

2012 Sagan Exoplanet Summer Workshop

Hands-on Session
“Transit Timing Variations”

Presenters

- Dan Fabrycky
 - Hubble Fellow at UC Santa Cruz (UCSC)
 - Assistant Professor at University of Chicago in fall
- Darin Ragozzine
 - Postdoctoral Researcher at University of Florida with Eric Ford
- Both Dan and Darin have expertise in working with Transit Timing Variations from Kepler as active members of Kepler's TTV/Multiples Working Group

Other support and Thanks

- Stefano Meschiari (UT Austin McDonald Fellow)
- Josh Carter (SAO Hubble Fellow)
- Eric Agol (UW Professor)
- Carolyn Brinkworth (NExScI Science Affairs)
- Technical Support from IPAC/NExScI
- Sagan Workshop Organizers

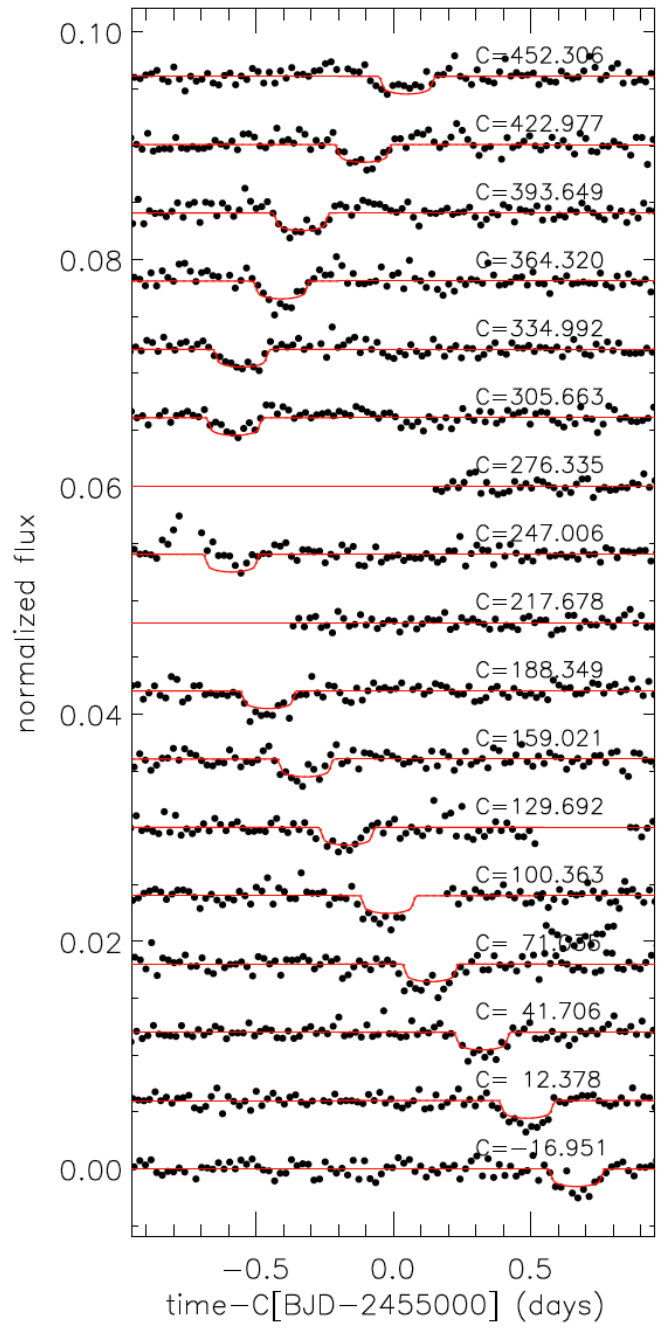
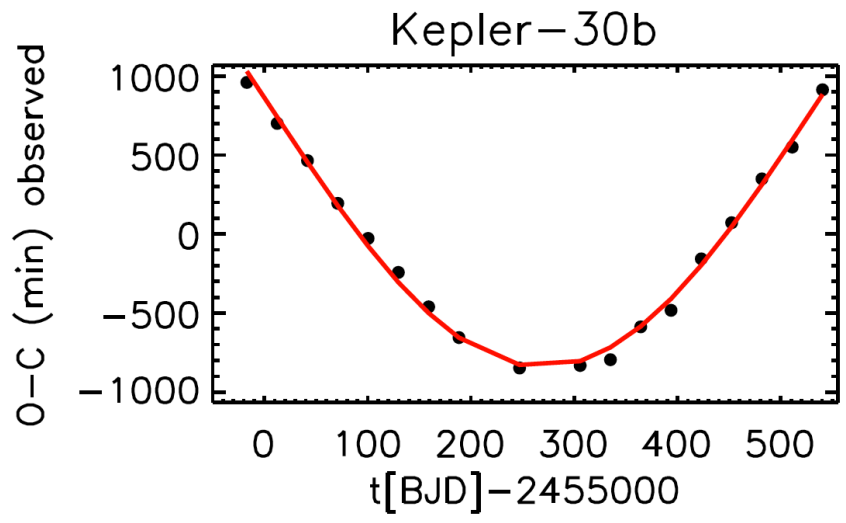
Get some files to follow along...

- Open up a web browser within the VNC viewer (world and mouse icon at the top), and navigate to:
<http://www.ucolick.org/~fabrycky/saganws/>
- You can get TTV_handson.pdf, to follow along this presentation.
- Open up QATS_notes.txt which we'll use later.

Review: Transit Timing Variations

- An idealized star+planet system undergoes perfectly periodic (“Keplerian” or two-body) motion
 - The times of transits are perfectly periodic
 - The times of transits follow a linear ephemeris:
$$T_N = T_0 + N * P$$
 - No significant deviations from a best-fit line
 - Even with ultra-precise, continuous, long-duration measurements from Kepler, the majority of planets have periodic transits
- Planet-planet gravitational interactions induce:
 - Non-periodic, non-linear transit times
 - Variations remain after subtracting best-fit period
- Transit Timing Variations = TTVs

Strong Example



Power of TTVs

- Already present in Kepler data
 - $\sim 10\%$ of Kepler candidates (Ford et al. 2012)
 - 100+ planets and growing fast!
- Can be used to confirm candidates (currently most numerous method for Kepler) or detect new planets
- Measures interacting planets and hence **MASSES**:
 - Complementary to radius measurements
 - Allows for density measurements, yielding insights into Composition, Formation, and Dynamics
 - *Currently best method for small planets*
- Best used in systems with more than one transiting planet = Multi-Transiting Systems (MTSs). ~ 900 such valuable systems already known from Kepler

Some TTV References

- Prediction of Method and Theory (Miralda-Escude 2002; Agol et al. 2005; Holman & Murray 2005; many others)
- Early Detections (Kepler-9, Holman et al. 2010; Kepler-11, Lissauer, Fabrycky et al. 2011; etc.)
- Planet Confirmations (Ford et al. 2012a; Fabrycky et al. 2012a; Steffen et al. 2012a)
- Non-Transiting Planet Detection (Ballard et al. 2011; Nesvorny et al. 2012)
- Planet Discovery issues (Garcia-Melendo & Lopez-Morales 2011; Carter & Agol 2012; etc.)
- **TTV Data** and Statistical Studies (Ford et al. 2011; Ford et al. 2012b; Steffen et al. 2012b,c; Rowe et al. 2012)

Main Goal for this Session

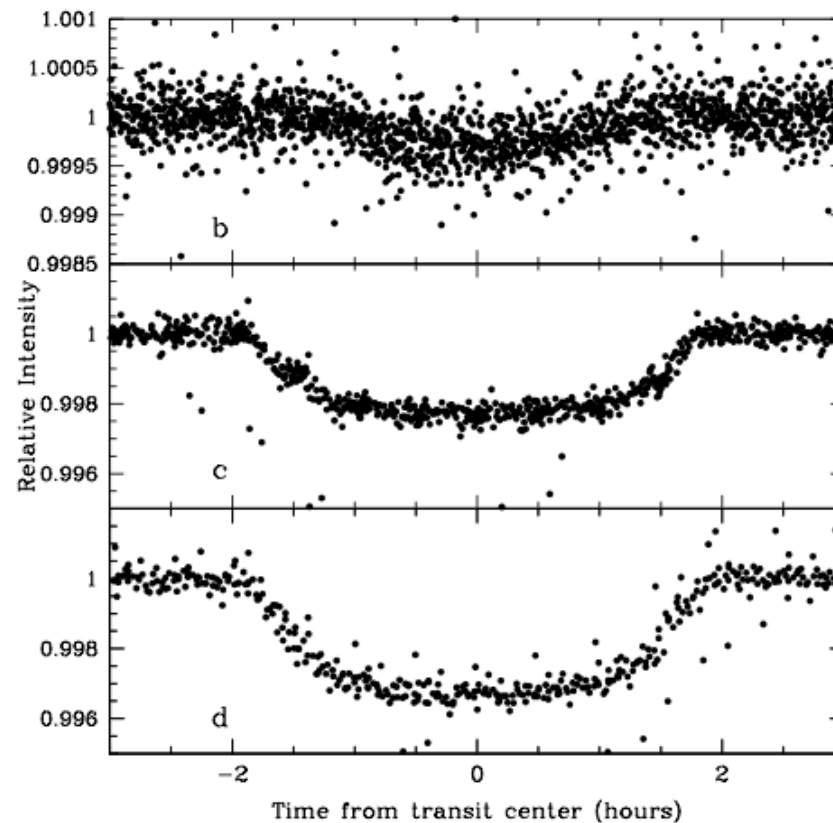
- **Analyzing transit times from a Kepler Multi-Transiting System to measure planet masses**
- Assumed starting point:
 - Individual transit times and errors for each planet
 - Other hands-on sessions
- *This is a very ripe area for research!*
 - 3 more quarters of Kepler data (Q7-Q9) are going public in a few days, very useful for TTVs
 - Kepler team regularly comes out with TTV data catalogs and identification of interesting systems
- (Note: for planets with transits too small to measure individual times, you need a different method, such as a photodynamical model (e.g., Kepler-36).)

Main Goal for this Session

- Reproducing mass measurements for Kepler-18c and Kepler-18d (Cochran et al. 2011 = C+11)
- Check that everyone is logged in and ready
- Let us know if you have any questions!
- At the end, we will use a non-linear transit search method (QATS) to search for the planets of Kepler-30

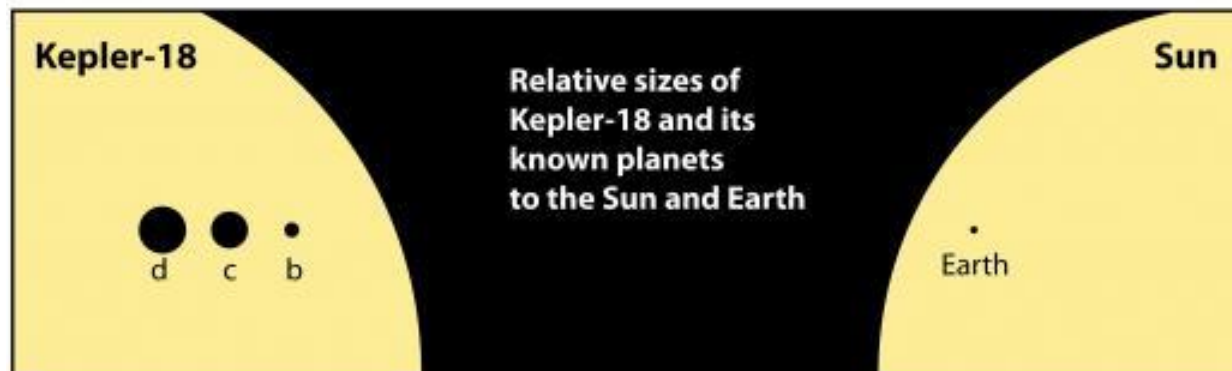
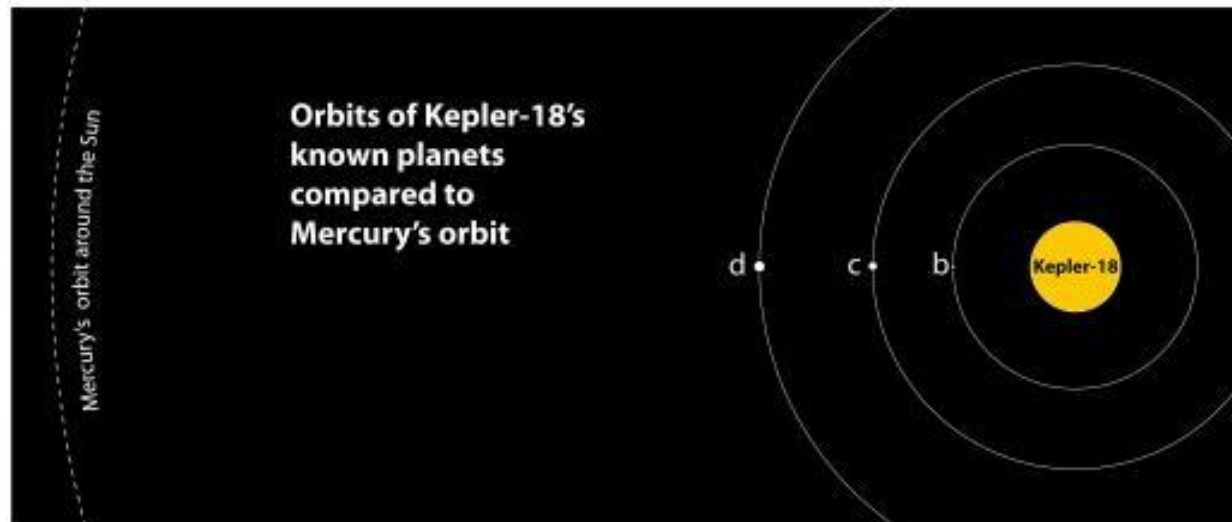
Kepler-18 (C+11 Fig 2)

- Kepler pipeline detected 3 planets



Kepler-18 Architecture

- System with Tightly-packed Inner Planets



Analysis using the Systemic Console

- Written by Stefano Meschiari and collaborators
 - Available at www.stefanom.org
- Extension from Console seen on Greg Laughlin's excellent exoplanet blog (www.oklo.org)
- Organized and efficient Java GUI
- Can fit a variety of datasets in flexible ways, very useful for fitting and studying TTVs.
- See Meschiari et al. 2009; Meschiari et al. 2010

Thank You Stefano!

Console Input Files

- Systemic keeps different datasets in individual column-delimited ASCII text files.
 - Transit timing data in “TDS” file.
- Investigate data for Kepler-18c
 - *more* `~/datafiles/k18c.tds`
- At the top is JD epoch (subtracted from all subsequent times)
- Also planet number (Kepler-18c is the 2nd planet)
- First column is transit time (since JD epoch) in days
- Second column is error (in days)
- Third column is 1 for primary or 2 for secondary eclipse.

