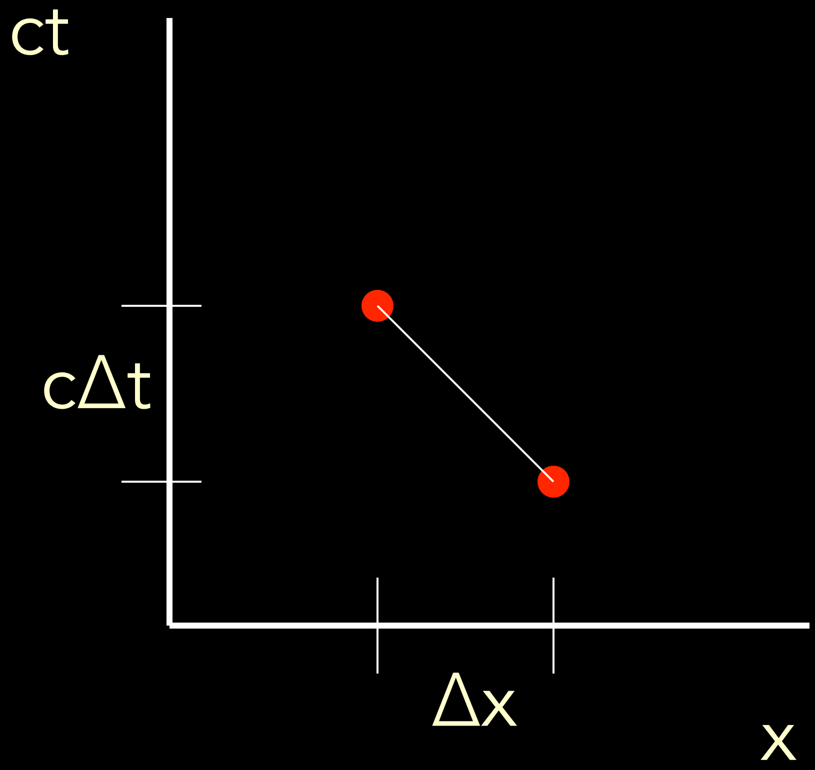
Spacetime Motion

Most of us travel almost exclusively in time (our speed through space is very small, v<<c).

But our motion can be shared between Space & Time: as we move faster through Space, our motion through Time slows down (our clocks slow down). In fact, all objects in the Universe travel through 4-d spacetime at a fixed speed: the speed of light.

Light-rays travel through Space at speed of light, so they have no motion through time: they don't age.

Spacetime Intervals



Observers in relative motion disagree on time and space intervals between 2 events but *agree* on the *spacetime* interval:

$$\Delta s=[(c\Delta t)^2-(\Delta x)^2]^{1/2}$$

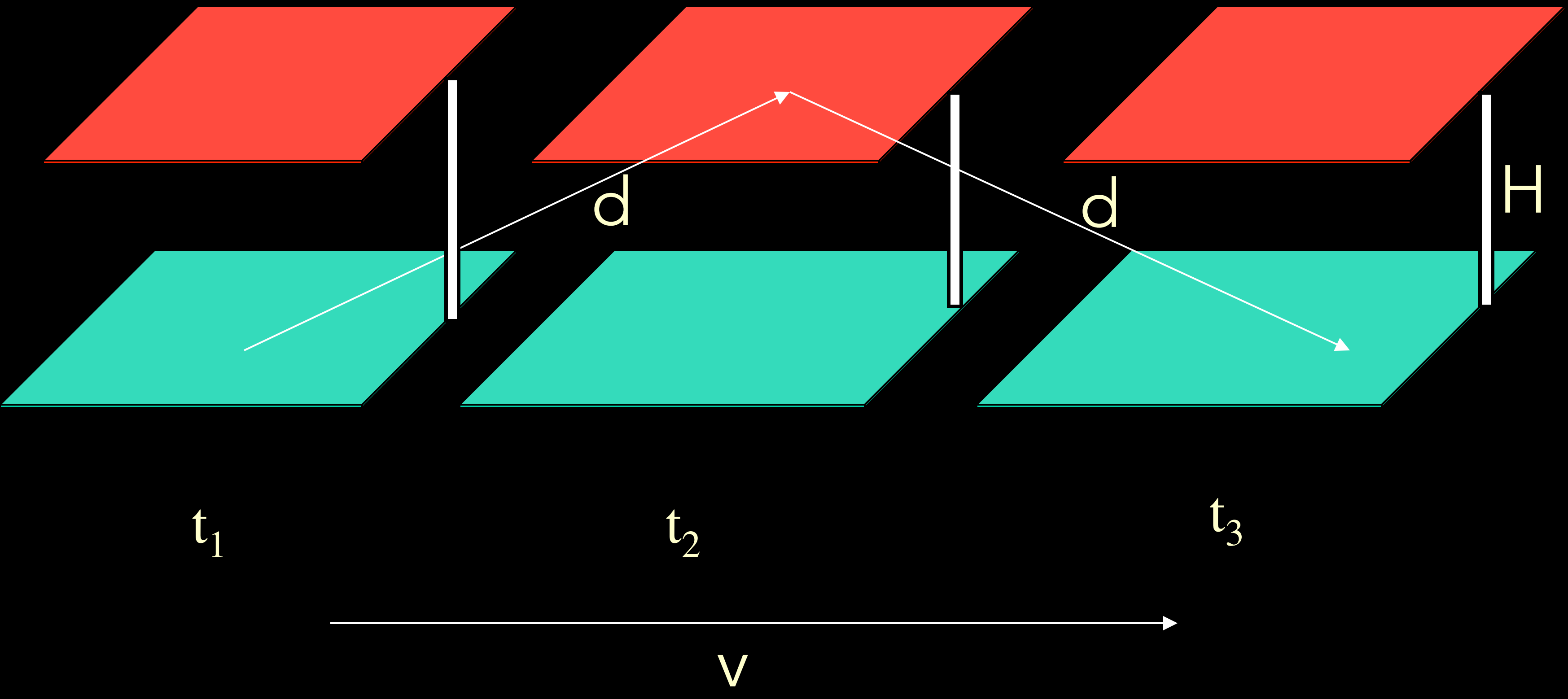
Note change in sign compared to Euclidean distance intervals

Time between ticks of A's clock according to A:

$$\Delta t_A = 2H/c$$

so spacetime interval between ticks of A's clock is

$$\Delta s = c\Delta t_A = 2H$$



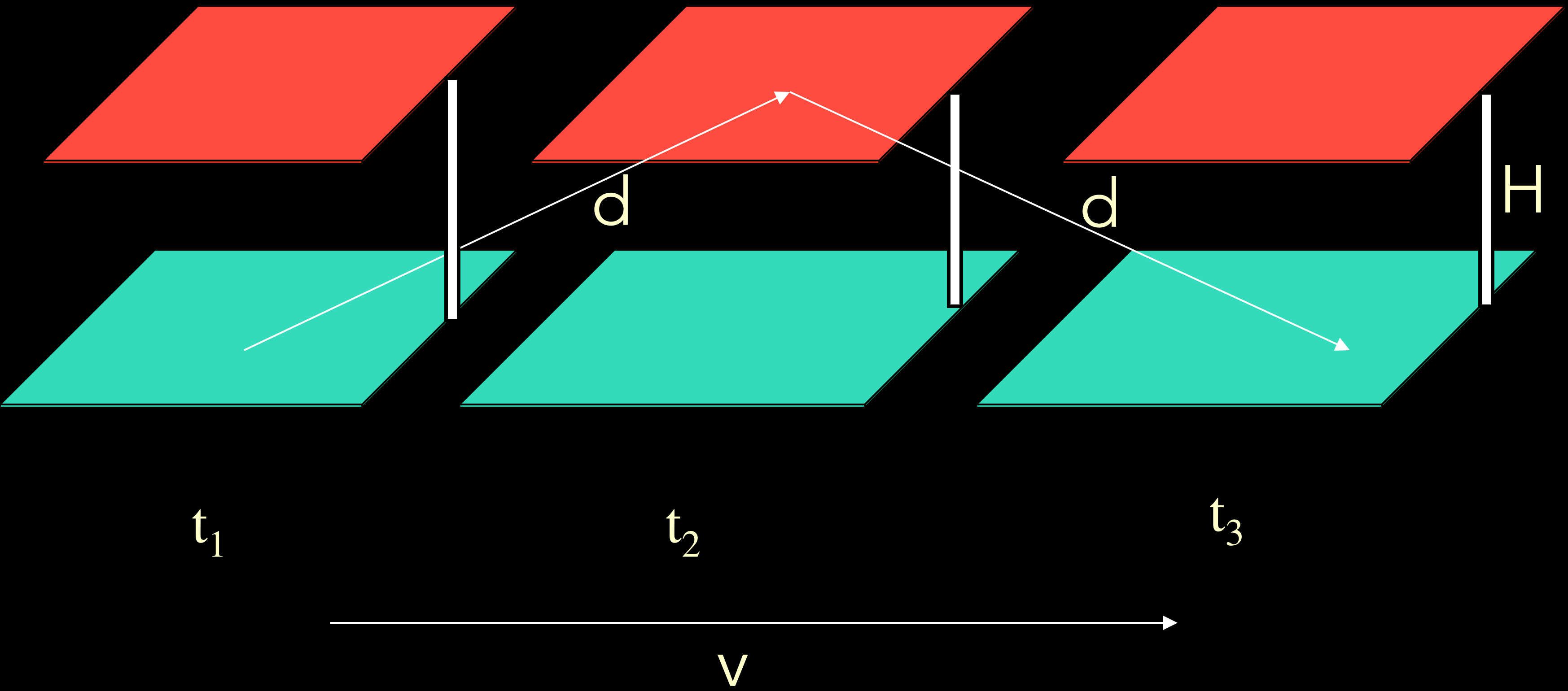
Time between ticks of A's clock according to C:

