ASC/Alliances Center for Astrophysical Thermonuclear Flashes

Verification of Type Ia Supernova Flame Model

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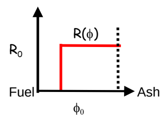
The initial phase of thermonuclear burning in Type Ia supernovae (SNe) is thought to occur via a deflagration wave; i.e., a thermonuclear flame. The thickness of the thermonuclear flame is initially ~ 10^{-3} cm, whereas the size of the white dwarf star is 2,300 km – a difference in scale of 10^{12}. For this reason, it is necessary to employ a sub-grid model of the flame in order to do 3-D, full-star simulations of Type Ia SNe. We and several other groups doing such simulations have employed an advection-reaction-diffusion (ARD) sub-grid model for the flame, in which the width of the flame is spread out over several grid points. Here we describe the work we have done to verify our ARD flame model.

ARD Flame Model

$$\partial_t \phi + \mathbf{v}\cdot\nabla\phi = s_0\delta_0\nabla^2\phi + \frac{s_0}{\delta_0}R(\phi)$$

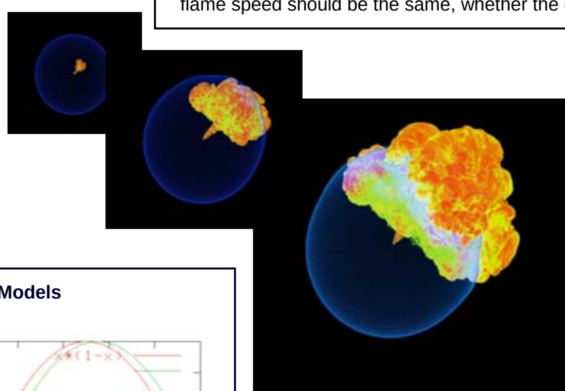
- ARD is a pseudo-physical model that allows the flame to be resolved: the numerical flame is ~4 zones wide.



Previous Type Ia SN simulations used a reaction term $R(\varphi)$ described by a top-hat (TH) function. Our studies show TH model is *noisy* and its *speed is strongly affected by curvature.*

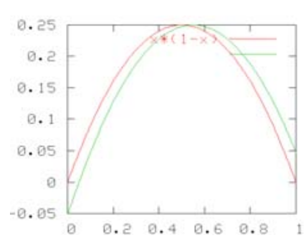
Essential Properties of the Flame Model

The speed of the flame model must be the same as that of fully resolved numerical simulations of thermonuclear flames. The flame model must also be sufficiently smooth (quiet) and stable. Finally, since the width of the actual nuclear flame is infinitesimal compared to the size of the finest grids that can be used in Type Ia SN simulations, the flame model should show little "curvature effect;" i.e., the flame speed should be the same, whether the curvature of the flame surface is positive or negative.



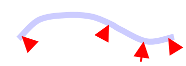
KPP and SKPP Flame Models

A known alternative to the TH reaction term is the KPP formulation (with symmetric form and zero values for pure fuel or ash) The flame profile from this model is symmetric (for inert flames). The flame speed one gets from this model is a constant.



However KPP flame model has some undesirable features: long "tails" in profile, and sensitivity to numerical details. These undesirable features can be eliminated by using a shifted (or sharpened) KPP model. In this model, the "tails" of the profile are truncated. As the shift is increased, the flame becomes narrower, but the variability increases; after a survey of parameter values, we chose a shift of 0.001.

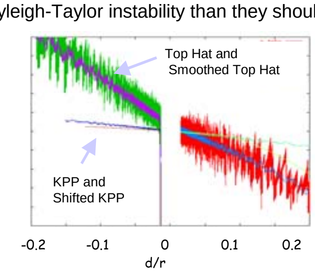
Curvature Effects



Flames described by an ARD model have curvature effects: flame speed depends on the local curvature – the speed is higher when the flame converges and lower when the flame diverges

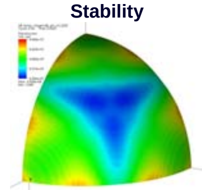
This effect is larger when the flame is wide. The width of the flame in the ARD flame model is unphysically large, and therefore the effect is magnified.

This effect *stabilizes* the flame. As a result, lower resolution models are more stable to the Rayleigh-Taylor instability than they should be.



Flame speed vs. ratio of width to radius for diverging and converging TH flames

Stability



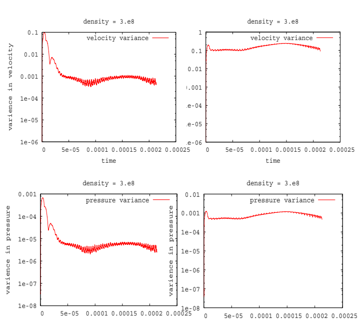
Sphericity has been checked in 2-D and 3-D by evolving an initially spherical laminar flame

Even when the initial radius of the flame is not much larger than the resolution, the flame remains spherical after propagating for a long time

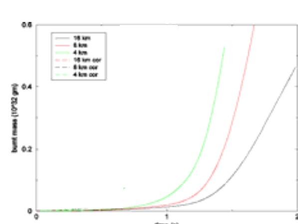
Coupling to Hydrodynamics

Variability of the TH flame produces noise in the fluid velocity and the pressure

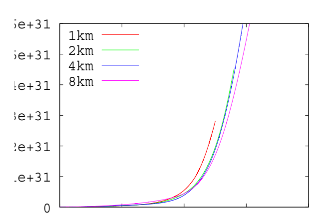
The noise is two orders of magnitude smaller with 0.001 shifted KPP



Results



Top Hat Flame Model



SKPP Flame Model

Simulations using the SKPP flame model show that the flame is quieter; the convergence with resolution is much better; thus the burned mass vs. time is more accurate than for the TH flame model

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