Questions for Discussion

- What is the approximate period of this planet?
 - 7.5 days, just subtract two adjacent times
- What is the difference between Transit Times and TTVs?
 - TTVs have the “average period” subtracted (O-C)
- What is the approximate period of planet d (*k18d.tds*)?
 - 15 days
- What is the typical TTV error in minutes for this planet?
 - 1 minute (from 0.0006 days and 1440 mins/day)
- What are the main factors in determining TTV errors?
 - SNR of single transit: depth and star brightness; ingress/egress timescales compared to cadence
 - K18c/d are large and K18 bright: excellent precision

Simultaneous Data Fitting

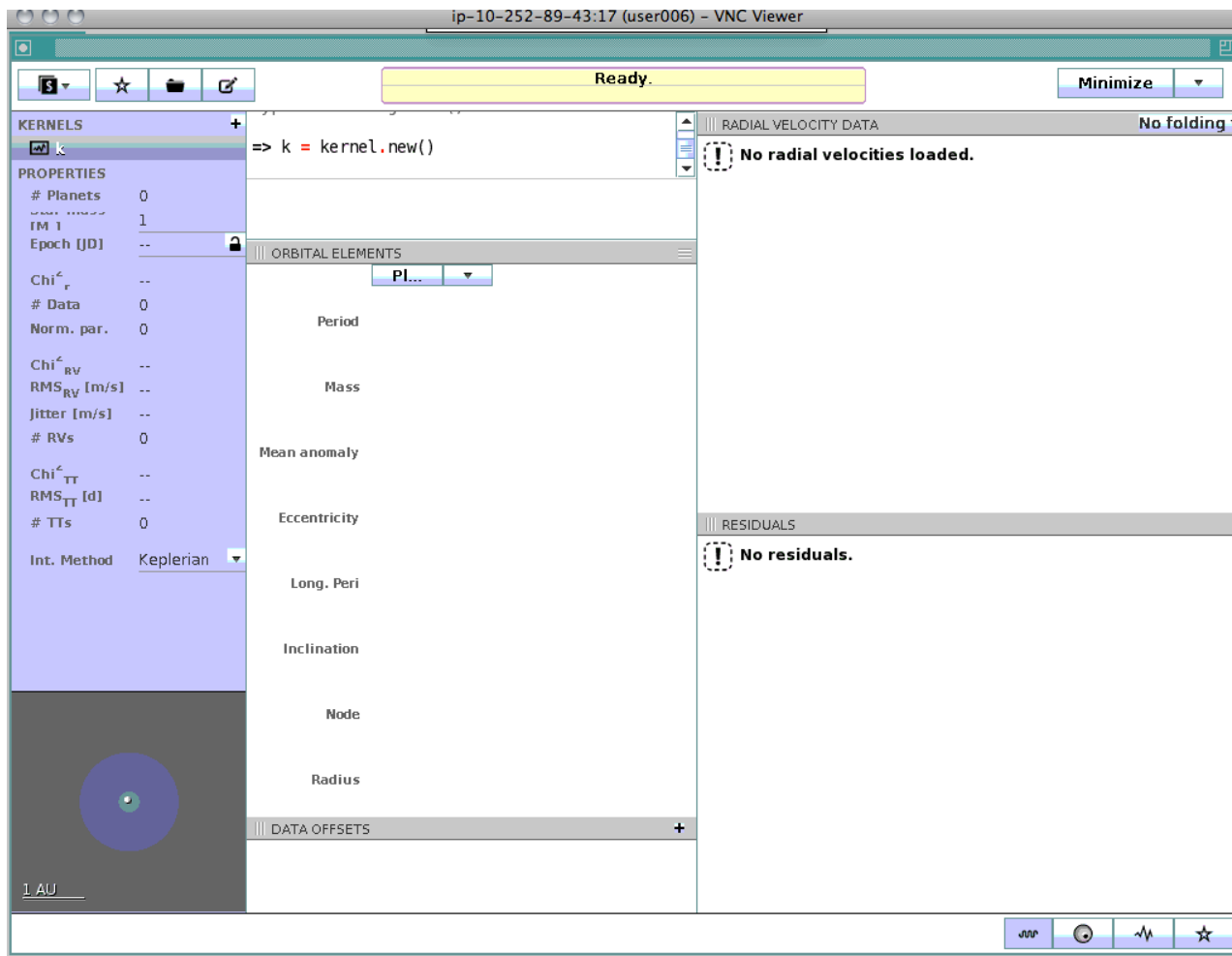
- Systemic can bundle multiple datasets together for simultaneous fitting by using a **.sys* file
- Kepler-18 datasets:
 - *more ~/datafiles/k18.sys*
- TD means transit data
 - We'll be fitting both *k18c.tds* and *k18d.tds*
- Kepler-18 also has RV measurements (*k18.vels*) that we'll be fitting as well
- *k18.sys* also includes the mass of the star and “Epoch”
 - In dynamically interacting systems, the orbital properties of the planets *change as a function of time*. Therefore, we choose a specific epoch at which time we will determine the orbits.

Load Systemic Console

- Get a terminal (from Applications, select `'xterm'`)
- Load the Systemic Console Java GUI
`$ java -jar SystemicGui.jar`
- Resize the window to fill the screen.
- Let us know if you have any trouble loading this up.
- For this workshop, we will be using Systemic v1.8.
 - Versions for various systems are available, e.g. Mac.
 - <http://www.stefanom.org/?systemic> for the latest version, tutorials, etc.

Introduction to the Console

- Panels ("left", "center", "right"); drag subpanels to resize



Load Kepler-18 datasets

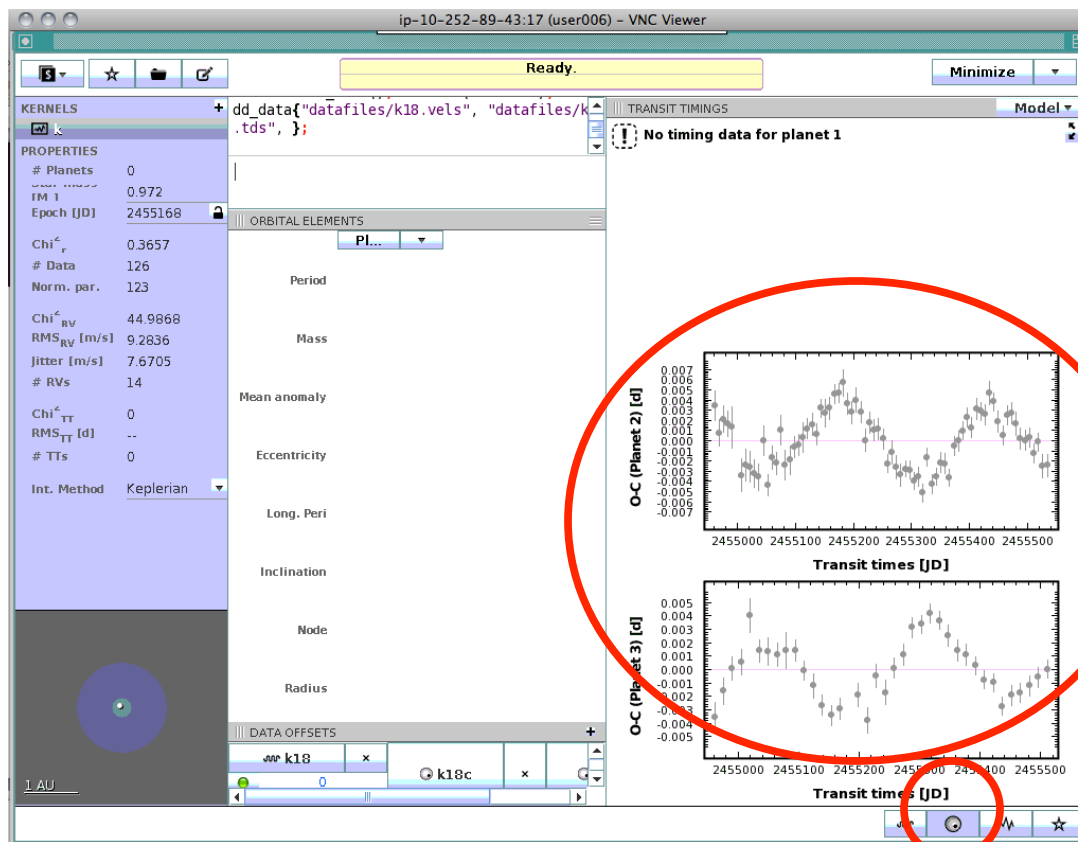
- Click on the star in the upper left or lower right
- Right panel shows filenames and data; "Load" *k18.sys*

The screenshot shows a VNC Viewer window with the following components:

- Kernel Panel:** A star icon is circled in red. Below it, the text `=> k = kernel.new()` is visible.
- Properties Panel:** Lists parameters such as # Planets (0), Epoch [JD], χ_r , χ_{RV} , RMS_{RV} [m/s], Jitter [m/s], # RVs (0), χ_{TT} , RMS_{TT} [d], # TTs (0), and Int. Method (Keplerian).
- Orbital Elements Panel:** Fields for Period, Mass, Mean anomaly, Eccentricity, Long. Peri, Inclination, Node, and Radius.
- Data Browser Panel:** A list of files: `k18.sys` (circled in red), `k18d.tds`, `k18c.tds`, and `k18.vels`.
- Plot Panel:** A graph of Radial Velocity [m/s] vs Time [JD]. The y-axis ranges from -25 to 25, and the x-axis ranges from 245511245518 to 2455322455390. Three data series are plotted: `k18.vels [14]` (blue), `k18c.tds [75]` (red), and `k18d.tds [37]` (green).
- Bottom Panel:** A 'Load in: k' field and a 'Load' button circled in red.

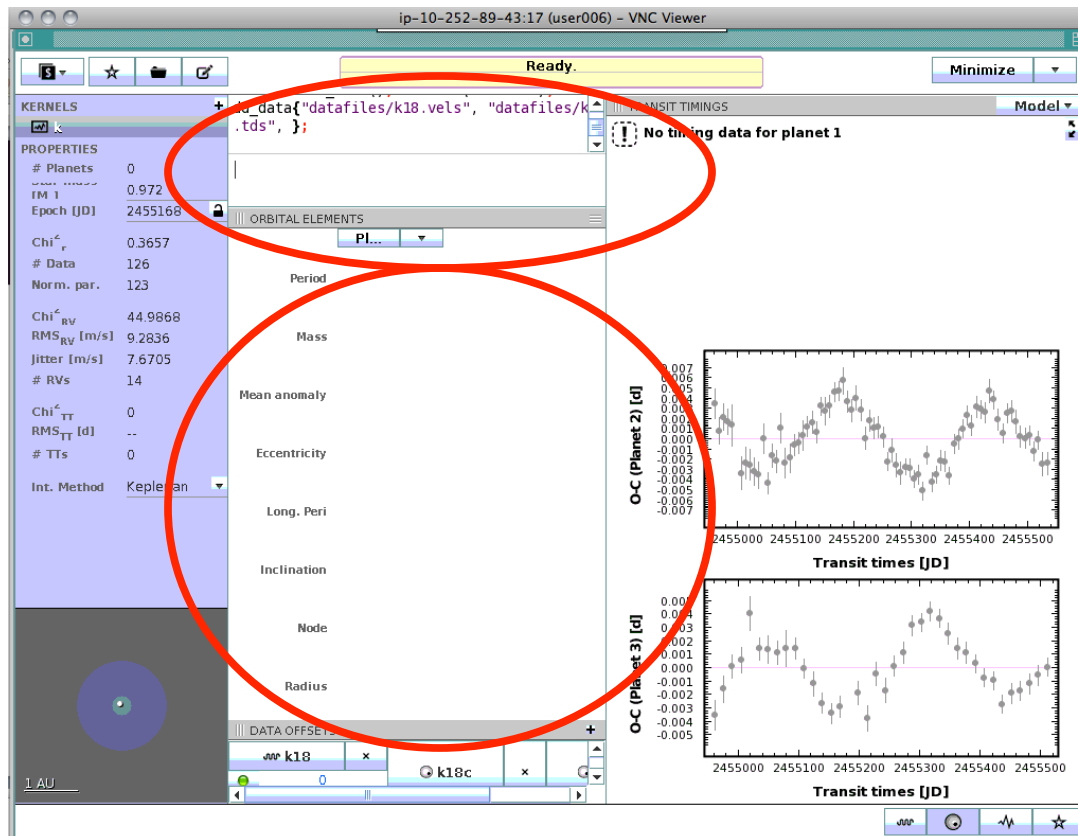
Look at TTV curve

- Click on transit symbol in lower right
- TTV data with best-fit period subtracted is shown



Center Panel

- Top subpanel: terminal echoing commands; we'll ignore
- Middle subpanel: Set planets' parameters: *SCIENCE!!*

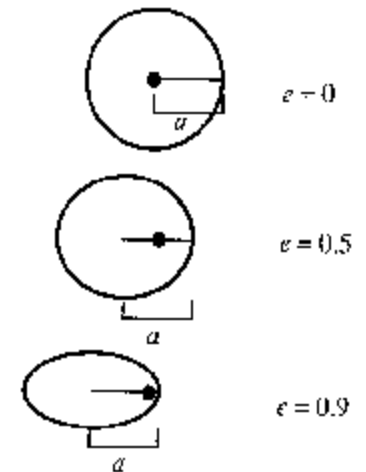


Planet Parameters

- Physical parameters: mass and radius
- Orbital parameters: period, mean anomaly, eccentricity, longitude of periapse (“long. peri.”), inclination, longitude of the ascending node (“node”)
 - In combination with stellar mass, these completely describe the position and velocity of the planet
 - Coordinate System: polar (radius, angle from reference line); centered on star; reference plane is the plane of the sky; reference line (in reference plane) is Astronomical North

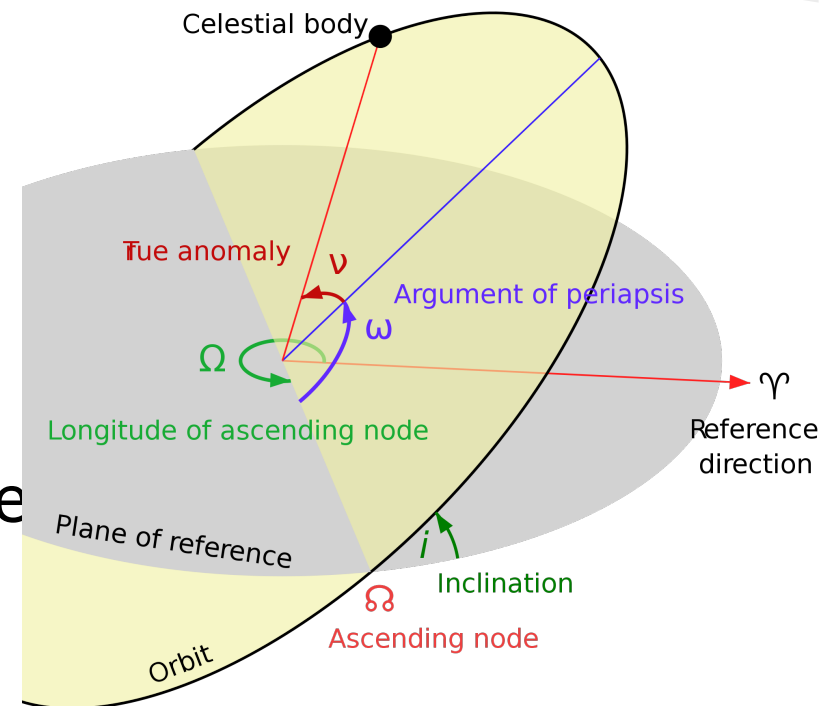
Orbital Elements: Size and Shape

- Period (P): time to make one complete revolution
 - If the masses are known, Kepler's Third Law directly relates the period to the semi-major axis (a), the physical size of the orbit (usually measured in AU)
 - Since we measure times and don't know masses perfectly, better to solve for periods
- Eccentricity (e): a measure of how circular an ellipse is
 - Transiting planets give very weak constraints
 - TTVs can help a lot!



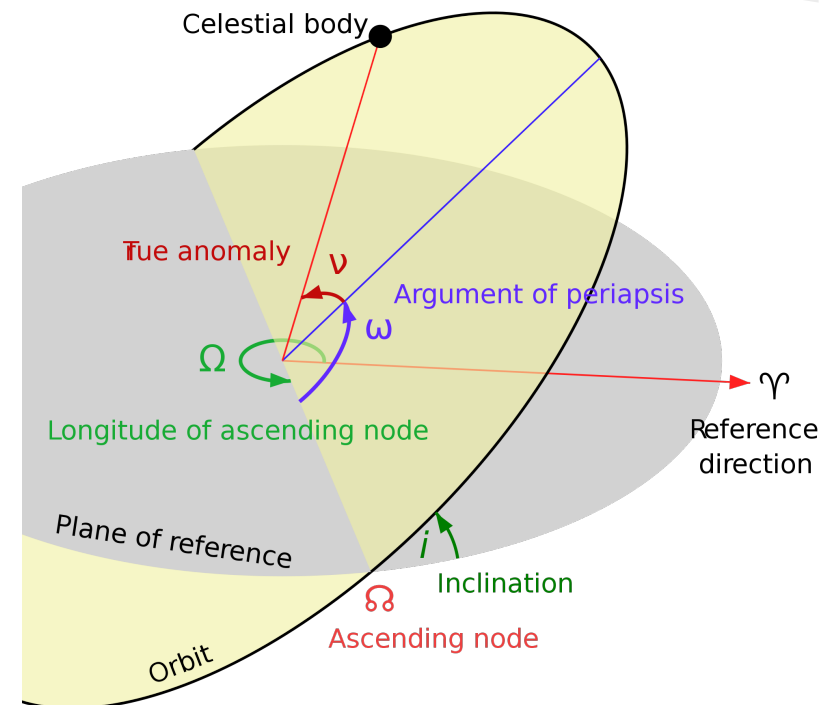
What do we need to know?

- Inclination (i): angle relative to sky plane
 - Edge-on (transiting) = 90°
- Longitude of the Ascending Node (Ω): usually not measurable (except for relative nodes between planets)
- Argument of periapse (ω): angle in plane of the orbit from nodal line to periapse
- Longitude of periapse ($\tilde{\omega}$): $\Omega + \omega$



Orbital Elements: Orientation Angles

- True Anomaly (ν or f): Angle in plane from periapsis to current position of planet
 - Non-uniform as a function of time due to Kepler's 2nd Law
- Mean Anomaly (MA): An averaged angle from periapsis directly related to the true anomaly by Kepler's Equation
 - Uniform as a function of time
- Etc.