$$\Delta t_C = \Delta t_A /(1-v^2/c^2)^{1/2}$$

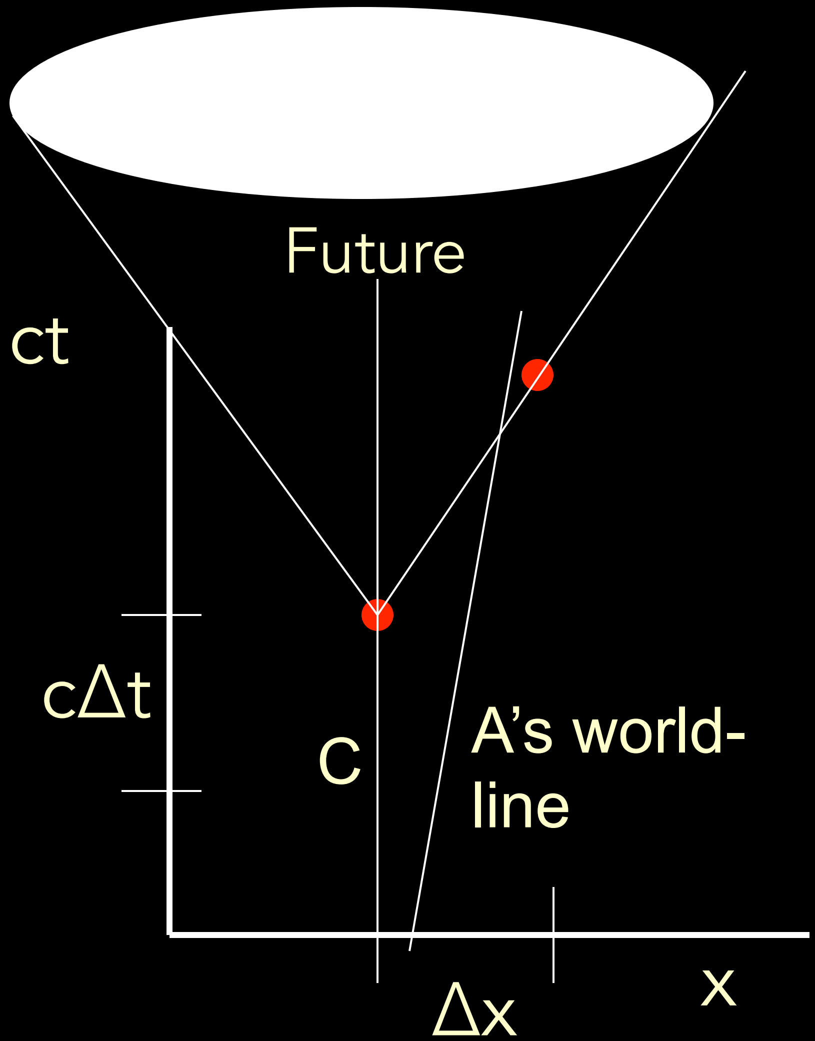
so spacetime interval according to C is

$$\Delta s = [(c\Delta t_C)^2-(\Delta x)^2]^{1/2} = \{(c\Delta t_C)^2 - (v\Delta t_C)^2\}^{1/2}$$
$$= c\Delta t_C\, (1-v^2/c^2)^{1/2} = c\Delta t_A = 2H$$

Spacetime interval is *invariant*.



Worldlines



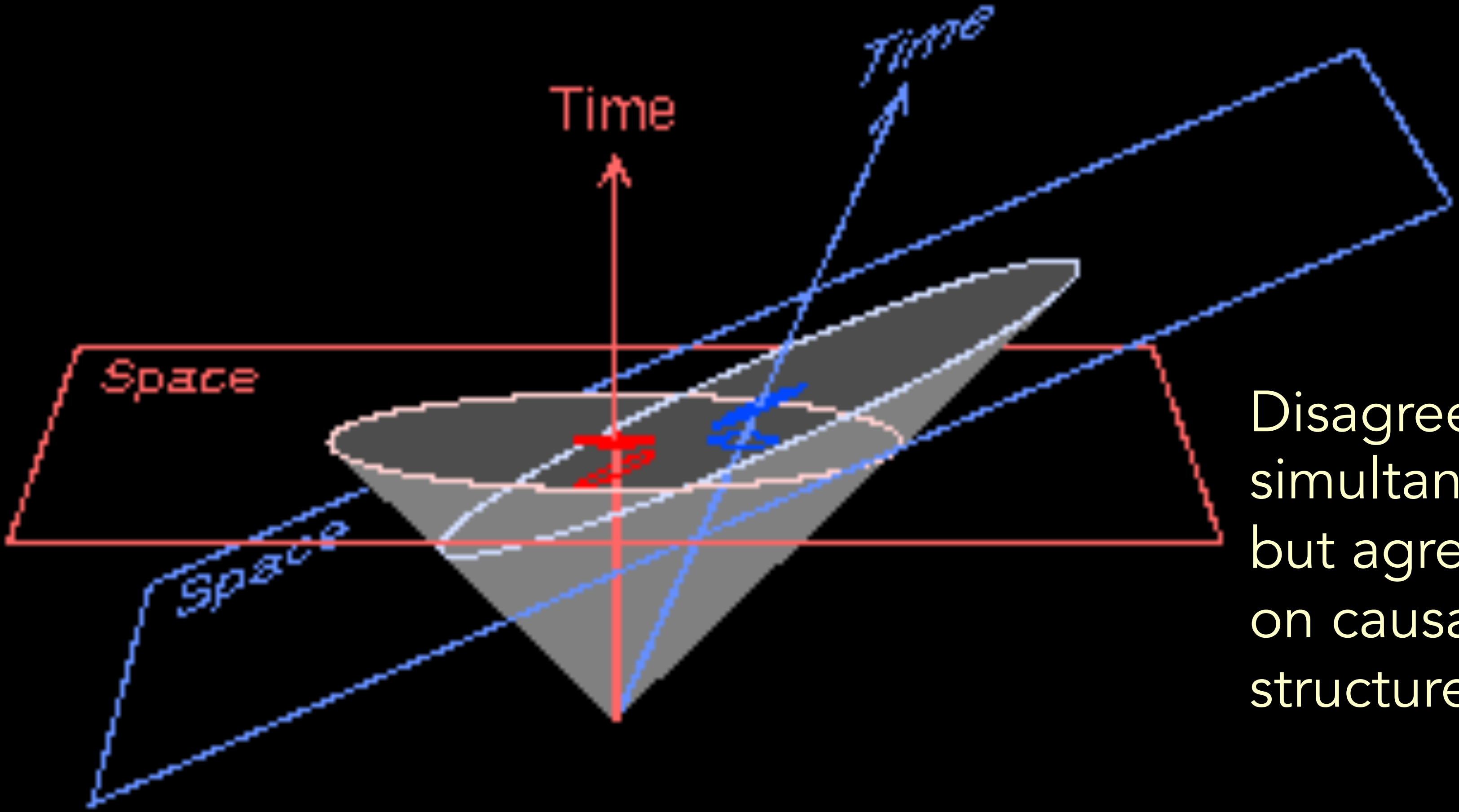
Observers trace out world lines in spacetime.

Light rays are always at 45 deg in spacetime coordinates:

$$\Delta s=[(c\Delta t)^2-(\Delta x)^2]^{1/2}=0$$

The causal future (past) of an event lies inside the forward (backward) 'light cone' traced out by light rays converging at that point, since nothing travels faster than light.