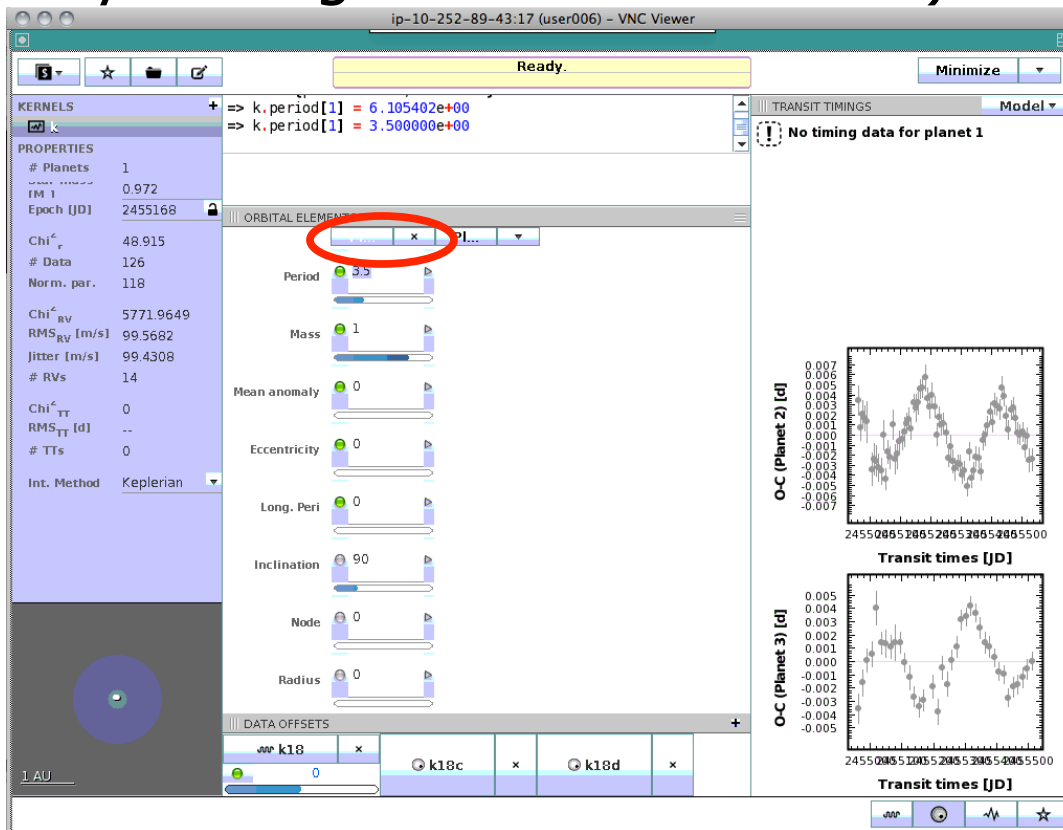


What do we need to know?

- Mean Anomaly (MA) is like orbital phase and is directly related to the time of transit
 - Regardless of TTVs, to get transit times right, we must have very correct values of P and MA
- Even in the geometrically simple case of circular motion, changing some elements (e.g., Ω) can change the geometric meaning of other elements (e.g., MA), sometimes in non-intuitive ways. Basically, MA (and often P) should always be left as a free parameter.
- We'll start with circular, coplanar, edge-on ($i=90^\circ$) planets, a very good approximation for Kepler-18 (C +11)

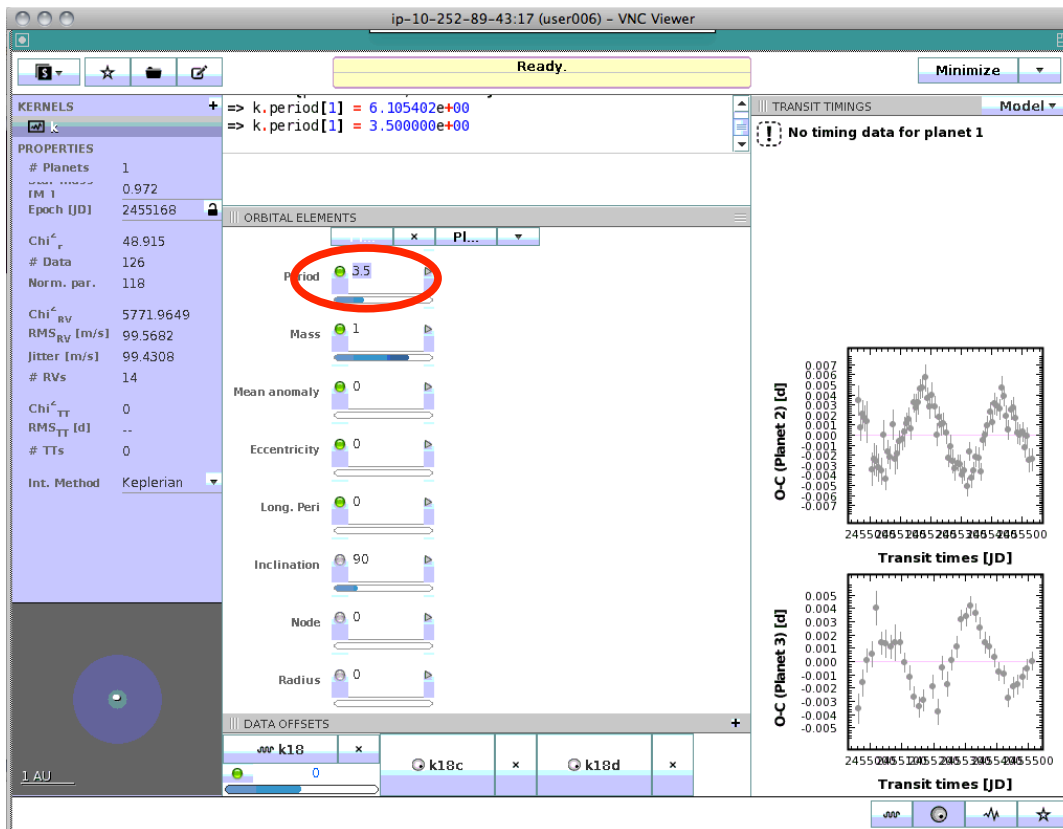
Console: Add Planet

- Add a planet by clicking on the "Planet N" button
- (Delete by clicking x next to the name.)



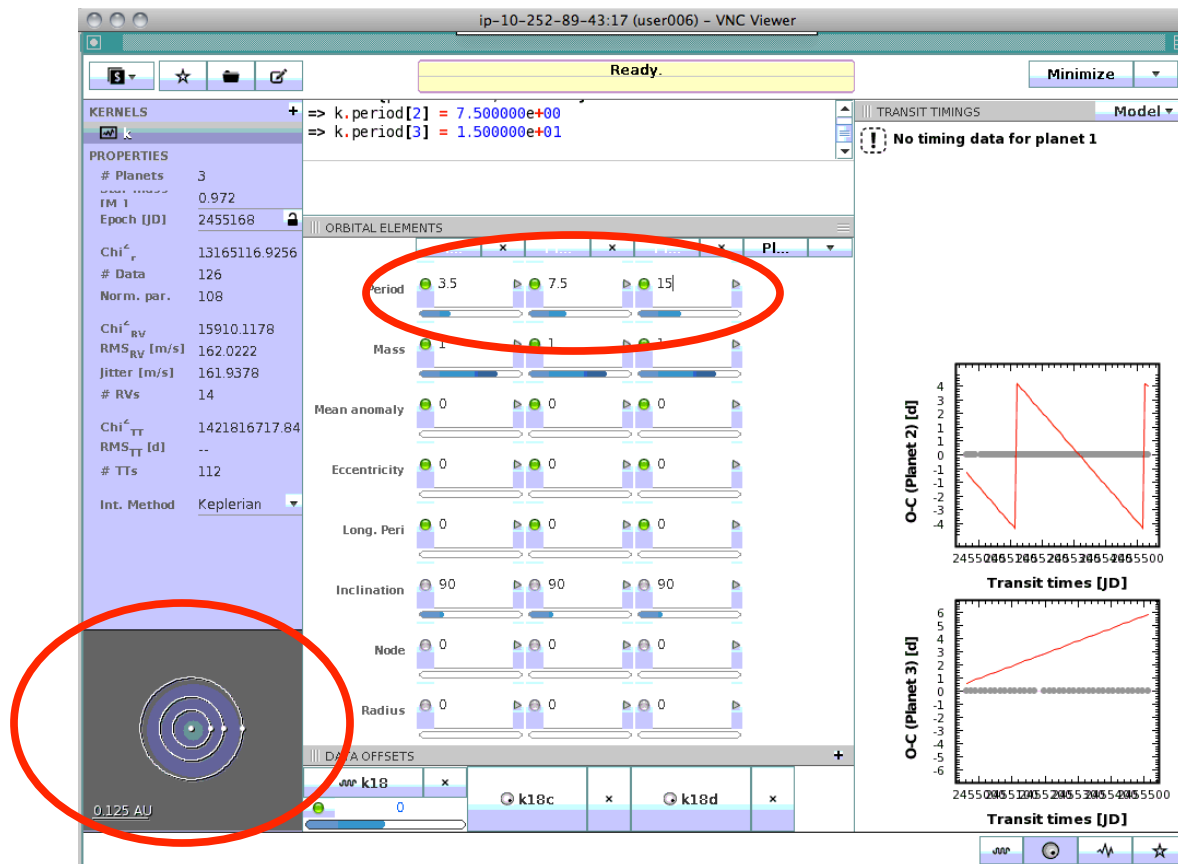
Console: Change Planet Parameters

- Click on text, edit, and hit enter
- Can also use slider underneath text



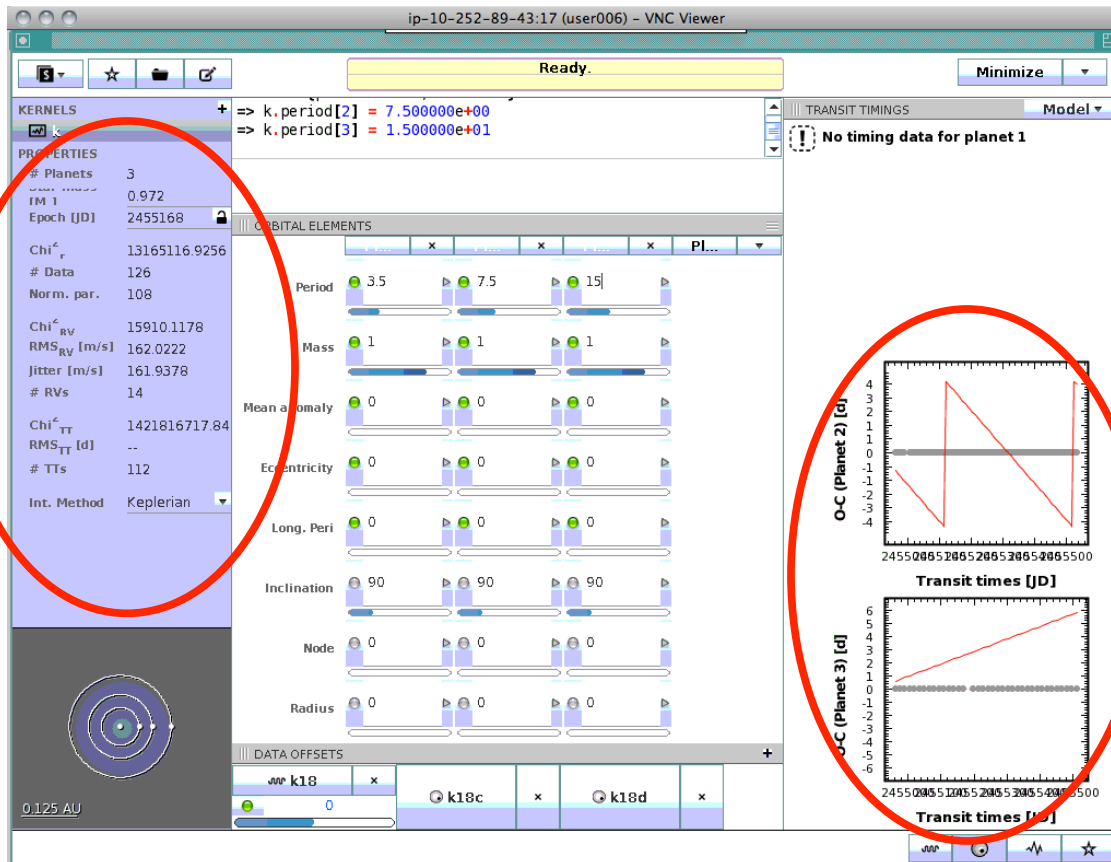
Console: Add Kepler-18 Planets

- Type approximate periods: 3.5, 7.5, and 15 days; “enter”.
- Lower left subpanel: orbit diagram; manipulate with mouse



Console: TTV Model

- Right panel auto-updates: shows current model as red line
- Left panel auto-updates: shows current goodness of fit (χ^2)