Light Cones of Observers in relative motion



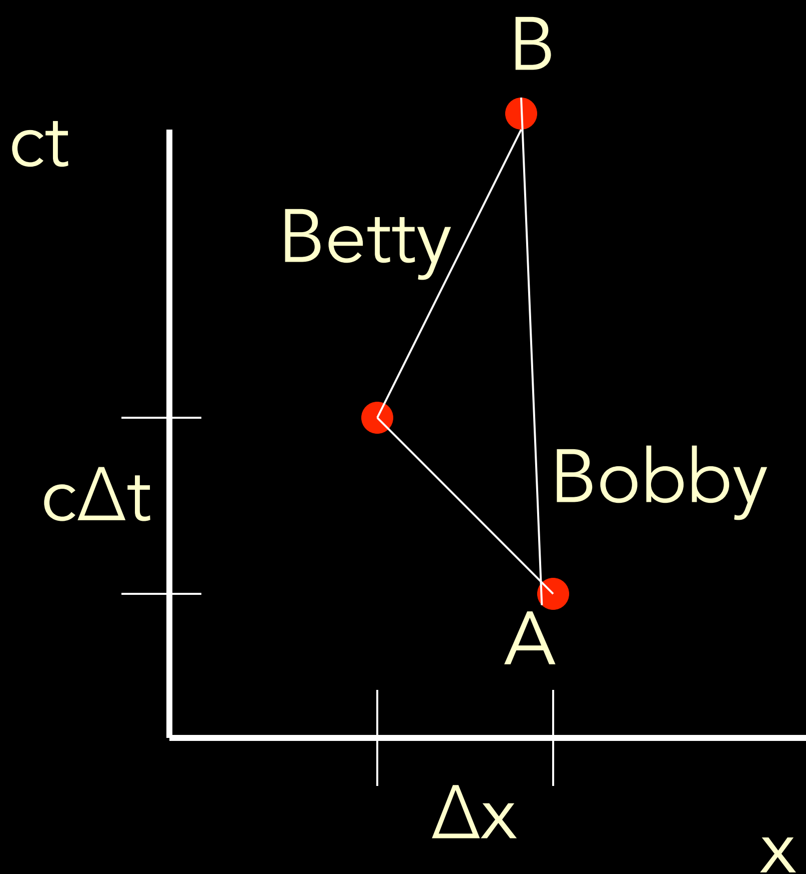
Disagree on simultaneity but agree on causal structure

http://casa.colorado.edu/~ajsh/sr/timebig.html

Back to the Future Day

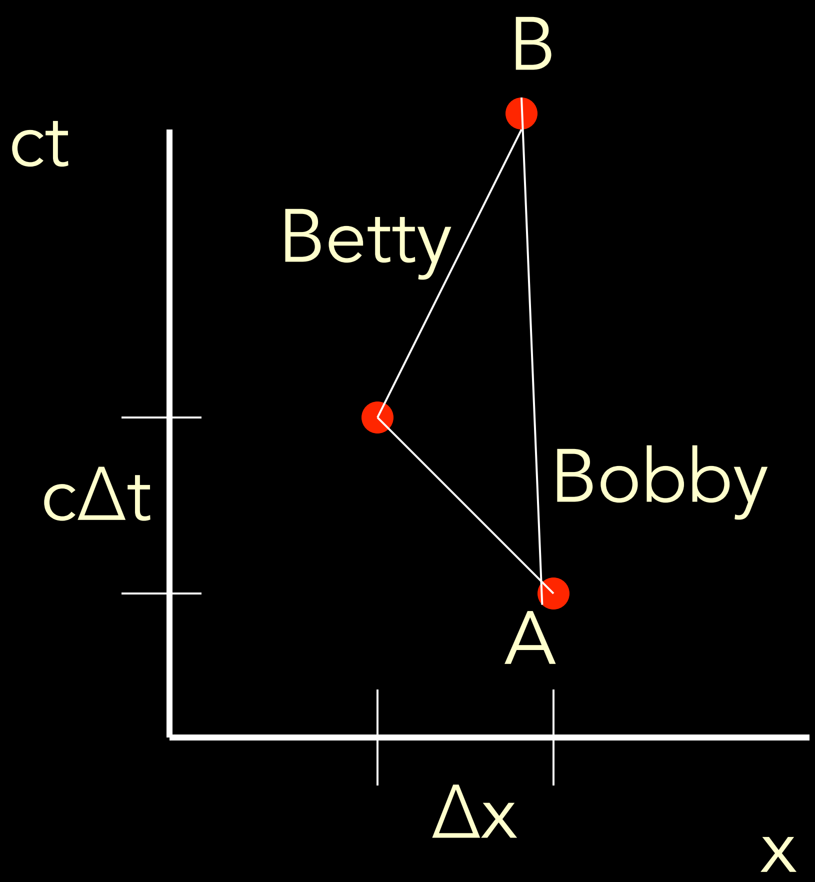
Back to the Future, Part II: 1989 film in which Marty McFly (Michael J. Fox) time travels from 1985 to October 21, 2015 at 4:29 pm, a time in which drones walk dogs, cars fly, and the Cubs have just won the World Series.

Twin Paradox



Betty travels to distant planet at nearly speed of light, turns around, and comes back. Bobby stays on Earth and meets her on her return. Since his spacetime interval from A to B is longer, his elapsed proper time is longer: he will be older than Betty when they reunite. Doesn't violate Relativity since she doesn't stay in inertial frame (she accelerates).

Twin Paradox



If Betty travels outward for a year of her clock time at v/c=0.99995, then back at the same speed for another year, she will find that those on Earth have aged 200 years while she has aged only 2 years.

$E=mc^2$

Equivalence of mass and energy:

$$E=mc^2=m_0c^2/(1-v^2/c^2)^{1/2}$$

where m_0 is the ‘rest mass’

- As something moves faster, its (kinetic) energy increases and according to Einstein its `mass’ increases: it gets heavier. As it approaches speed of light, its `mass’ m grows without bound: it would take an infinite amount of energy to accelerate a massive body to the speed of light. That’s why it is a fundamental barrier. (N.B.: light itself has no mass.)

$E=mc^2$

Equivalence of mass and energy:

$$E=mc^2=m_0c^2/(1-v^2/c^2)^{1/2}$$

where m_0 is the ‘rest mass’

- Atomic bomb: converts less than 1% of Uranium rest mass into tremendous kinetic energy (the speed of light is `big’).
- Fermilab, LHC: matter and anti-matter particles annihilate to create Energy, which is then converted into other particles.

Einstein's General Relativity (1915)

Special Relativity: Space & Time are not universal, but depend on the state of motion of the observer.

General Relativity: Space & Time are dynamical quantities that respond to (are distorted by) the presence of mass-energy: these distortions are what we call gravity.

SR: considered observers in constant relative motion wrt each other (inertial observers); Observers cannot distinguish rest from motion at constant speed.

GR: considers observers in non-constant (i.e., accelerated) motion; Obs. cannot distinguish uniform acceleration from the (local) effects of gravity. (Principle of Equivalence)

Principle of Equivalence

- Force of Earth's gravity on you:

$$W = \frac{GM_E m}{R_E^2}$$

- If you release a body near the surface of accelerates downward with:

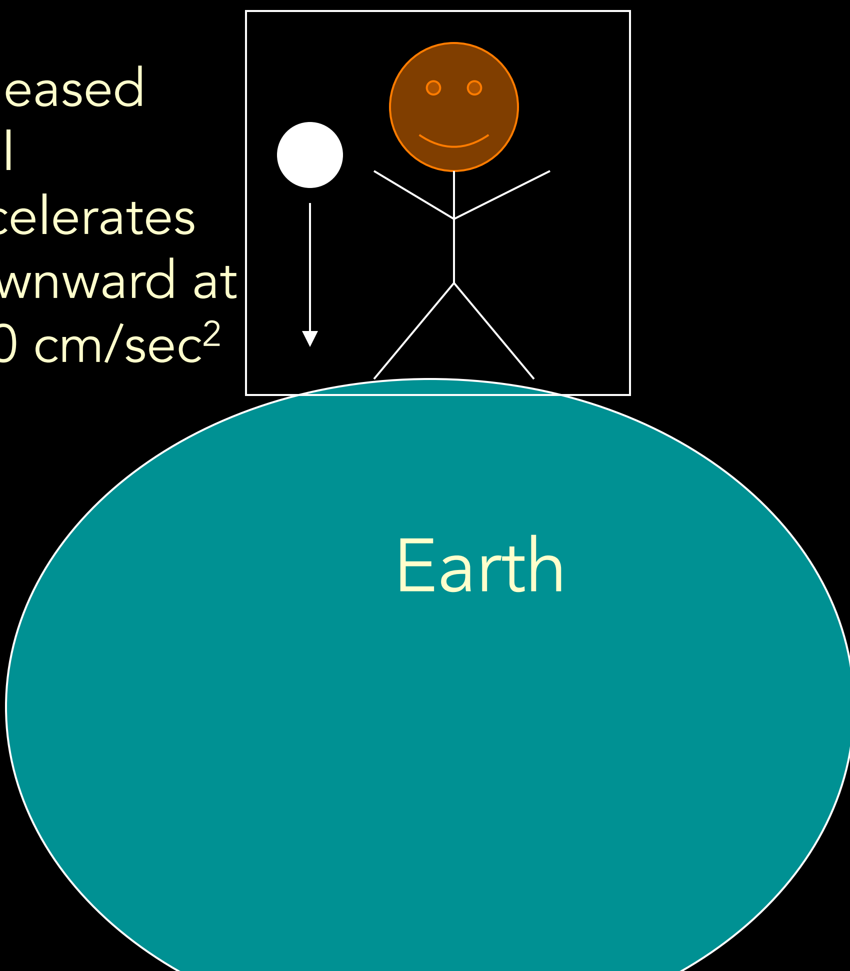
$$a = \frac{F}{m} = \frac{GM_E m}{R_E^2 m} = \frac{GM_E}{R_E^2} = 980\mathrm{cm/sec}^2$$