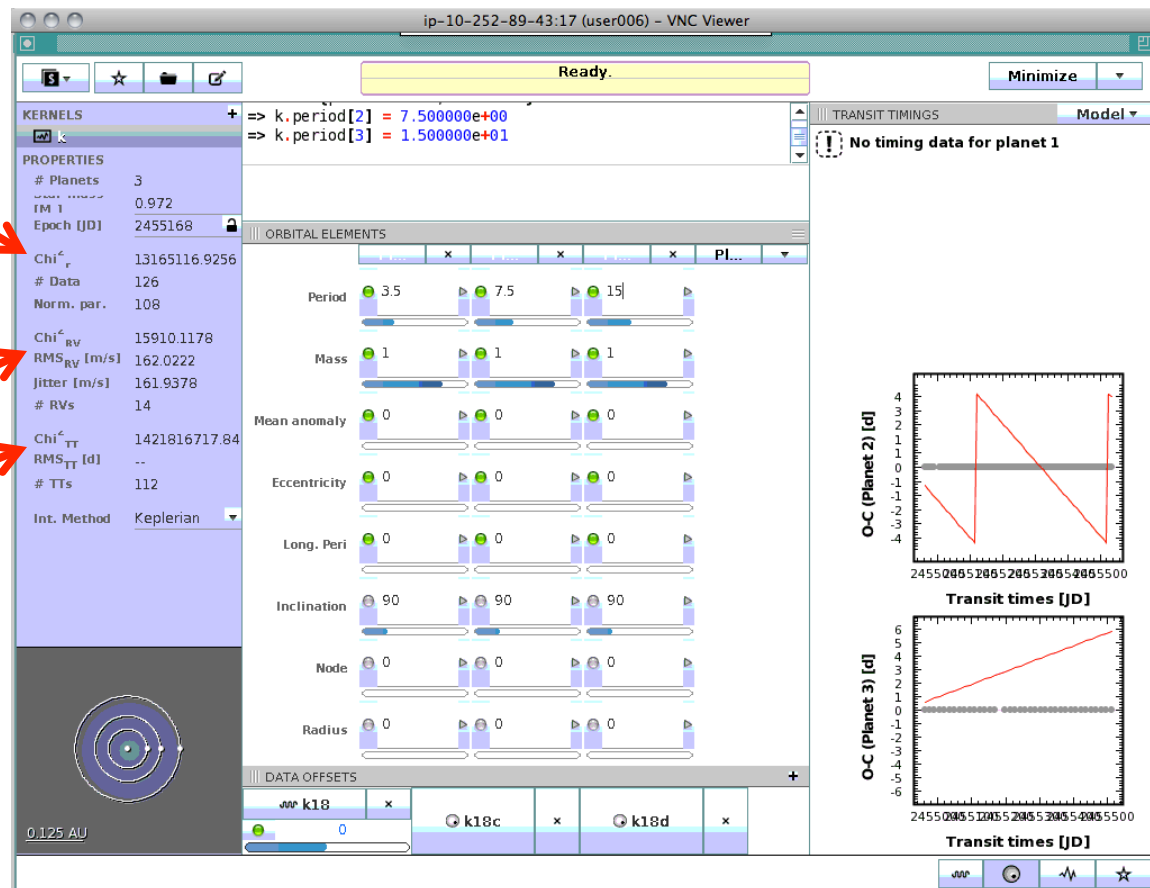


Console: Built in Minimizer

- Console has built-in minimizers to automatically improve fit

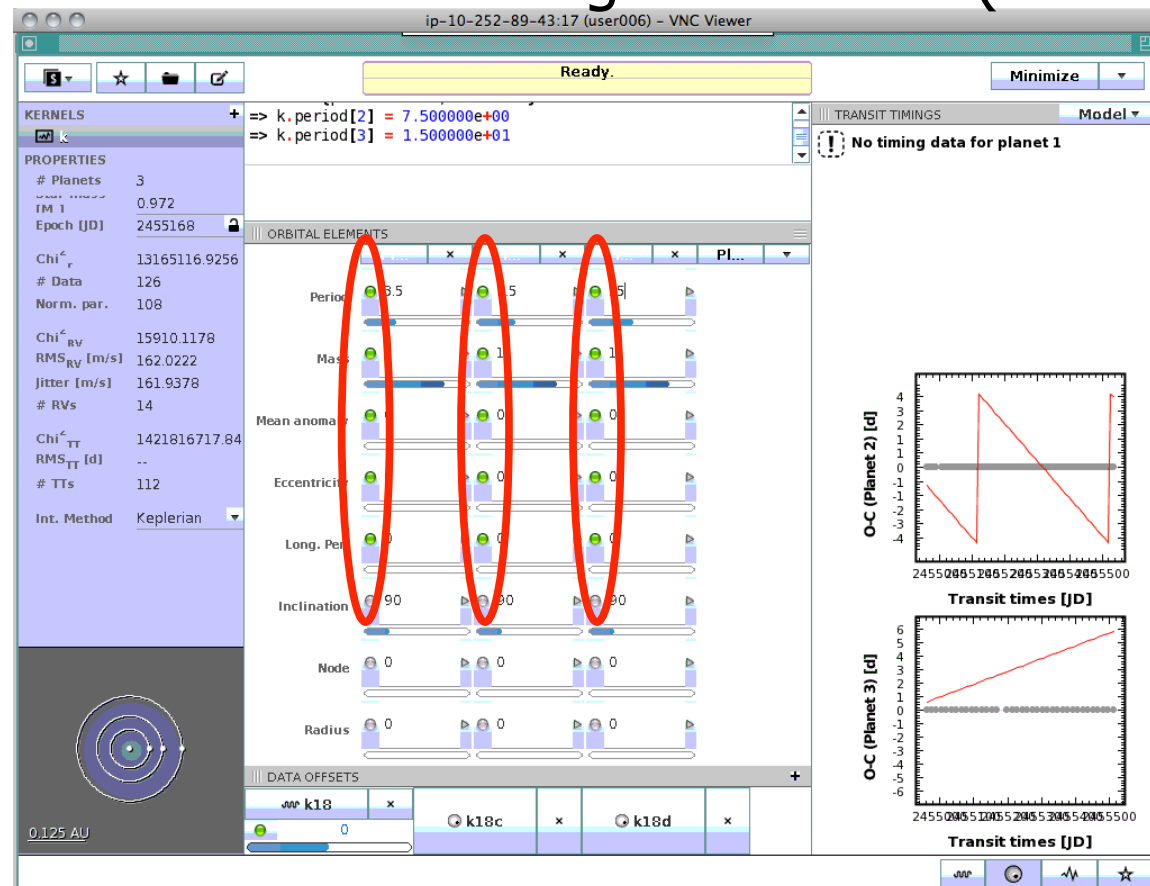
Figure of merit is reduced chi-square (χ^2_r), ~ 1 = "good fit"

χ^2_{RV} : RV
 χ^2_{TT} : Transit times



Console: Built in Minimizer

- Radio buttons near parameters determine what is included in the multi-variate minimization: green = "on" (included)

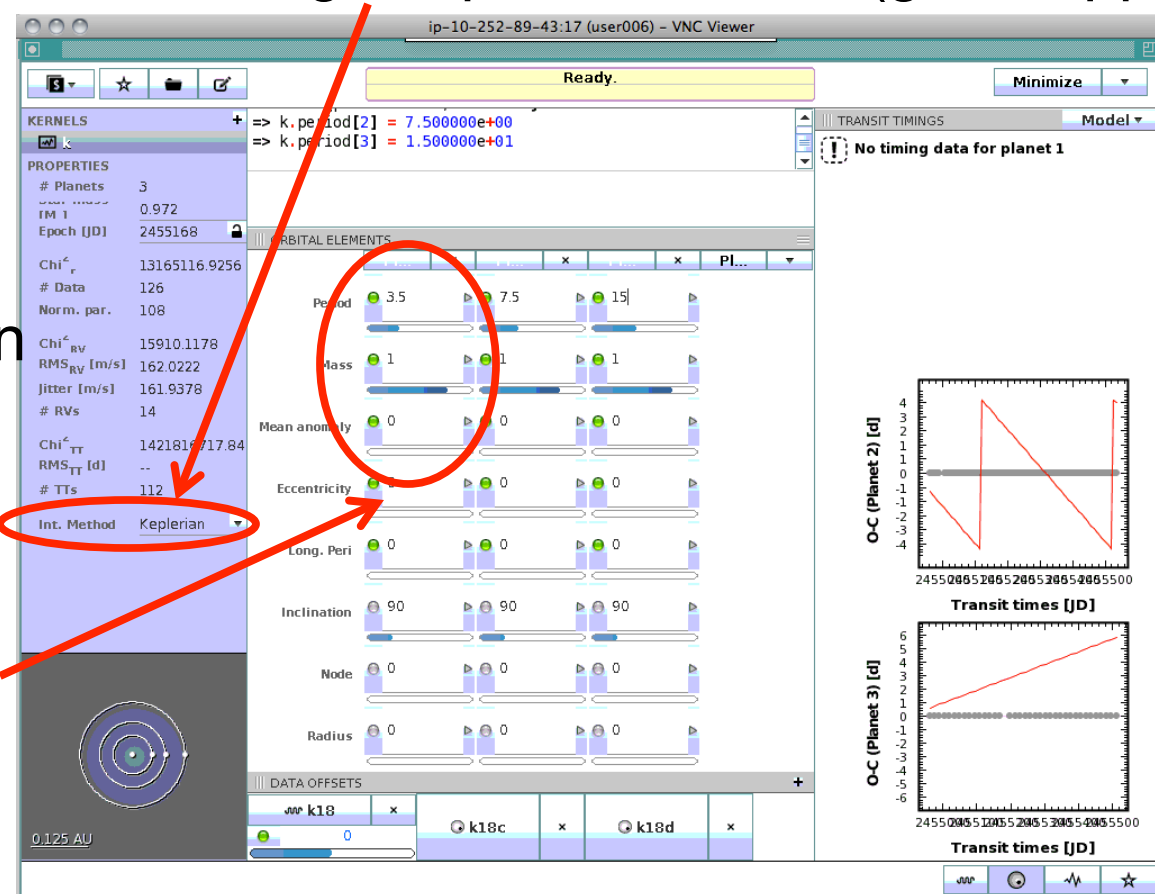


Console: Start Slowly, and fix b

- Minimizers need a good initial guess to converge correctly
- Start with non-interacting “Keplerian” orbits (good approx.)

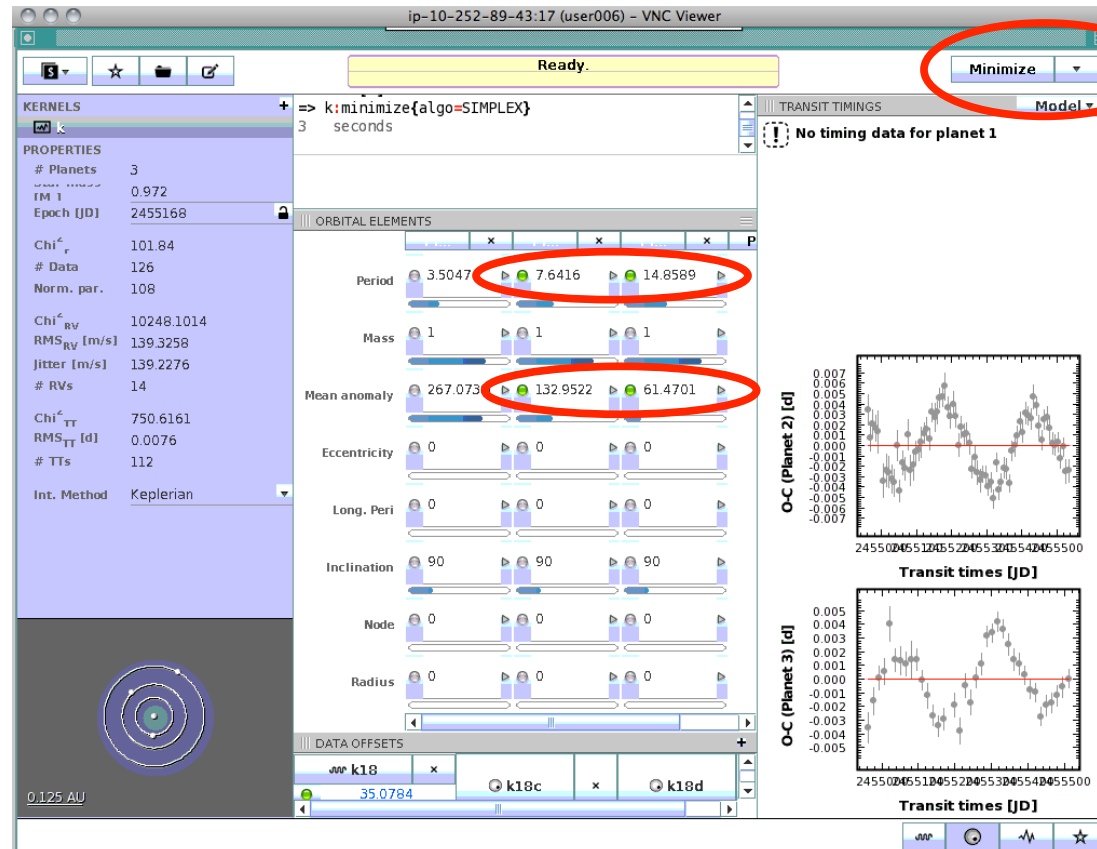
- Planet b is not our focus, use properties known from transit:

– $P=3.504725$
and
 $MA=267.0738$



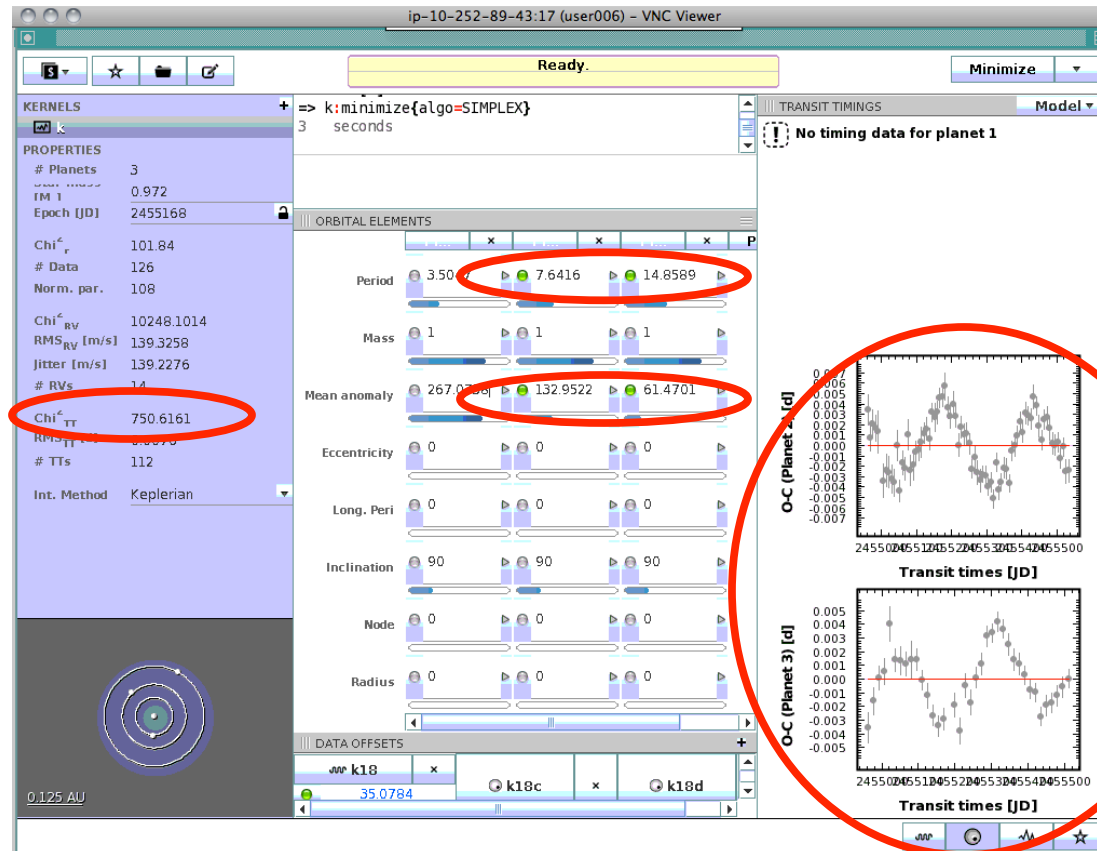
Console: Better Periods and Phases

- Deselect all buttons for minimization except Period and Mean Anomaly of Planets 2 and 3. Hit "Minimize"!



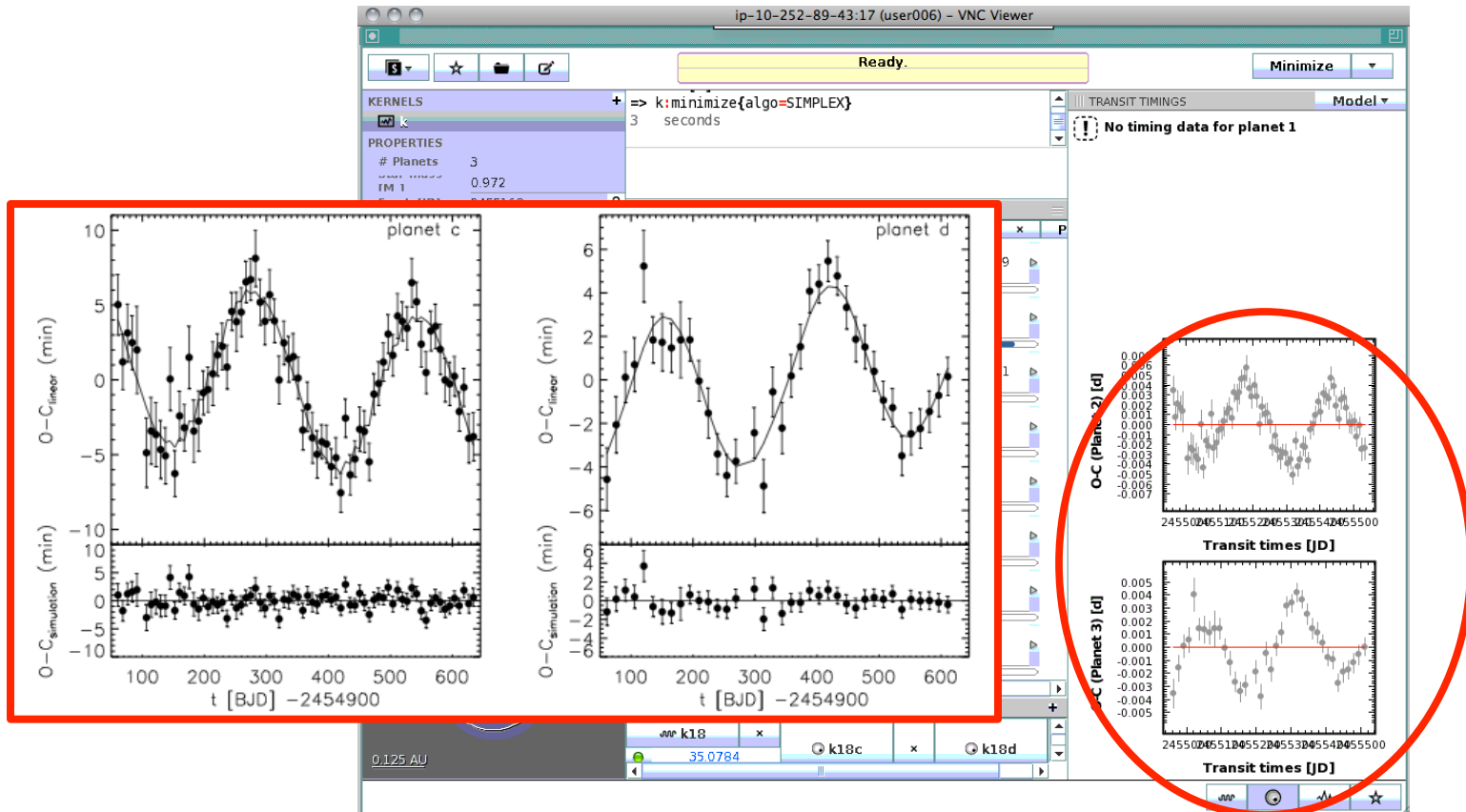
Console: Better Periods and Phases

- Systemic determined best-fit periods and phases.
- Red model line and Chi^2_{TT} are much much better.



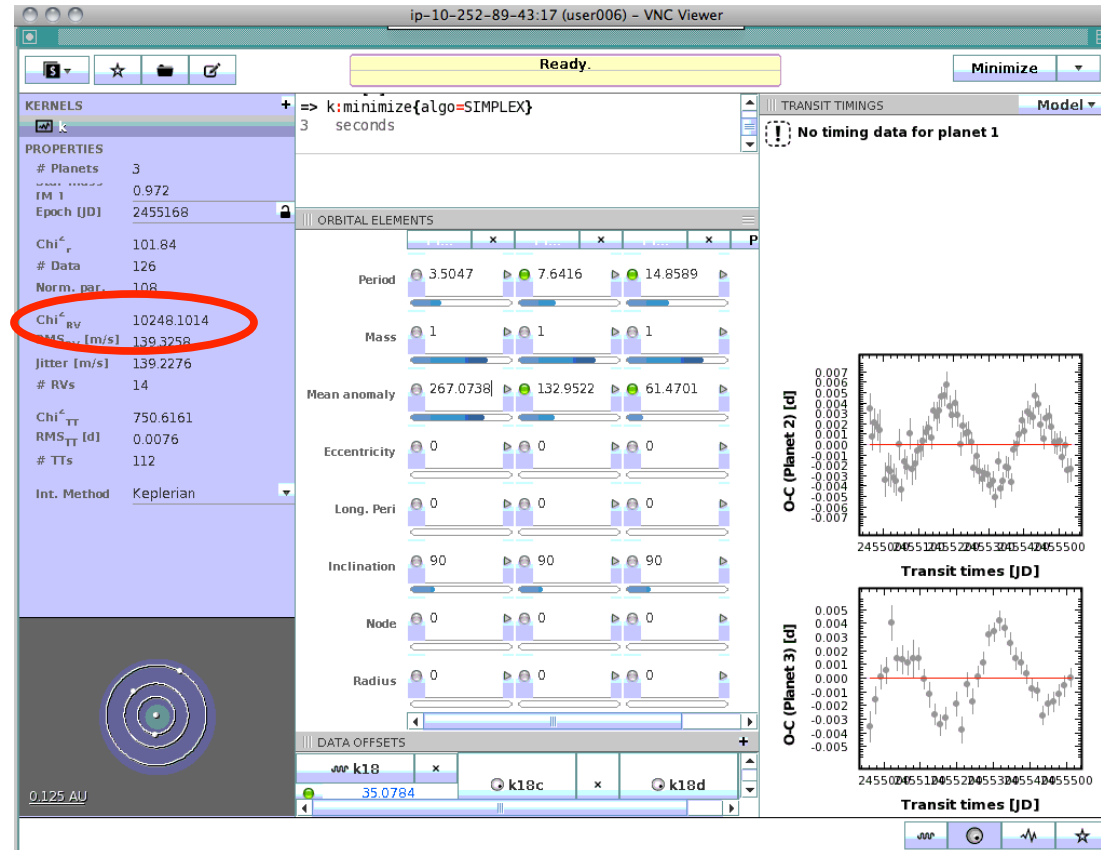
Compare to C+11 Values

- TTV (or O-C) is now similar to C+11 Figure 7
- C+11 Periods: 7.64159 and 14.85888 (Table 4)



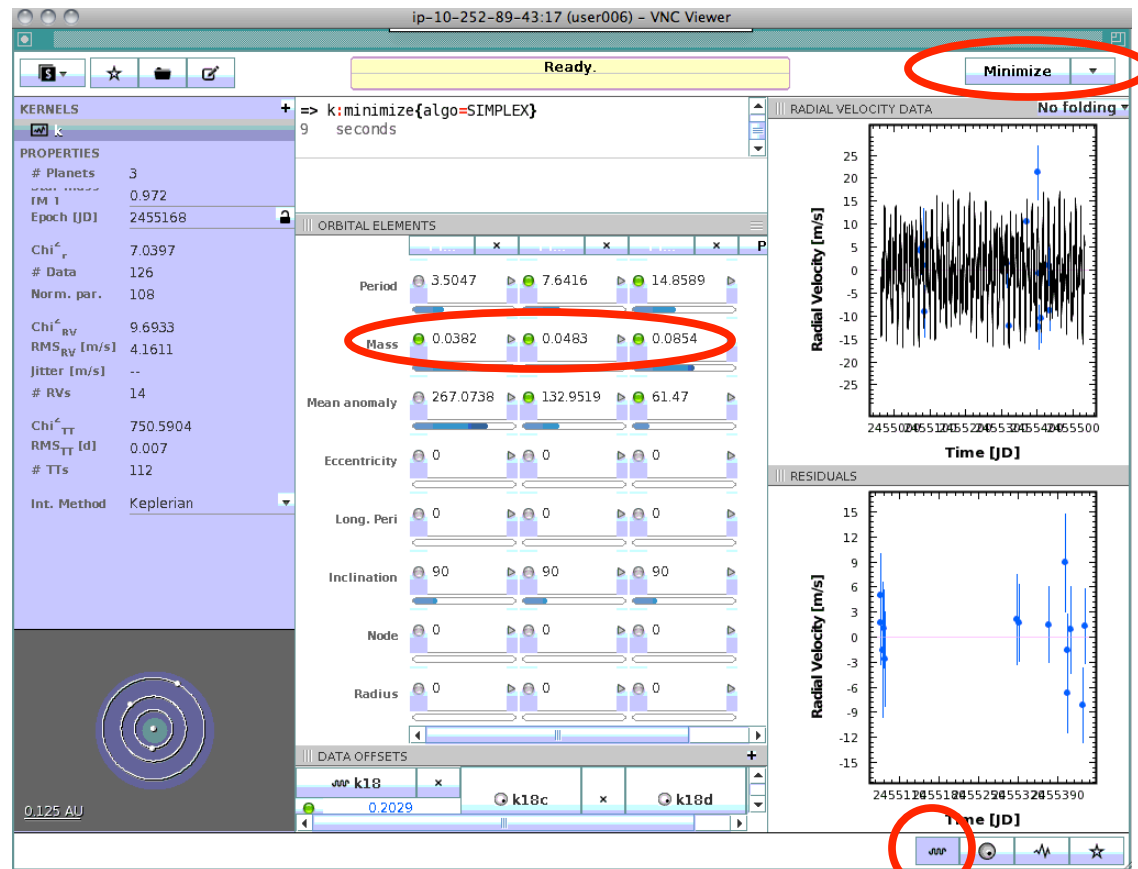
Console: Radial Velocities

- Both data (Chi^2_{RV}) and typical planet densities suggest that 1 Jupiter mass is way too big for these planets (2, 5.5, and 7 Earth radii).



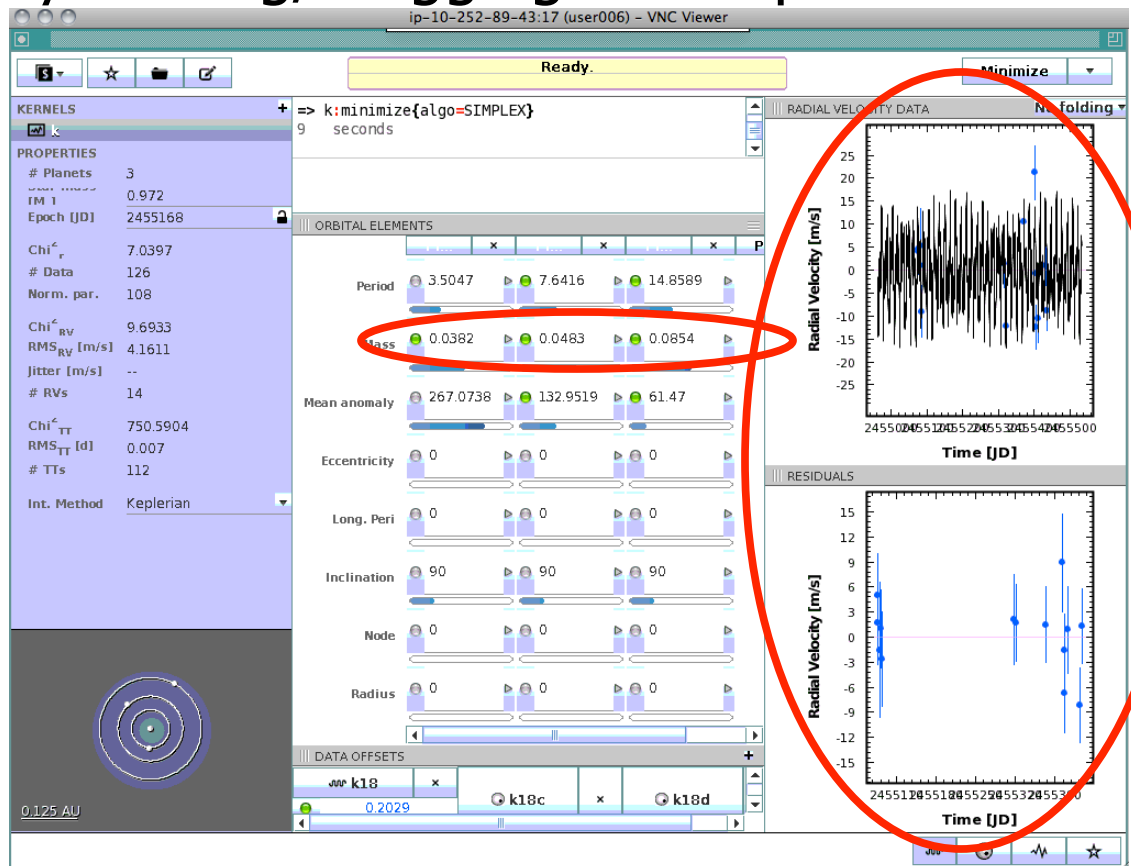
Console: RVs to estimate masses

- Add all 3 masses to minimizer by clicking their green buttons on. Hit the sine-curve button to view the data. "Minimize"!



Console: Better mass estimates

- New masses are more reasonable (0.01 Jup \approx 3.2 Earths)
- Zoom in and out by clicking/dragging on RV panel.
- Note that TTVs will also determine masses



Effect on TTVs?

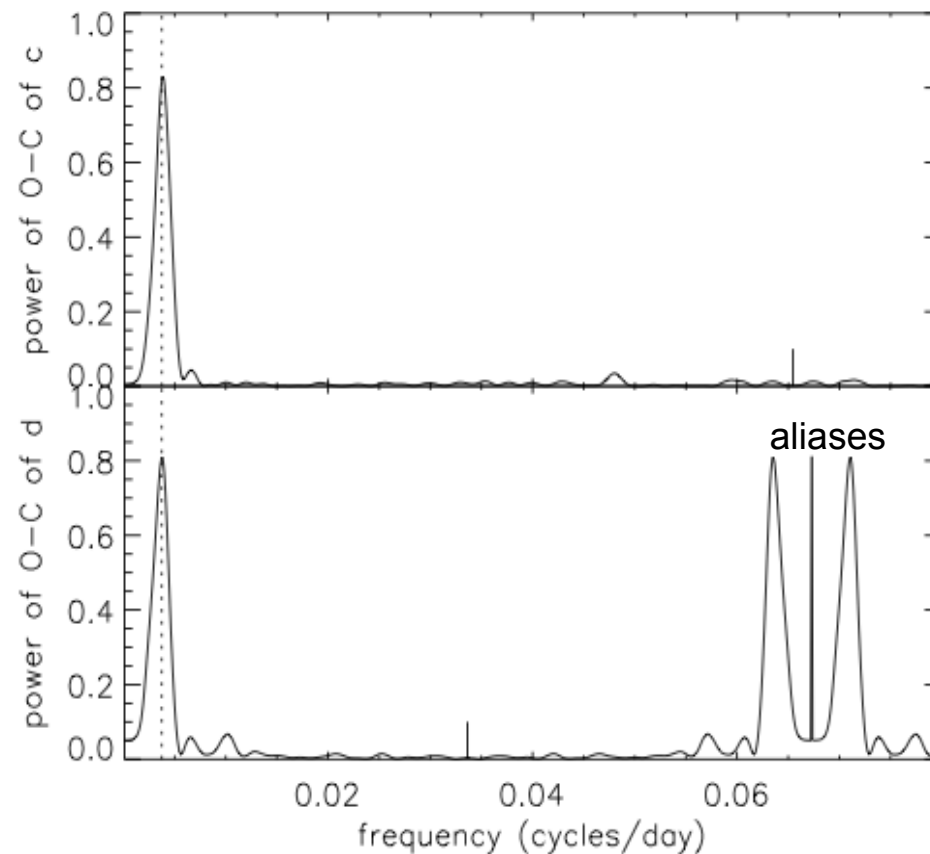
- Click the transiting planet button. With our new mass estimates, is the TTV fit (χ^2_{TT}) improved? Why or why not?

Interpreting TTV Curves

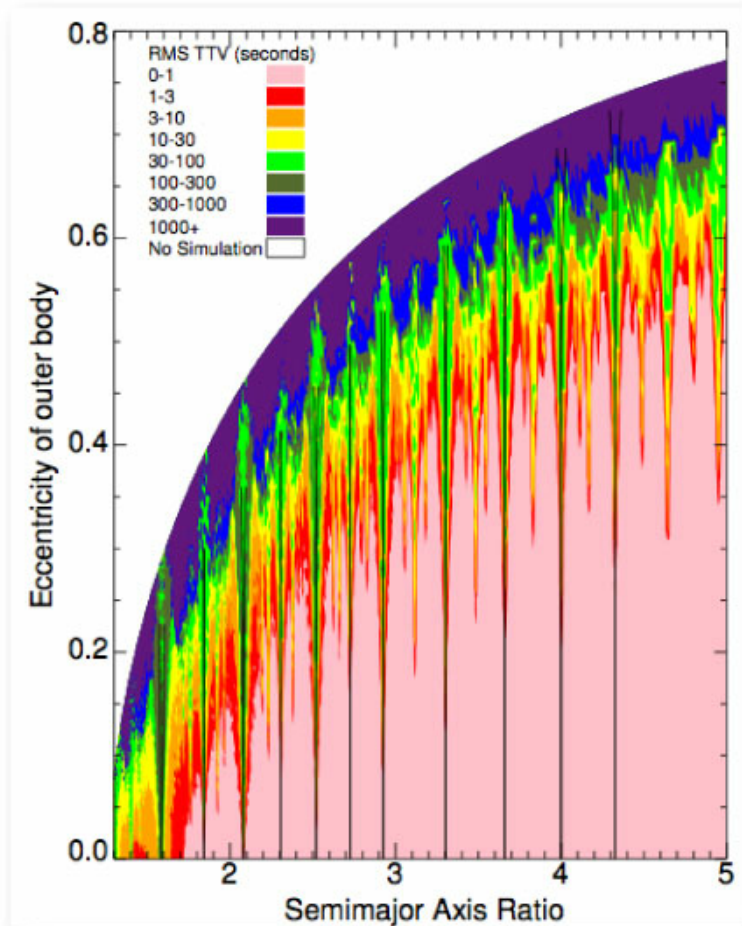
- The ratio of the period of the planets is 1.944
 - Near a ratio of small integers, i.e., a commensurability or (mean motion) resonance
 - At/near resonances, interactions can add coherently
 - May or may not be actually in resonance
- Near (but not deep in) resonance has predicted timescale:
$$\mathbf{P_{TTV} = 1/(2/P_d - 1/P_c) = 268 \text{ days}}$$
- (Period of the sweeping of the “line of conjunctions” across the line of sight)
- Ratio of amplitudes is related to ratio of planetary masses (smaller planets get pushed around more)

TTVs at Expected Period

- C+11 Figure 8 shows a power spectrum of the TTV signals for both planets and a dashed line at the predicted period



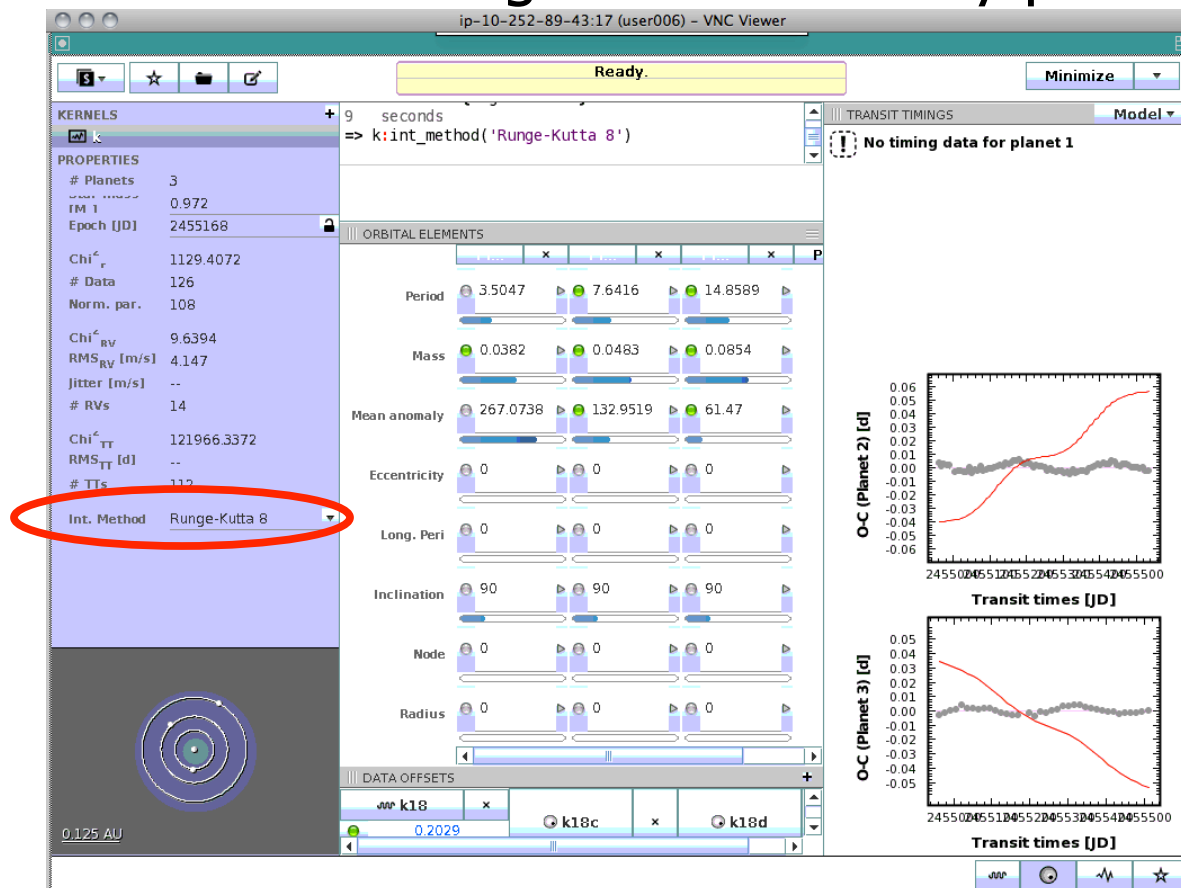
TTV Amplitudes



- Amplitude of TTV signals is very sensitive to various orbital parameters, particularly period ratio (e.g., Veras et al. 2011, left)
- Especially when the perturbing planet is non-transiting it can be *difficult to impossible to determine the properties of the perturber* from TTVs alone (Meschiari et al. 2010, Ballard et al. 2011, but see Nesvorny et al. 2012).