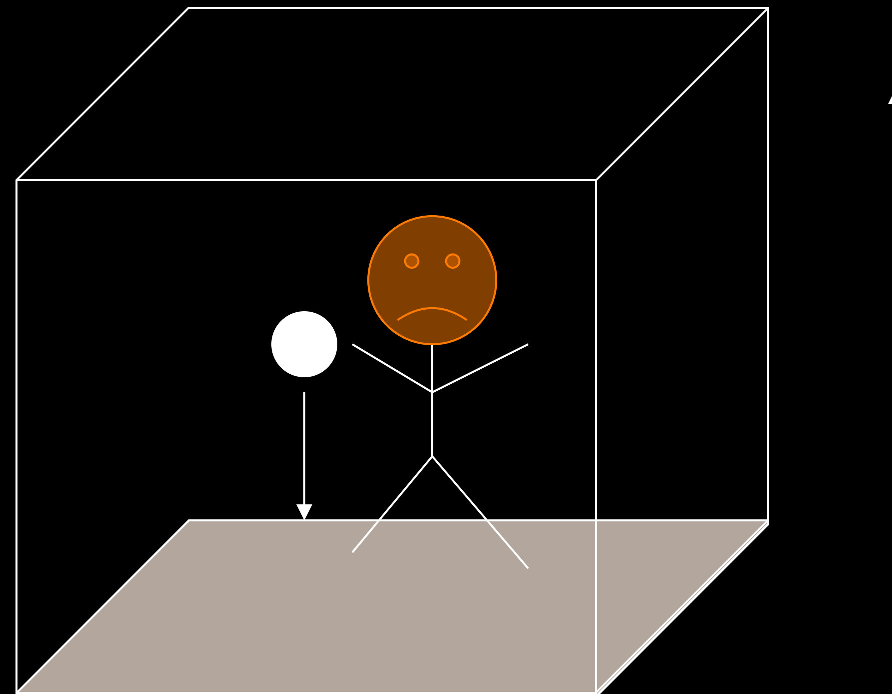
independent of the mass m of the body (Galileo). To Einstein, the fact that all bodies in a gravitational field fall at the same acceleration is a deep principle: freely falling observer feels no gravitational force.

Observer B in compartment moving upward with acceleration of 980 cm/sec^2 in outer space (no gravity)

Observer A at rest on surface of Earth (in its gravitational field)

Released ball accelerates downward at 980 cm/sec^2





Einstein's Thought Experiment

Acceleration and Gravity

These two observers feel the same physical effects. By observing the motions of bodies in their compartments, they cannot tell which of the two physical situations they are in: no (local)* experiment either could do to distinguish which of the two states s/he is in.

The observer in outer space could mistakenly believe s/he's at rest on the surface of the Earth.

*(because we're assuming the Earth's grav. field is uniform in direction, a very good approximation for nearby objects)

Corollary

If you're in a closed elevator above the Earth and the supporting cord is cut, you go into free fall (similarly if you jump out of an airplane). If you perform the same experiment (release a ball in your hand), it remains at rest relative to you or moves away from you at constant speed* (recall movies of astronauts). By these observations, you would conclude that you are not in a gravitational field at all, since you don't see released objects accelerate in one direction the way they do when you're at rest on Earth.

Thus, the observable effects of a gravitational field can be `turned off' by accelerating at the free-fall rate.

*so Theory of Special Relativity (locally) applies

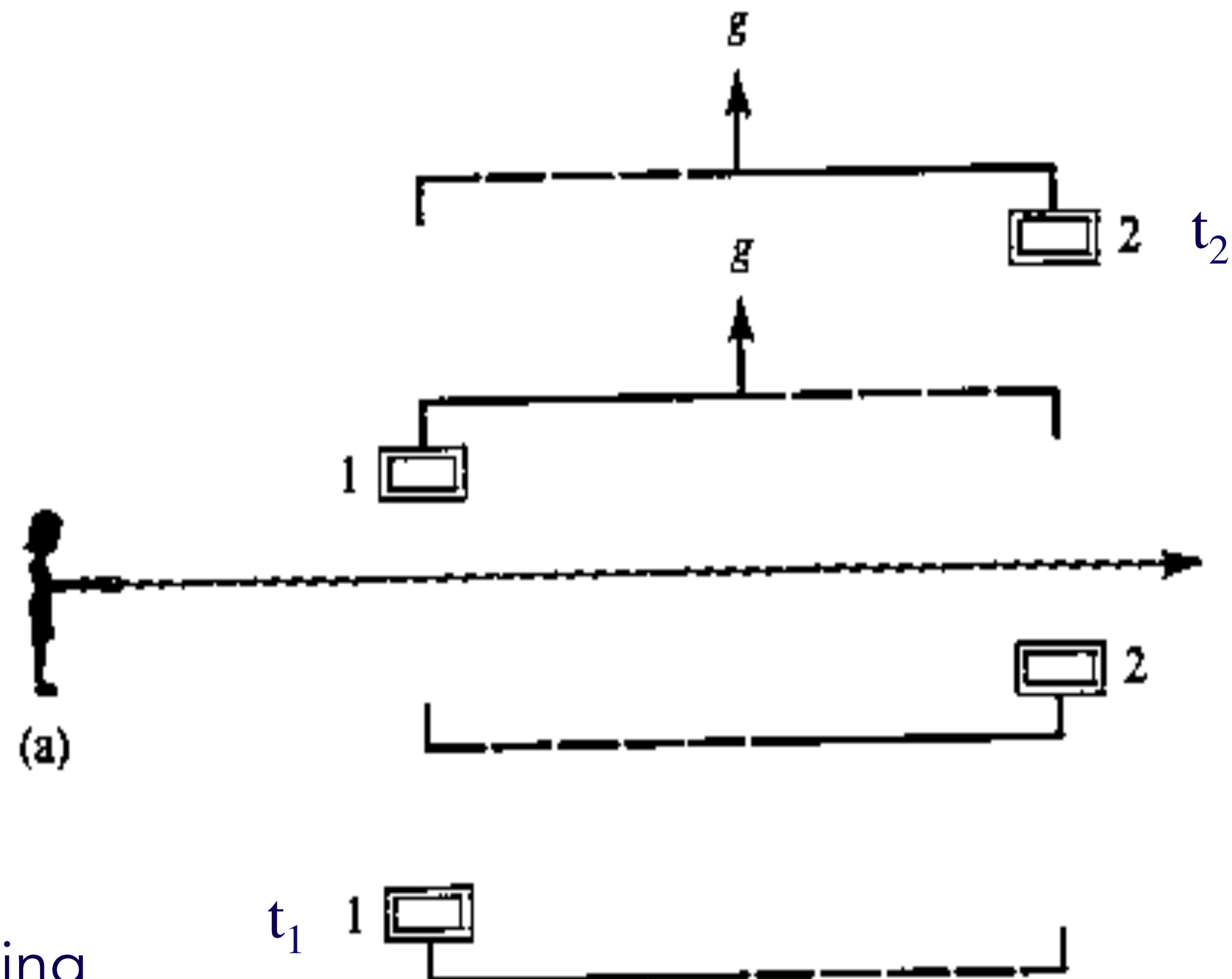
Gravity and Acceleration

Hence we can use the properties of accelerated motion to (partially) understand gravitation.