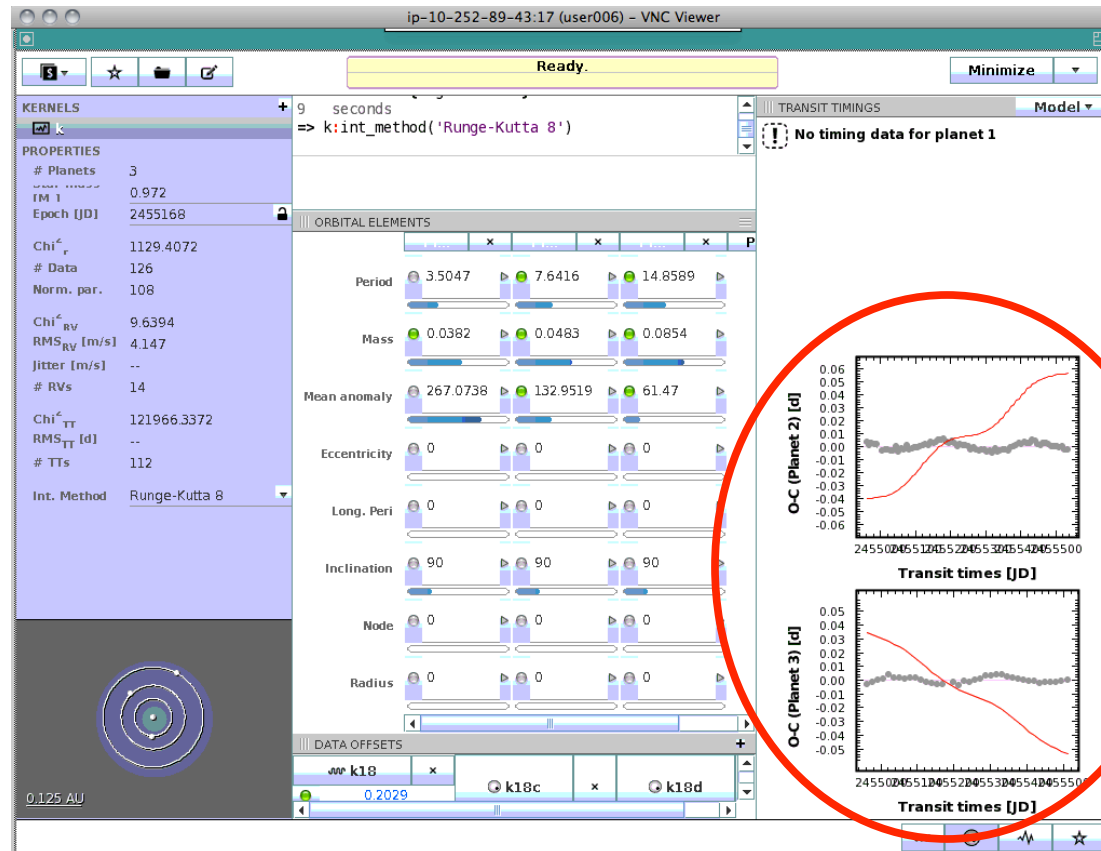
Console: Interacting Planets

- In the middle of the left panel, change the “Int. Method” to “Runge-Kutta 8”: this integrates the 4-body problem



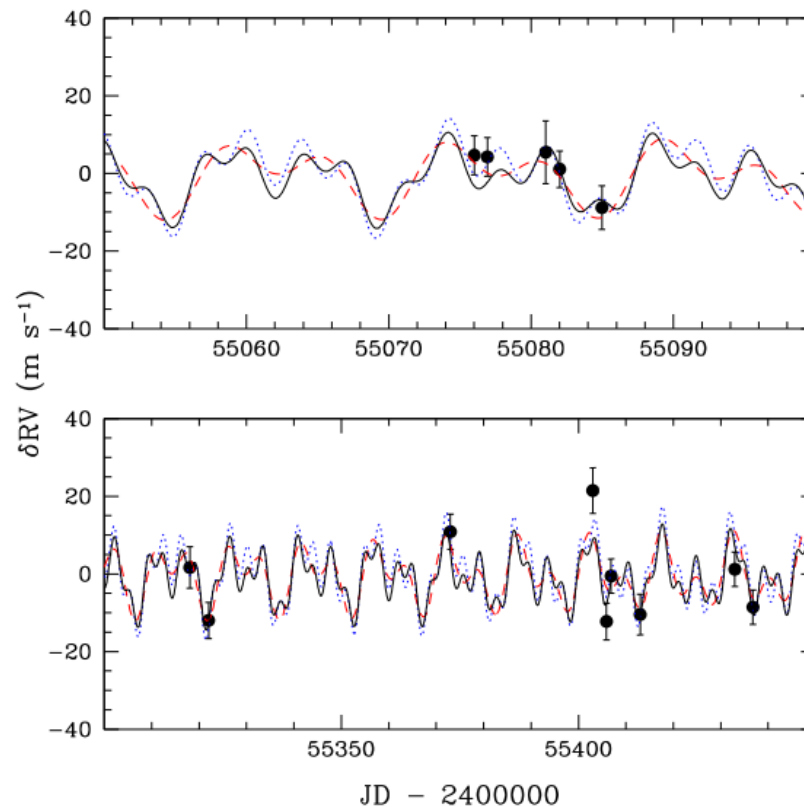
Console: Interacting Planets???

- The red model line is now curved, indicating non-Keplerian
- What happened to our nice TTV plot? (Why?)



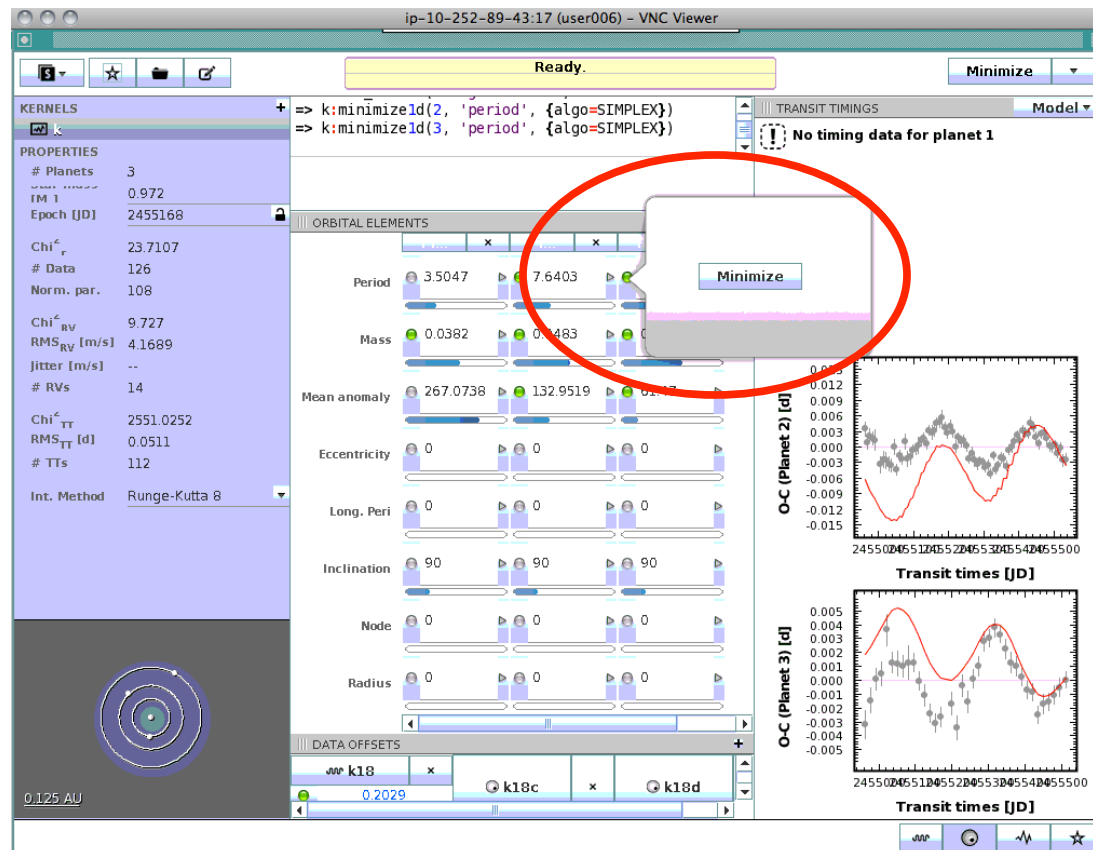
RVs of Interacting Planets

- Are the RVs any different for interacting planets?
- C+11 Figure 4: Keplerian = dashed red, interacting = black



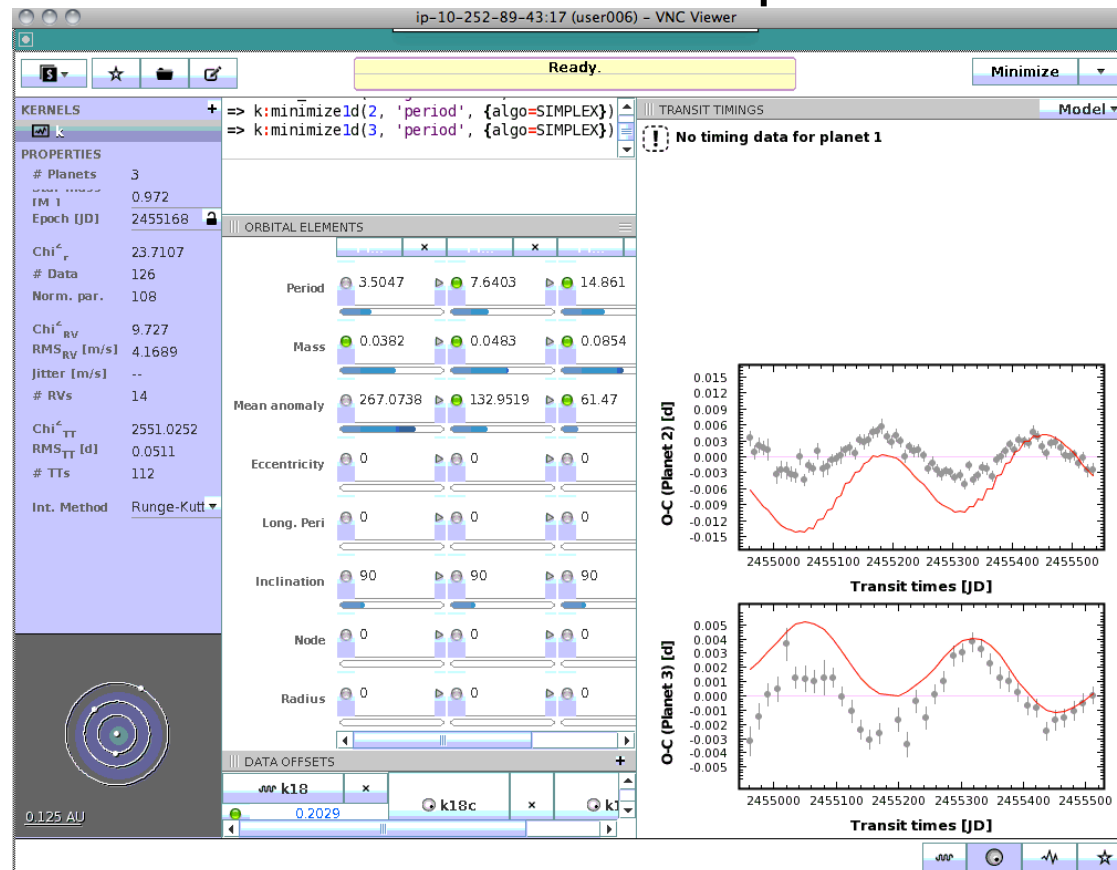
Console: Single parameter minimizer

- Click on the arrow (▶) to the right of Period of Planet 2
- Hit "Minimize"! Do the same for Period of Planet 3



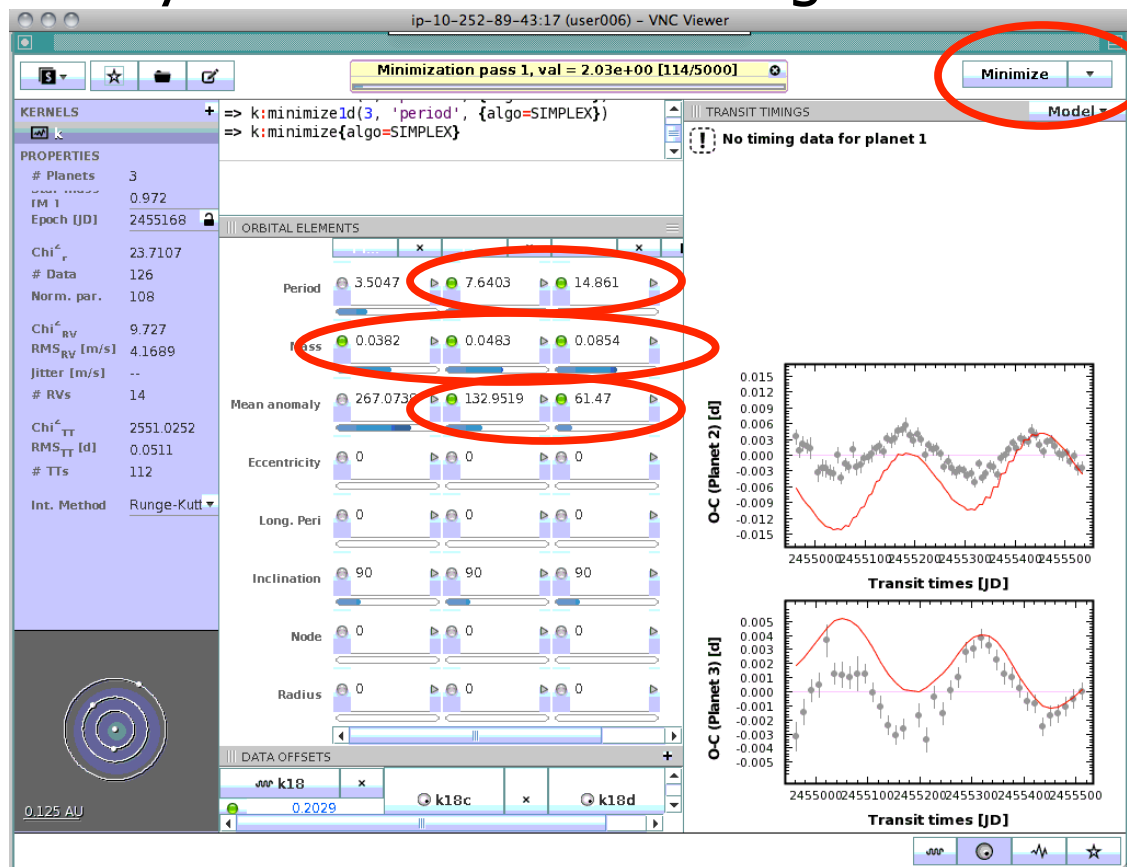
Console: Single parameter minimizer

- Improved average period estimate a lot.
- Single parameter minimization helps refine initial guess