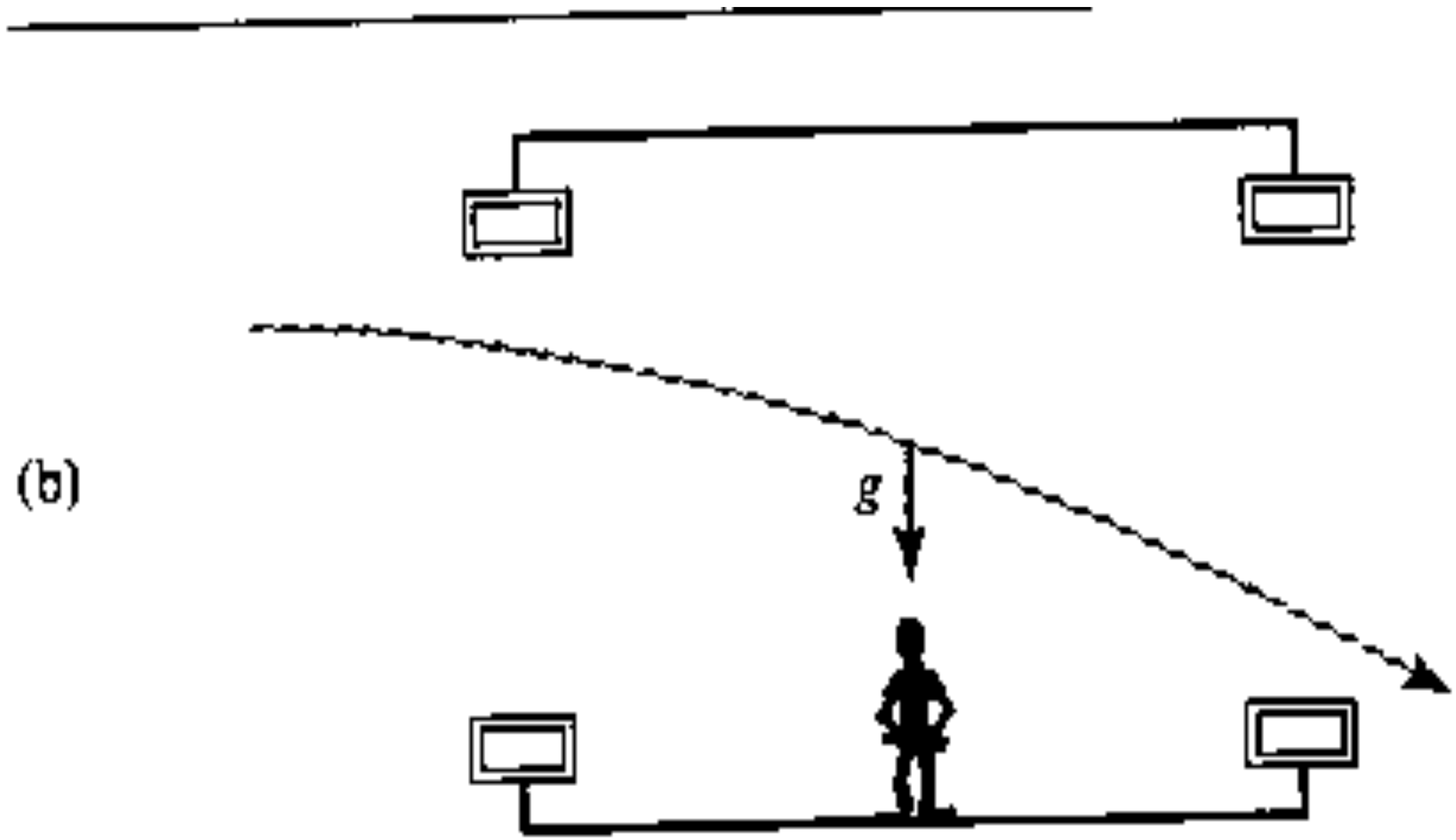
By extending the arguments we used in Special Relativity (for non-accelerated motion), we can consider what happens to observations of Space & Time for accelerated observers. By the Principle of Equivalence, this tells us the

Observational Effects of Gravity on Space & Time

Another thought experiment



Inertial Observer Ida shines light through a compartment that is <u>accelerating</u> upward (in outer space) with $a = g = 980\ cm/sec^2$



Accelerating observer Alex sees the light follow a curved path. Since he mistakenly believes he's at rest on the surface of the Earth, he attributes this light bending to the Earth's gravity. By the Principle of Equivalence, he is correct:

Gravity bends light rays.

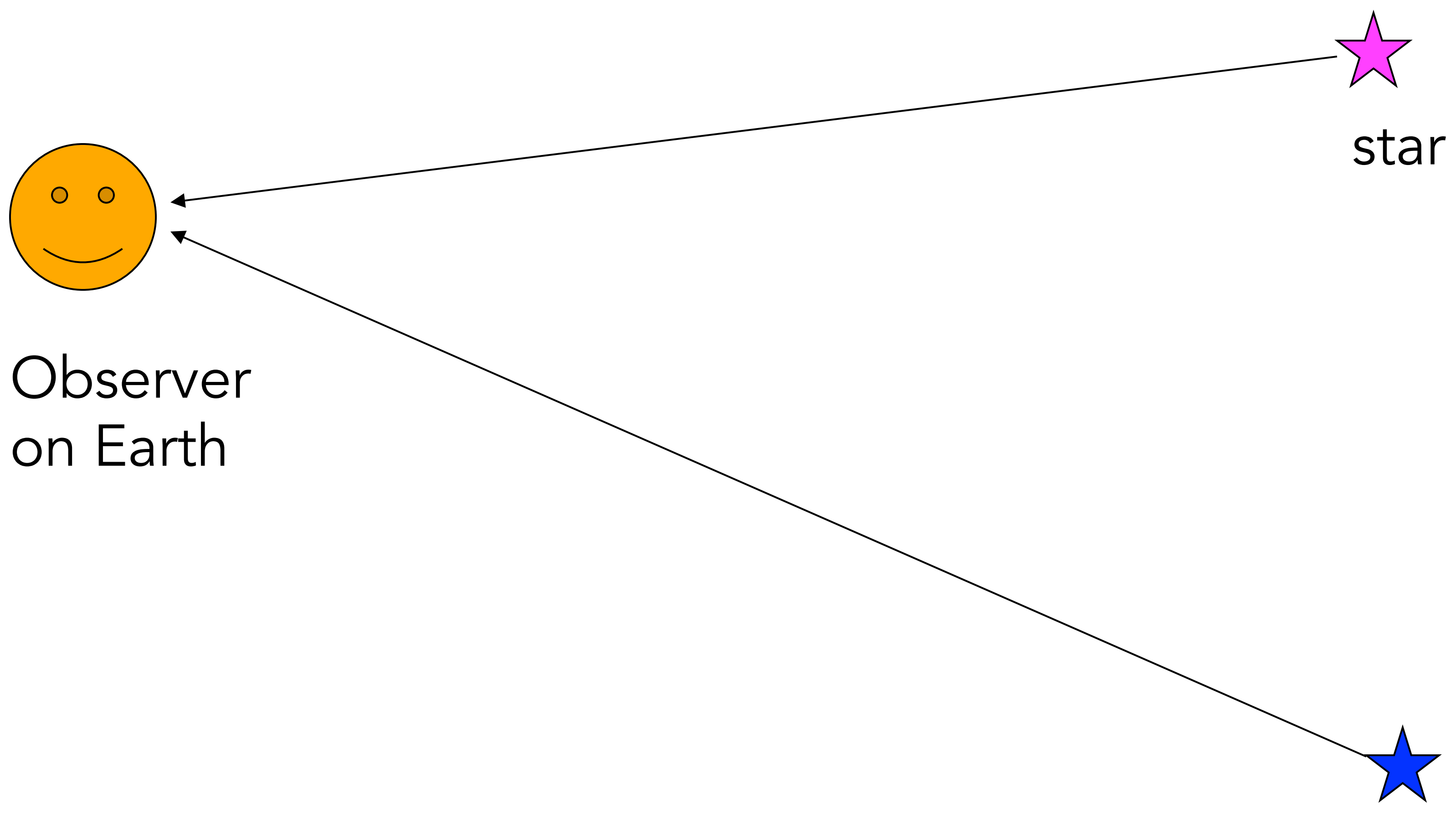
Gravitational Bending of Light

Expedition to observe solar eclipse of May 29, 1919 in Northeast Brazil and Principe Island to test this prediction of Einstein's Theory of General Relativity.

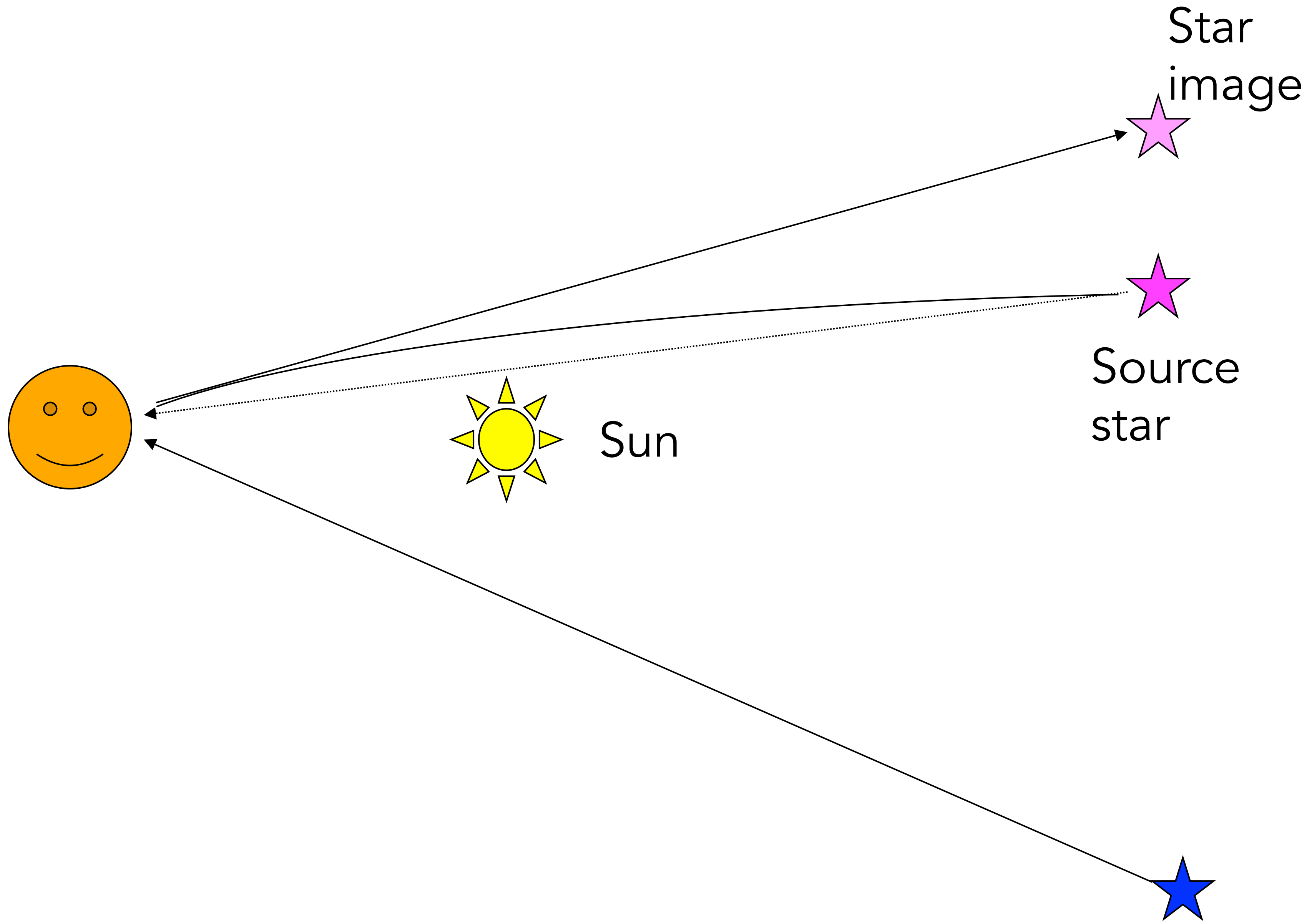
Measured angular positions of stars near the Sun and compared with positions of the same stars several months later