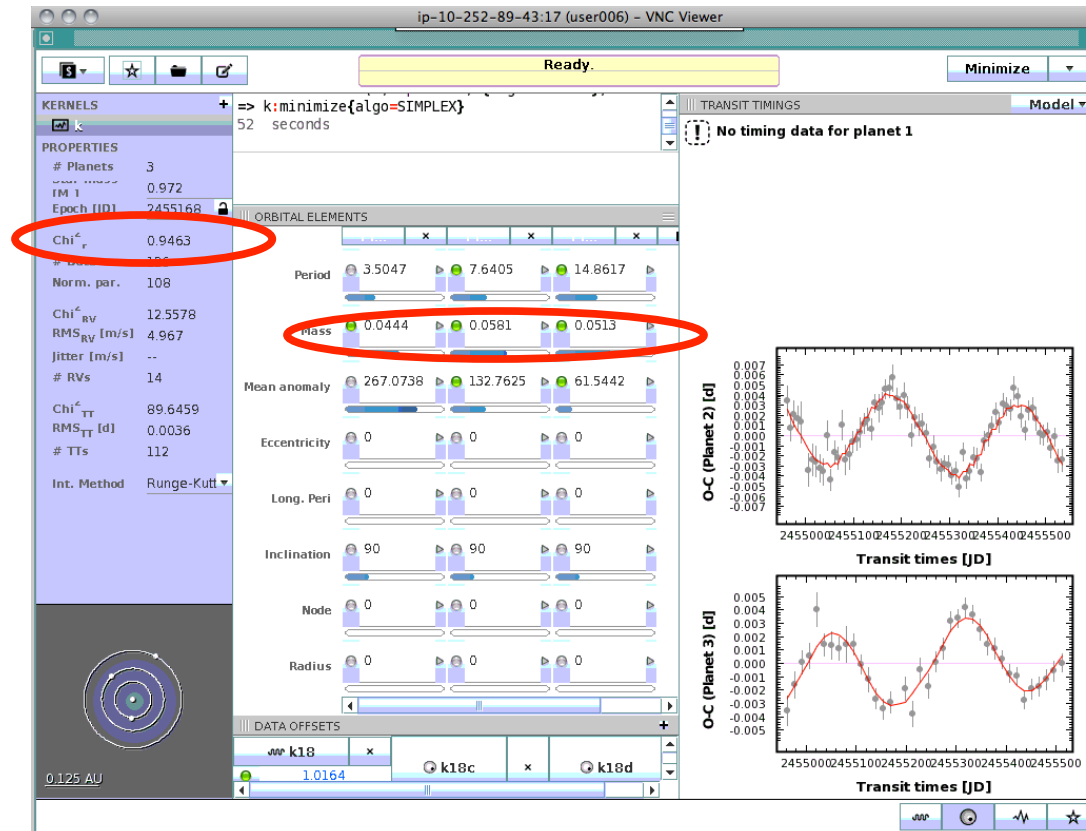
Console: Fit the TTVs!

- Activate masses of all 3 planets and P and MA of 2 and 3.
- Hit Minimize! Why does this take so long?



Console: Final Fit!

- $\chi^2_r \approx 0.94$ suggests we have an excellent fit!
- C+11: 0.022 ± 0.011 , 0.054 ± 0.006 , 0.052 ± 0.004

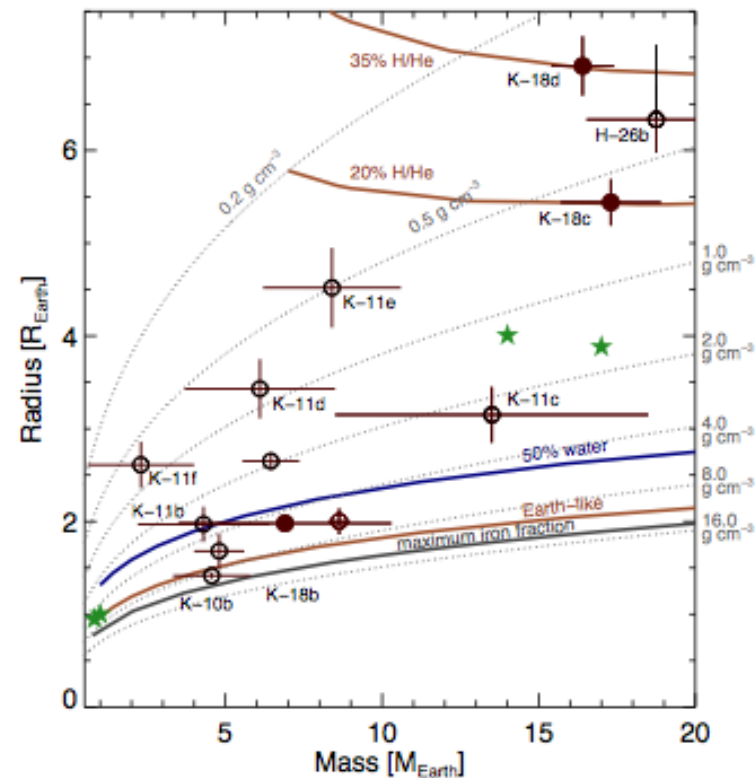
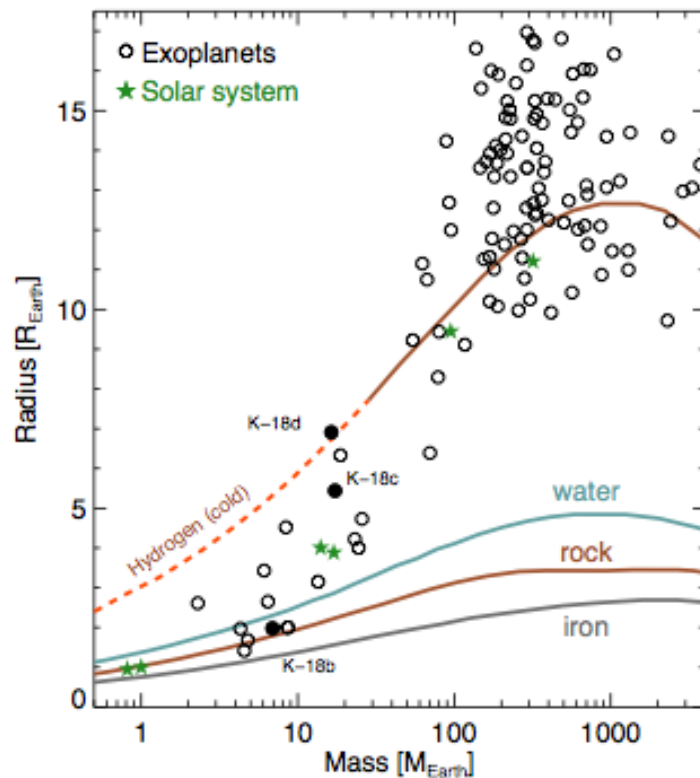


Thoughts on the final fit

- Minimizers often go to local minima instead of global minima. Even slight differences in the ordering of fitting can lead to different results: you may have different reduced chi-square or masses. For well-posed problems, the differences will usually be within the noise, unless you've gotten stuck in the wrong local minimum (usually due to a poor initial guess).
- Note also that C+11 had some small differences in analysis
- With practice, you can measure masses and other properties due to TTVs rapidly and efficiently.

Thoughts on the final fit

- We just measured the masses of two extra-solar planets!
- Now we can put our results on a mass-radius diagram
 - Kepler-18c and d must have significant fractions of H/He!



Additional Exercises

- Play with (or read about) other aspects of the Console (e.g., saving kernels, other minimizers, etc.)
- Fit for planet masses without RV data
- Fit for eccentricities and longitudes of pericenter for Kepler-18 c and d
- Determine upper limit on mutual inclination from TTVs (Hint: use sky-inclination determined by lightcurve shape, which are published)
- Think about how to use the Console to measure errors
- Fit your favorite TTV data! Fill in the small planet Mass-Radius plot!
- Publish results!

Discovering planets with TTVs

- New algorithms:
 - Widen the duration and use BLS
 - Others in development (non-linear ephemerides)
 - **Quasiperiodic Automated Transit Search (QATS)**
Carter & Agol
 - Ready for general use
 - Already discovered Kepler-36b (Carter, Agol, and the Kepler Team 2012)
 - *Let's use it today!* **(Thanks to Josh Carter!)**

Pre-QATS

- Compiled in your directory: C++ code with an IDL wrapper
- We'll apply it to Kepler-30 ("Strong Example" above)
- Before running QATS: cleaned, detrended data (provided for you in *~/qatscpp/koi806data.sav*)

Run QATS!

- *cd ~/qatscpp/*
- Start IDL by typing '*IDL*' at the command prompt
- *.compile searchQATS* (prepares the IDL file to be run)
- *restore, 'koi806data.sav'* (loads in Kepler-30 data)
- *searchQATS, x, sigma=robust_sigma(f), pmin=250, pmax=22000, fraction=[0.025], q0=1.08d0, ql=0.75, qr=1.25, deltabg=[2], threshold=7.1d0, output='KOI806test', header=['KOI-806test'], tstart=min(time), dt=0.0204335d0, maxDetect=6*
- See also:
http://www.ucolick.org/~fabrycky/saganws/QATS_notes.txt

QATS options

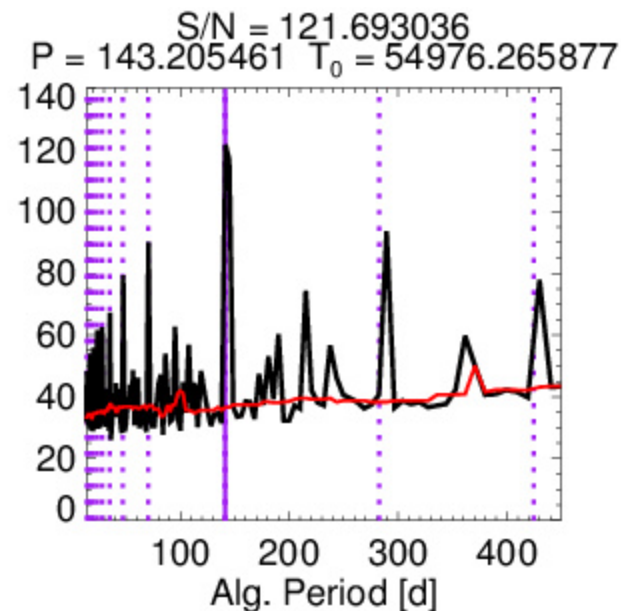
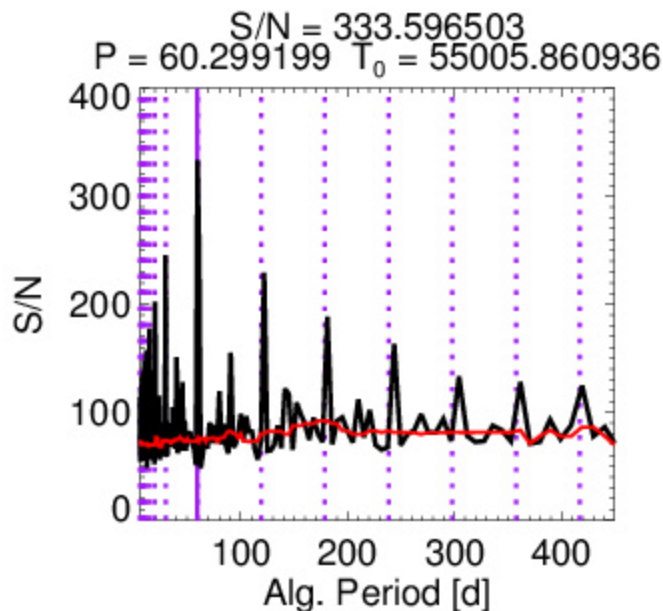
- $pmin=250, pmax=22000$: min and max periods searched (in cadences)
- $fraction=[0.025]$: transit-to-transit slop in orbital period
- $q0=1.08d0, ql=0.75, qr=1.25$: specifies transit durations
- $tstart=min(time), dt=0.0204335d0$: cadence length in days
- $output='KOI806test', header=['KOI-806test']$: for the plot
- $deltabg=[2], threshold=7.1d0$: measures of significance
- $maxDetect=6$: how many signals to propose as planets

Investigate QATS Results!

- QATS gives some command line output from which we see that 4 planets were detected with varying periods and significances
- QATS also returns a results and diagnostics file: *KOI806test_QATS_0.eps* (or a similar filename)
- Let's take a look at this: *gv KOI806test_QATS_0.eps*
- Each column corresponds to a single candidate and has four main parts: spectrum, "river plot" (with TTVs), and co-added transit plot
- (Additional informative text about the file and thresholds, etc. not belonging to any column)

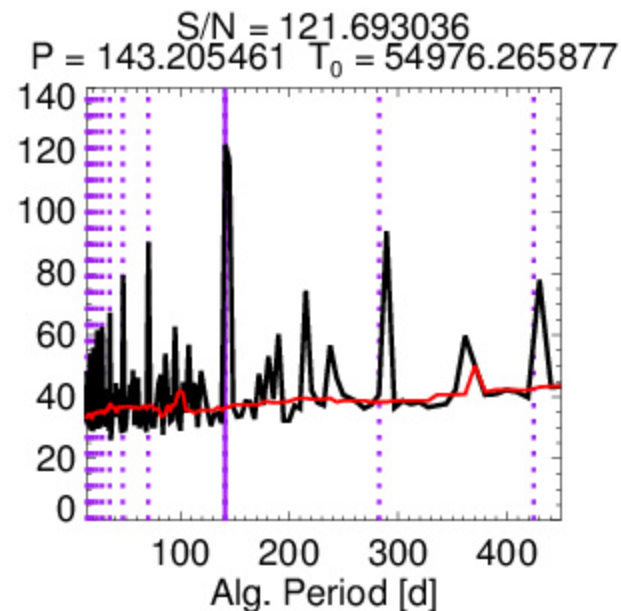
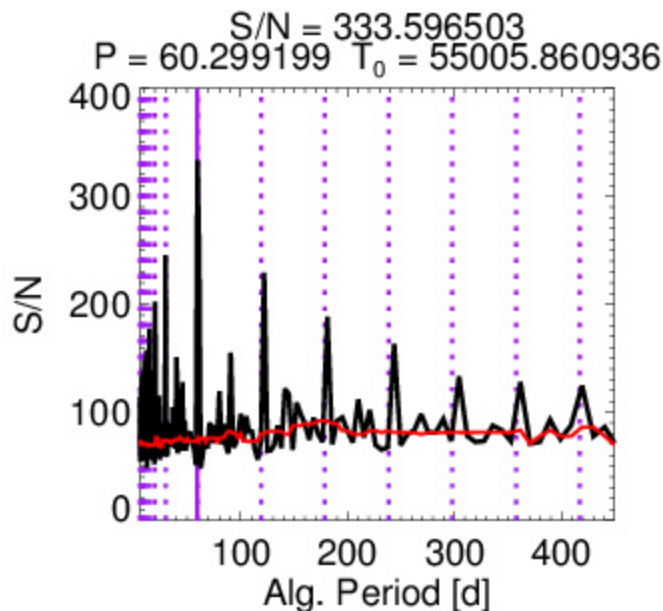
QATS Report: Spectra

- Plot: Signal/Noise vs. Algorithm [\sim average] Period
- Black line is the “spectrum” of power detected by QATS
- Solid purple vertical line is best-fit period; dotted vertical lines are aliases (with a predictable pattern)
- Red line is “Stochastic background”, i.e., the noise floor