Note: if light is like `ordinary' matter, then it should be deflected by gravity according to Newton as well. However, <u>General Relativity predicts light deflection twice as large as Newtonian gravity</u>.

Light follows straight paths through empty space

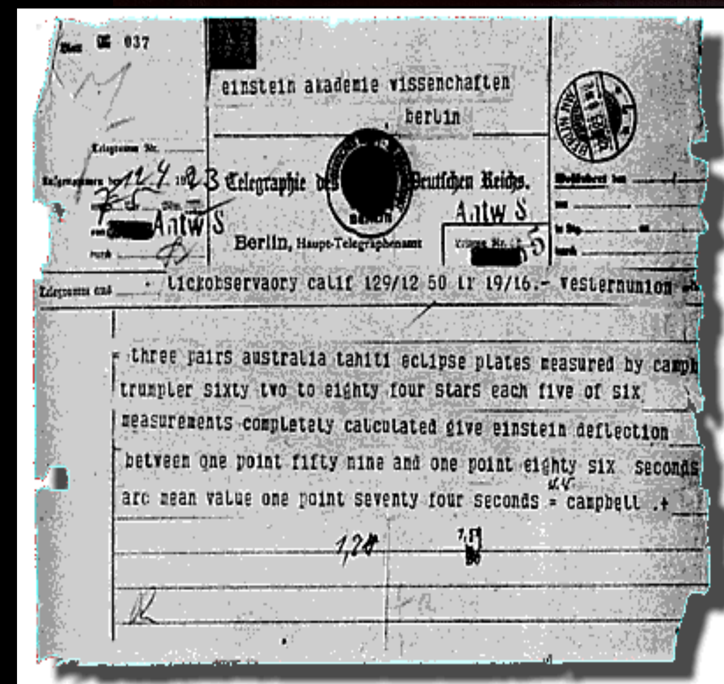


Light path is bent by gravitational field of the Sun



In 1919, Eddington led expedition to observe solar eclipse ...

Leading to confirmation of Einstein's theory ...



einstein akademie vissenschaften berlin

Telegraphie des Deutschen Reichs.

Berlin, Haupt-Telegraphenamt

lickobservaory calif 129/12 50 lr 19/16.- vesternunion

three pairs australia tahiti eclipse plates measured by camp[bell] trumpler sixty two to eighty four stars each five of six measurements completely calculated give einstein deflection between one point fifty nine and one point eighty six seconds arc mean value one point seventy four seconds = campbell .+

And widespread adulation ...

London Times Nov 8, 1919

The Revolution in Science:

Einstein vs. Newton

According to Ida, the light ray follows a straight path. According to Alex, the accelerated observer, the light path is curved. How do we reconcile these two views?

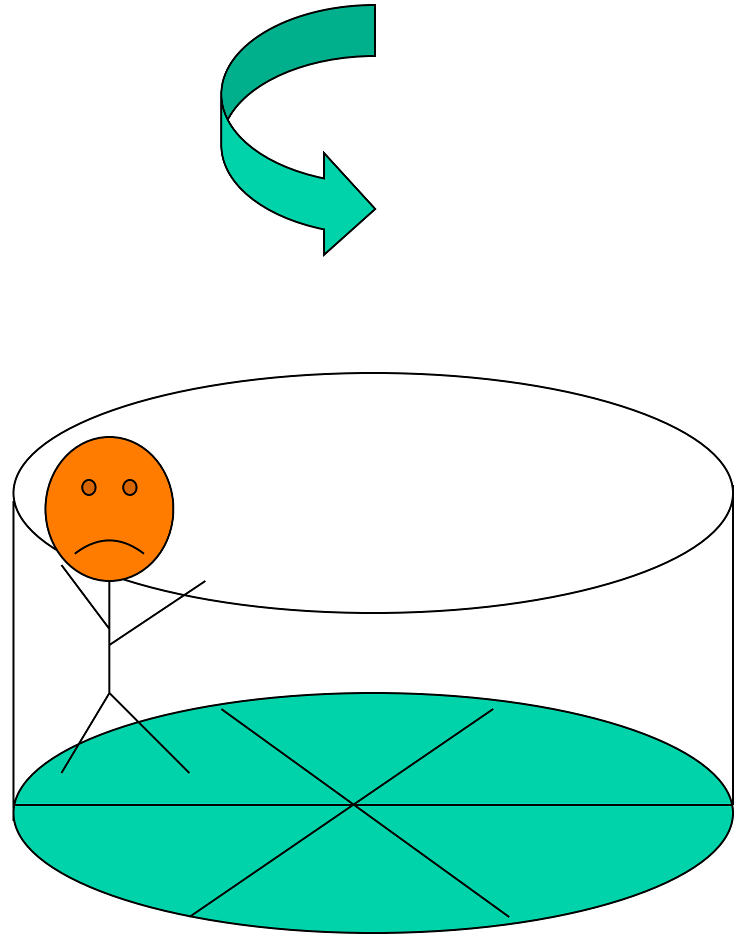
Einstein: according to the accelerated observer, space is curved, and light rays follow the `straightest possible' (aka geodesic) paths in this curved space.

Special Relativity: observers in relative motion have different notions of Space & Time intervals.

General Relativity: accelerated observers have different notions of the geometry of Spacetime.

By the Principle of Equivalence, this applies to gravity as well: A gravitational field corresponds to the curvature of Spacetime.

Tornado Ride



Fixed Observer
Ida measures circumference and radius of the rotating ride, then throws Alex a ruler and asks him to carry out same measurements

Rotating observer Alex moving at constant acceleration

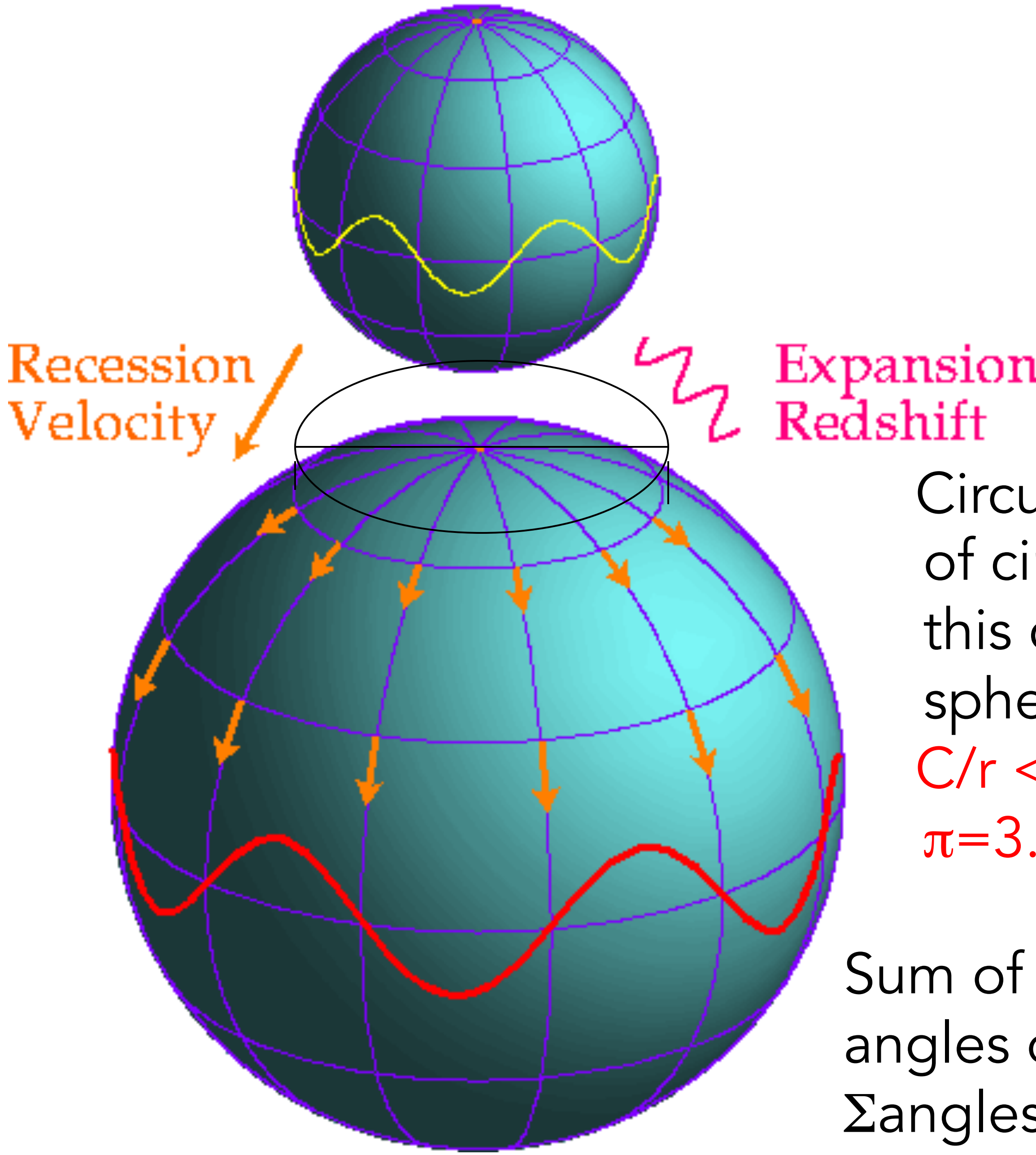
Ida finds the circumference C and radius r of the spinning ride are related by $C/r = 2\pi$, where $\pi = 3.1415926\ldots$

She throws a ruler to Alex and he measures the circumference and radius by laying down the ruler around the perimeter and across the center. According to Ida, when he measures the circumference, his ruler is contracted since it is aligned with the motion of the ride. Therefore, he measures a larger circumference C than she does. When he measures the radius, the ruler is perpendicular to his motion, so they agree on the value of r. As a result, he finds that $C/r > 2\pi$, so he concludes that he is in a curved space.

By the Principle of Equivalence, he concludes the same thing when he's at rest in a gravitational field:

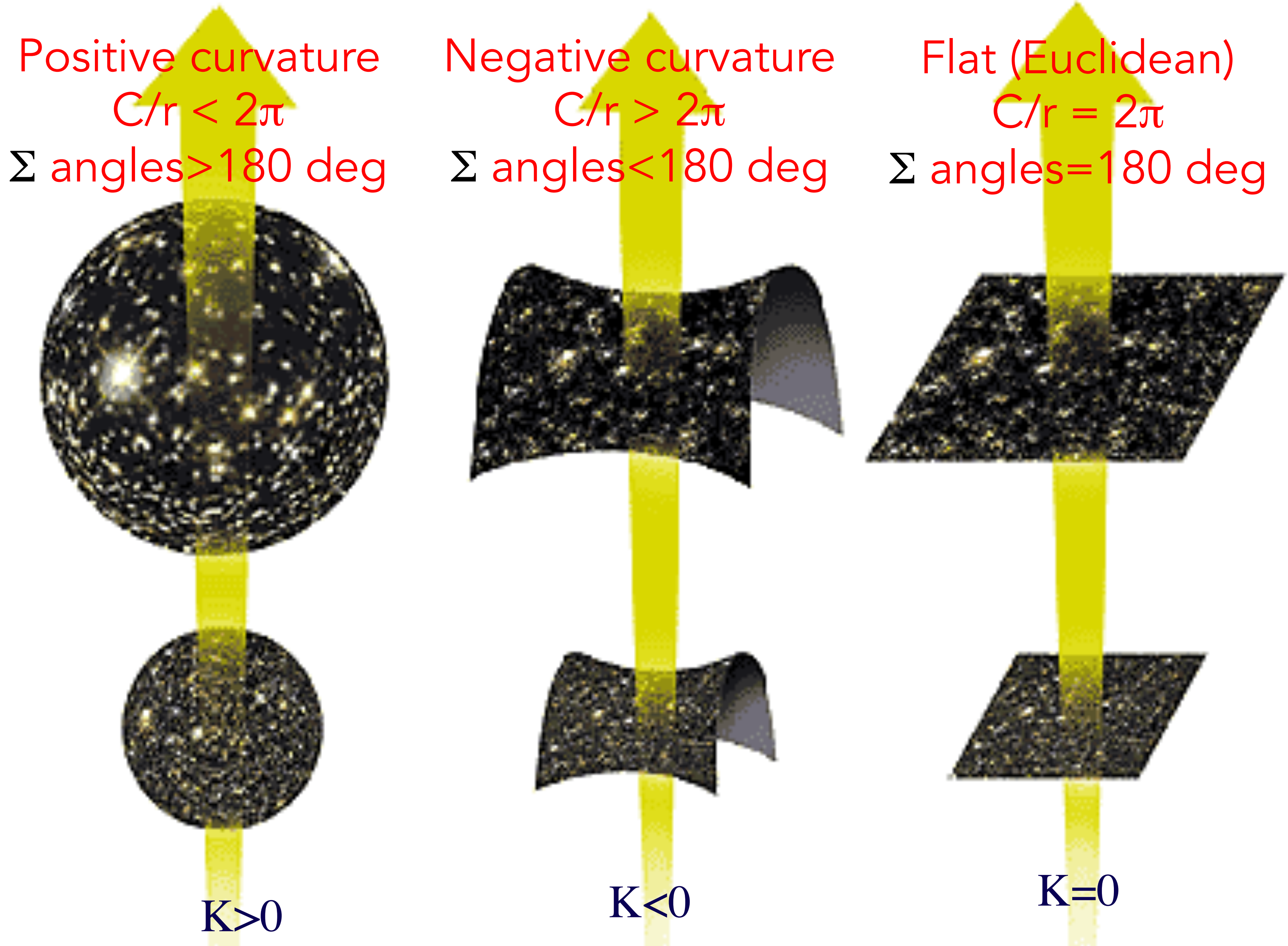
Gravity $\leftarrow\rightarrow$ Curved Spacetime

Cosmological Expansion



Circumference of circle on this curved spherical surface:
$C/r < 2\pi$ where $\pi = 3.1415926\ldots$

Sum of interior angles of triangle
Σangles > 180 deg


Positive curvature
C/r < 2π
Σ angles>180 deg
Negative curvature
C/r > 2π
Σ angles<180 deg
Flat (Euclidean)
C/r = 2π
Σ angles=180 deg
K>0
K<0
K=0

Space vs Spacetime Curvature

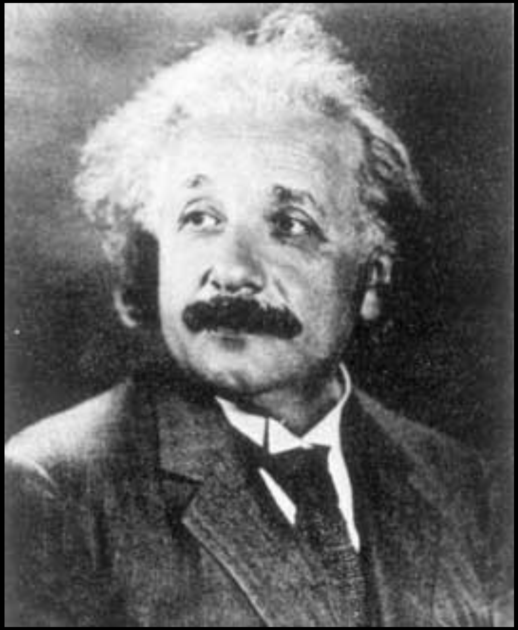
Curvature of 3-dimensional Space vs. Curvature of 4-dimensional Spacetime:

General Relativity: implies that Spacetime is generally curved.

Cosmology: mainly concerned with the curvature of 3-dimensional space (K) (i.e., of a `slice' through spacetime at a fixed time) since it is related to the density and fate of the Universe.