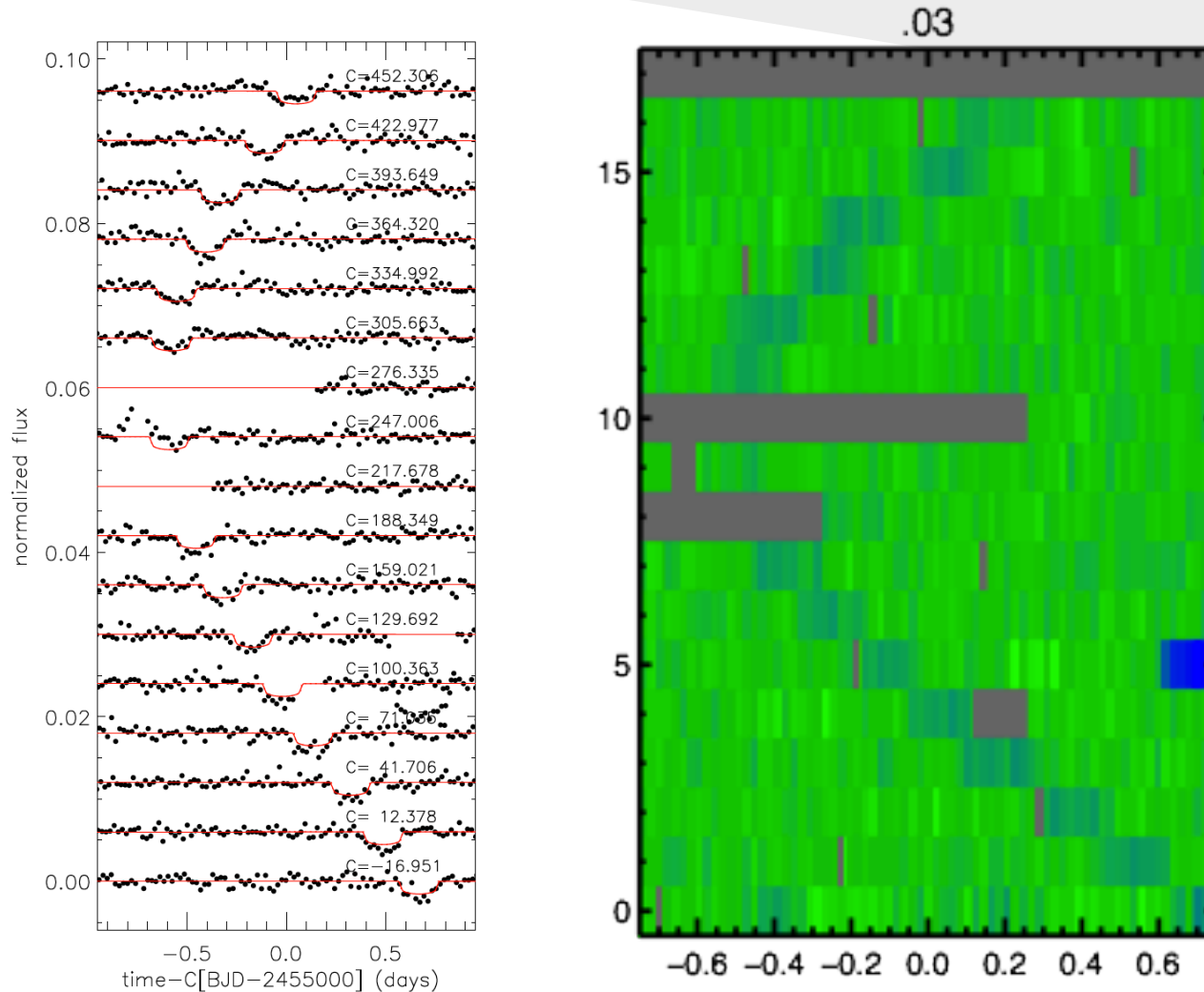


QATS Report: Spectra

- Text is about each planet candidate.
 - S/N is Signal to Noise of detection
 - P and T₀ are Period and Epoch of candidate
- First three columns correspond to Kepler-30c, d, and b.
- Last column shows candidate barely above noise



Kepler-30b river plots

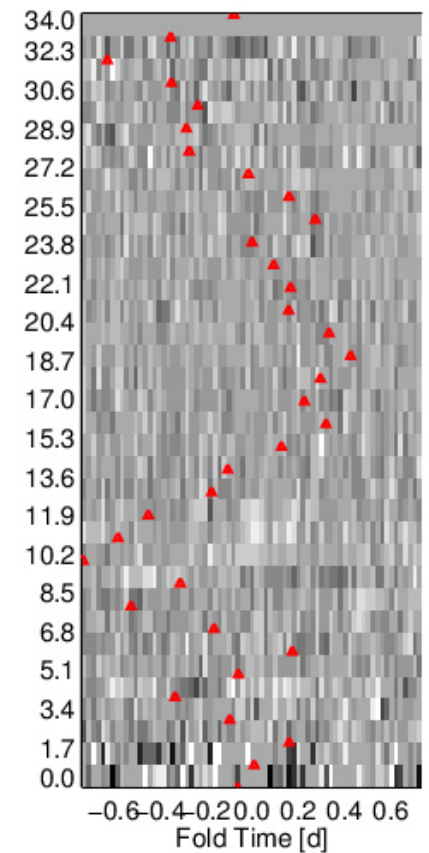
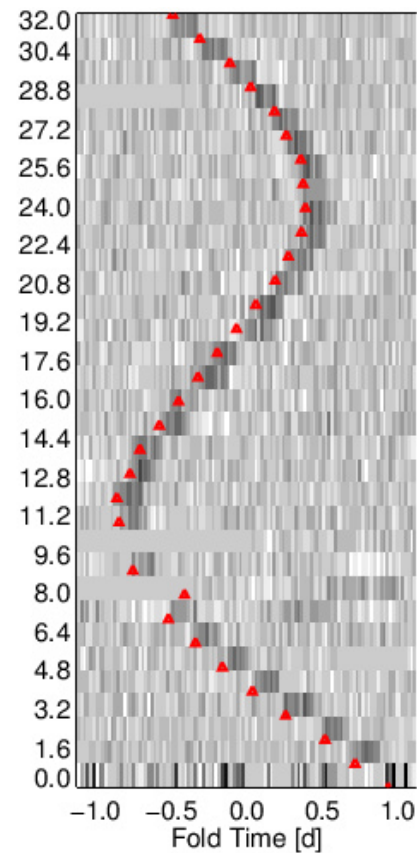
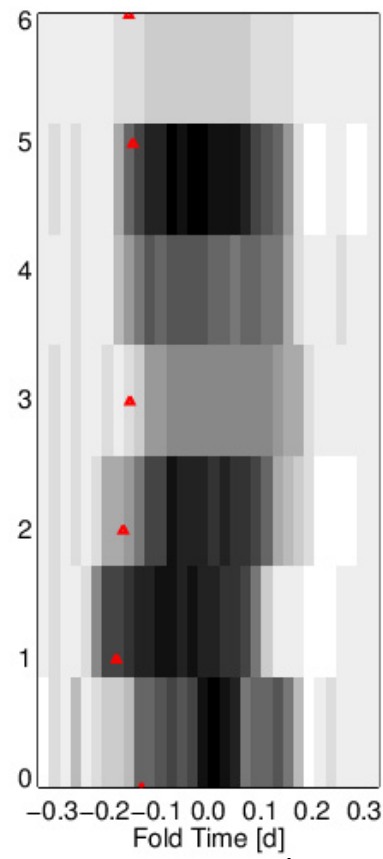
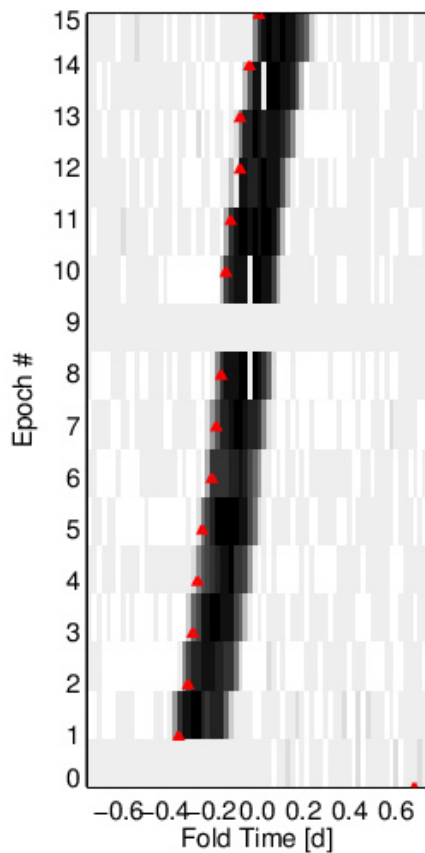


QATS Report: River Plots

- River Plot shows light curve folded on average period
- Great way to visualize TTVs, trends, anti-correlated signals, periodic transits, etc.
- Also on the river plot is a red triangle at the “transit time” detected by QATS

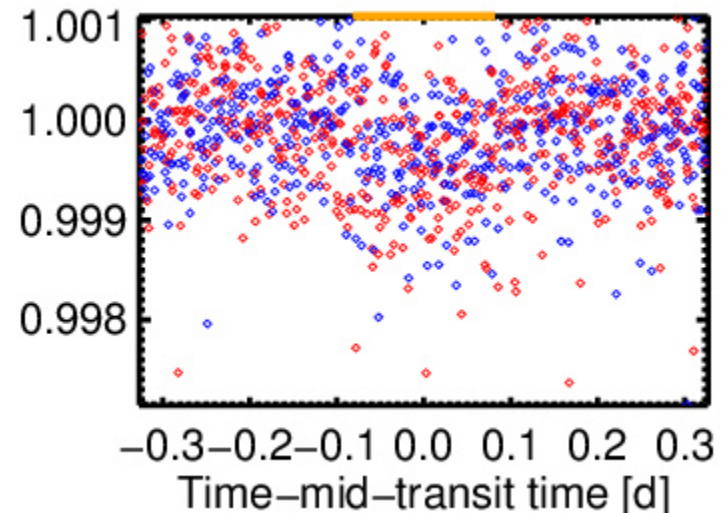
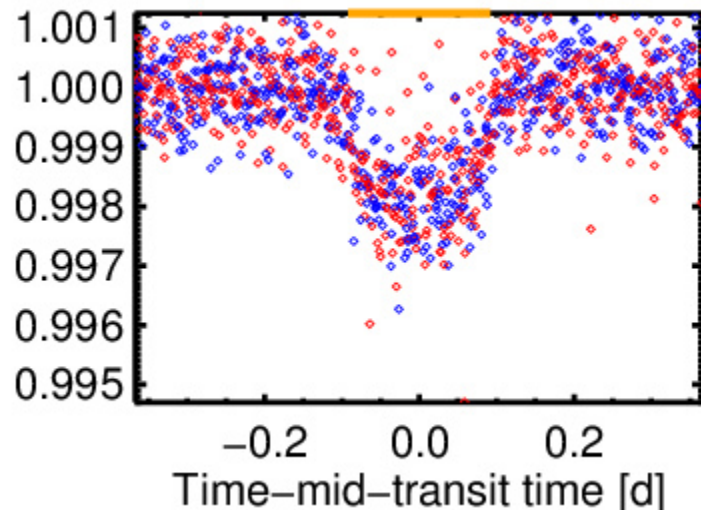
QATS Report: River Plots

- Kepler-30 results (planets c, d, b, and a poor candidate)
- (Red triangles mark the start of the transit)



QATS Report: Co-added Signal

- Folded, de-TTVed light curves, with colors alternating between even-numbered and odd-numbered transits, y-axis is relative flux
- Below: 3rd and 4th column (30b and poor candidate).
- Yellow bar shows estimated duration



QATS: Instrumental False Positives

- QATS has a lot of freedom to choose transit times, leading to higher chances of instrumental false positives, like the fourth candidate in the report
- The “Delta BG [BackGround]” should be significant to indicate that a candidate is $> \sim 2x$ above the noise floor
- TTV signal doesn't need to be smooth (e.g. circumbinary planets), but random-walk river plots are not a good sign
- Although QATS has already been used to investigate various Kepler populations, there are probably many planet candidates that are clearly above the background that remain to be discovered

Conclusions

- TTVs (especially in Kepler MTSs) are powerful ways to determine planetary properties with a very ripe observational dataset currently available from Kepler
- Systemic Console is a very useful tool for fitting a variety of exoplanet data, including MTS TTVs
- Reproduced mass measurements for Kepler-18 planets
- Detected planets of Kepler-30 using QATS, an algorithm designed to find planets with non-periodic transits

Acknowledgments: Stefano Meschiari, Josh Carter, Carolyn Brinkworth, Eric Agol, Sagan Workshop Organizers

Thank You!

Table 4 of Cochran+2011

TABLE 4
ADOPTED SYSTEM PARAMETERS

Parameter	Value
$M_{\star} (M_{\odot})^a$	0.972 ± 0.042
$R_{\star} (R_{\odot})^a$	1.108 ± 0.051
$\log L_{\star} (L_{\odot})^a$	-0.031 ± 0.035
Age (Gyr) ^a	10.0 ± 2.3
$\log g_{\star}^b$	4.31 ± 0.12
$\rho_{\star} (g/cm^3)^c$	1.01 ± 0.12
Kepler-18b = K00137.03	
T_0^c	2454966.5068 ± 0.0021
P (days) ^c	3.504725 ± 0.000028
Transit depth (ppm) ^c	254.0 ± 7.8
b (Impact Parameter) ^c	0.771 ± 0.025
R_p/R_{\star}^c	0.01656 ± 0.00032
$M_p (M_{\oplus})^d$	6.9 ± 3.4
$R_p (R_{\oplus})^c$	2.00 ± 0.10
i (deg) ^e	84.92 ± 0.26
a/R_{\star}^e	8.58 ± 0.37
$\bar{\rho}_p (g/cm^3)^{3,4}$	4.9 ± 2.4
a (AU) ^e	0.0447 ± 0.0006
K (m/s) ^c	5.2 ± 2.4
T_{14} (h) ^e	2.076 ± 0.036
T_{12} (h) ^e	0.0818 ± 0.0082

Kepler-18c = K00137.01	
T_0^d	2455167.0883 ± 0.0023
P (days) ^d	7.64159 ± 0.00003
Transit depth (ppm) ^c	2286.6 ± 8.6
b (Impact Parameter) ^c	0.593 ± 0.050
R_p/R_{\star}^c	0.04549 ± 0.00055
$M_p (M_{\oplus})^d$	17.3 ± 1.9
$R_p (R_{\oplus})^c$	5.49 ± 0.26
i (deg) ^e	87.68 ± 0.22
a/R_{\star}^e	14.43 ± 0.61
$\bar{\rho}_p (g/cm^3)^{3,4}$	0.59 ± 0.07
a (AU) ^e	0.0752 ± 0.0011
K (m/s) ^c	5.1 ± 1.9
T_{14} (h) ^e	3.488 ± 0.020
T_{12} (h) ^e	0.229 ± 0.022

Kepler-18d = K00137.02	
T_0^d	2455169.1776 ± 0.0013
P (days) ^d	14.85888 ± 0.00004
Transit depth (ppm) ^c	$3265. \pm 12.$
b (Impact Parameter) ^c	0.767 ± 0.024
R_p/R_{\star}^c	0.05782 ± 0.00069
$M_p (M_{\oplus})^d$	16.4 ± 1.4
$R_p (R_{\oplus})^c$	6.98 ± 0.33
i (deg) ^e	88.07 ± 0.10
a/R_{\star}^e	22.48 ± 0.96
$\bar{\rho}_p (g/cm^3)^{3,4}$	0.27 ± 0.03
a (AU) ^e	0.1172 ± 0.0017
K (m/s) ^c	7.3 ± 2.1
T_{14} (h) ^e	3.679 ± 0.036
T_{12} (h) ^e	0.459 ± 0.045

Tables 7 & 8

TABLE 7
OSCULATING JACOBIAN ELEMENTS, AT EPOCH 2455168.0 [BJD], FOR THE 3-PLANET TTV DYNAMICAL SOLUTION.

	Period (days)	T_0 (days)	$e \cos \omega$	$e \sin \omega$
b	3.504674 ± 0.000054	266.276996 ± 0.005453	0	0
c	7.641039 ± 0.000087	267.092502 ± 0.000262	0.000291 ± 0.000079	0.000173 ± 0.000233
d	14.860509 ± 0.000148	269.174850 ± 0.000253	-0.000076 ± 0.000019	0.000516 ± 0.000450

TABLE 8
MASSES AND DENSITIES OF THE PLANETS IN THE KEPLER-18 SYSTEM

Method		Kepler-18b (K00137.03)	Kepler-18c (K00137.01)	Kepler-18d (K00137.02)
MCMC (lightcurve + RV) Solution	(M_{\oplus})	13.4 ± 5.8	16.9 ± 6.1	29.9 ± 8.8
RV + transit time P and T_0	(M_{\oplus})	12 ± 5	15 ± 5	28 ± 7
TTV dynamical model	(M_{\oplus})	18 ± 9	17.3 ± 1.7	15.8 ± 1.3
TTV + RV dynamical model (adopted values)	(M_{\oplus})	6.9 ± 3.4	17.3 ± 1.9	16.4 ± 1.4
density from adopted mass	(g cm^{-3})	4.9 ± 2.4	0.59 ± 0.07	0.27 ± 0.03