Einstein's General Realtivity

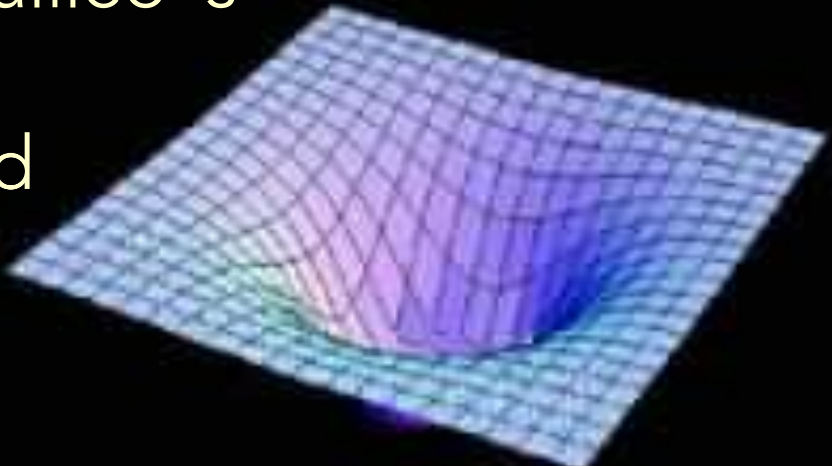
Describes how Matter (mass-energy) affects the structure of Spacetime



Replaces Newtonian forces acting at a distance

Automatically incorporates Galileo's observation that all objects fall at same rate in a grav. field

A massive star attracts nearby objects by distorting spacetime around it



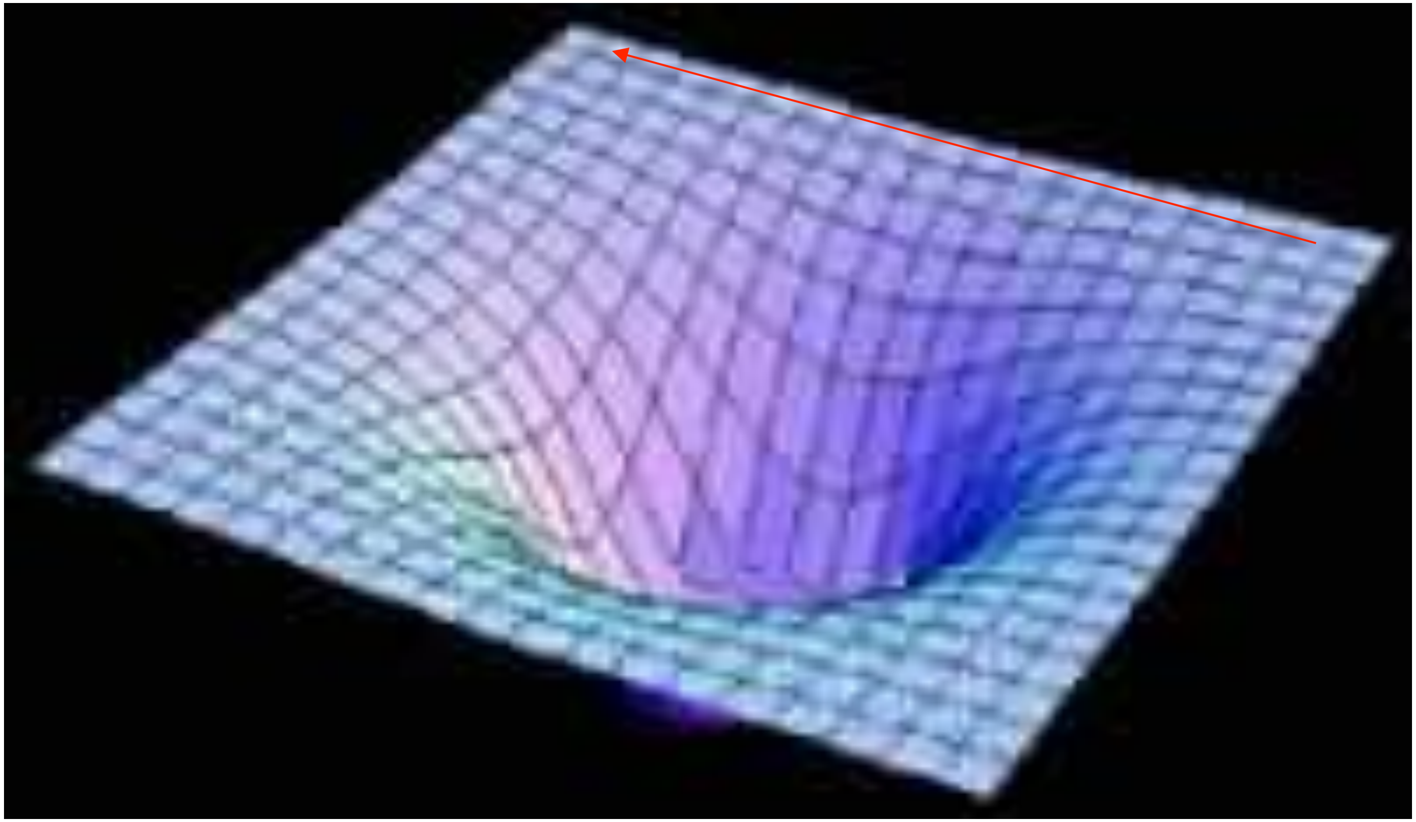
Gravity: Newton vs. Einstein

Newton:

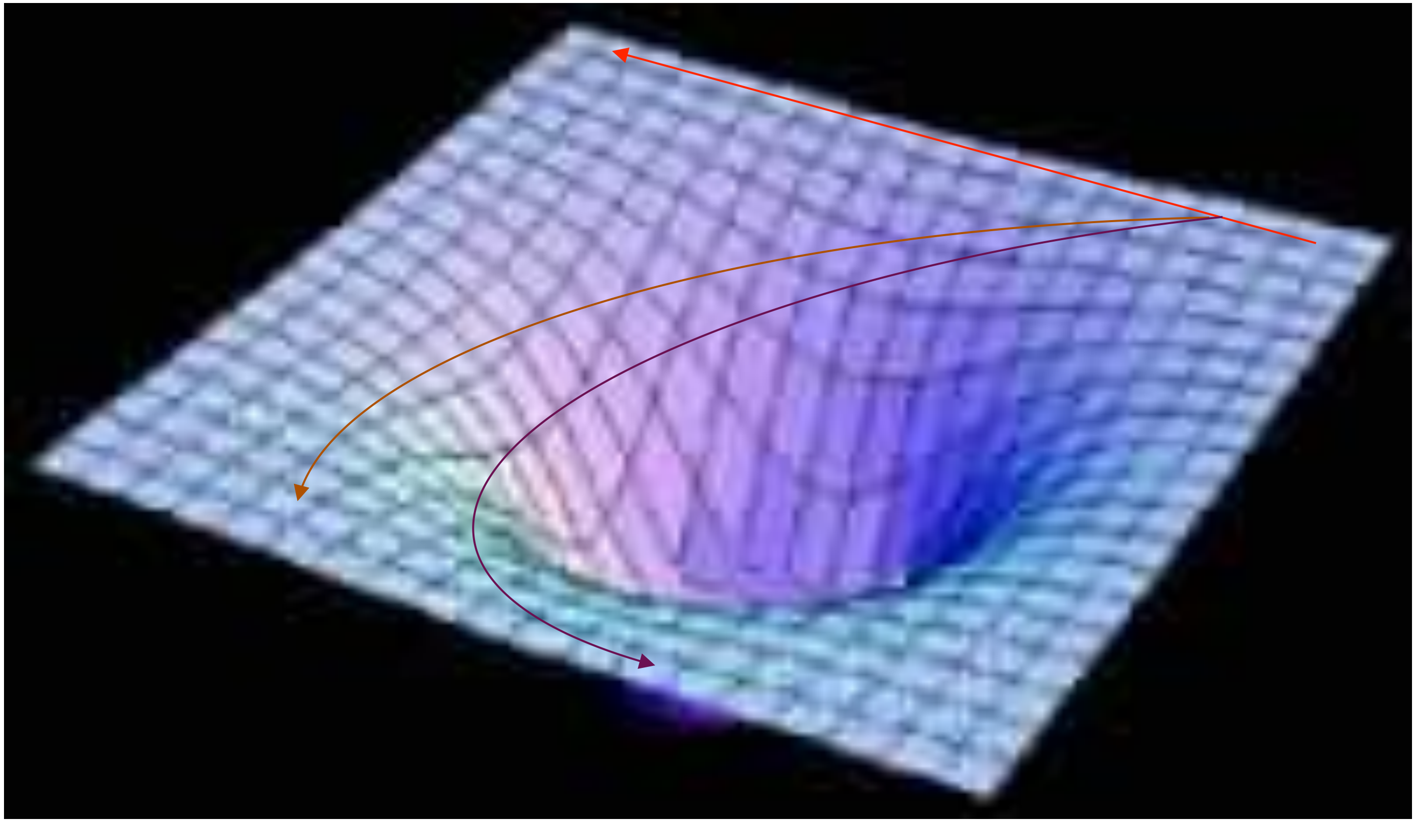
- Gravitation is a force exerted by one massive body on another.
- A body acted on by a force accelerates

Einstein:

- Gravitation is the curvature of spacetime due to a nearby massive body (or any form of energy)
- A body follows the `straightest possible path ‘ (aka geodesic) in curved spacetime



Geodesics:
straightest possible paths in curved spacetime



Geodesics:
straightest possible paths in curved spacetime