XEROS

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It is demonstrated that scalar particles can have the same radiation zeros as those found in electromagnetism provided that their couplings have a form related to a local chiral transformation. This is satisfied in extended $N = 2$ supersymmetry and leads to checks on scalar gluon calculations and to solutions for matter fields interacting with scalar plane waves.

Introduction. It is an interesting question whether or not the emission of scalar particles can ever be characterized by the same radiation symmetries [1–5] found for the photon. Furthermore, all previous discussions regarding the relationship between couplings in gauge theories and local spacetime transformations overlook chiral symmetry. Since many theories house scalar particles together with photons and photinos in a gauge supermultiplet, the occurrence of radiation zeros and szeros for photons and photinos, first discovered by Mikaelian et al. [6] and Barger et al. [7], respectively, leads us to suspect that under certain circumstances they may also "rotate" by extended supersymmetry into a null zone for scalar emission.

Gauge vector boson emission can be described as a first-order Poincaré transformation of the associated Feynman-diagram leg [1], while gaugino emission corresponds to a supersymmetry (SUSY) transformation [5]. Also, in the null zone, these transforms collapse to the identity, giving rise to the decoupling theorem for external plane waves [4]. This theorem is demonstrated by rewriting the interacting matter fields as Volkov–Taub-type transforms of "free" (non-gauge-interacting) matter fields [8], and is the covering theorem for radiation zeros.

We investigate these properties for gauge scalars. First, we find that amplitude zeros (dubbed "xeros", for extended SUSY and chirality transformations) do exist in a model with massless matter fields and are identical with those for photons and photinos. We find that gauge scalar emission induces first-order chiral transformations of the external legs, so that all the rigid symmetries are now shown to play a role locally: chiral for emitted scalars, SUSY for spinors, and Poincaré for vectors. We next extend the decoupling theorem to gauge scalars and present a transformation solving the equations of motion for the interactions of matter with gauge scalars. We lastly present a general proof of the extended SUSY (XSUSY) radiation theorem.

Tree graph analysis. Consider the massless XSUSY–QED model given by Fayet [9] [‡1] where a vector supermultiplet of gauge fields (consisting of a vector V_μ, a complex scalar ω, and two Majorana spinors λ and p) interacts with a scalar hypermultiplet of charged matter fields, containing a Dirac spinor ψ and two complex scalars ϕ_1 and ϕ_2, all with charge e and zero mass. In the Wess–Zumino gauge, the interaction portion of the lagrangian can be written as

$$L = L_e + L_I, \qquad (1)$$

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[‡1] We use Bjorken–Drell metric, $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, and the Majorana representation for γ^μ [10].

$$L_e = i\bar{\psi}\not{D}\psi + D_\mu\phi_1^+ D^\mu\phi_1 + D_\mu\phi_2^+ D^\mu\phi_2, \tag{2a}$$

$$L_I = e\sqrt{2}[\bar{\psi}(\gamma_-\phi_1 + \gamma_+\phi_2)\lambda + \bar{\lambda}(\gamma_+\phi_1^+ + \gamma_-\phi_2^+)\psi - i\bar{\psi}(\gamma_-\phi_2 + \gamma_+\phi_1)p + i\bar{p}(\gamma_-\phi_1^+ + \gamma_+\phi_2^+)\psi$$
$$- i\bar{\psi}(\gamma_-\omega - \gamma_+\omega^+)\psi] - \tfrac{1}{2}e^2(\Phi^+\Phi)^2 - 2e^2\omega^+\omega\Phi^+\Phi, \tag{2b}$$

where $D_\mu = \partial_\mu + ieV_\mu$, $\gamma_\pm = \frac{1}{2}(1 \pm \gamma_5)$, and $\Phi^+\Phi = \phi_1^+\phi_1 + \phi_2^+\phi_2$.

We examine the first-order tree diagrams of three processes $1 + 2 \to 3 + 4 + \omega$, in fig. 1, and obtain the corresponding Born amplitudes:

$$M(\psi\psi \to \psi\psi\omega) = \sqrt{2}e^3\bar{u}(p_3)\Bigg\{(p_2 - p_4)^{-2}\left[\frac{1}{2}\left(\frac{\not{q}\gamma^\mu}{p_3\cdot q} + \frac{\gamma^\mu\not{q}}{p_1\cdot q}\right)\gamma_+ u(p_1)\bar{u}(p_4)\gamma_\mu - \not{q}\left(\frac{\gamma^-}{p_3\cdot q} + \frac{\gamma^+}{p_1\cdot q}\right)u(p_1)\bar{u}(p_4)\gamma_+\right.$$
$$\left. + (p_1 - p_3)^{-2}\left[\tfrac{1}{2}\gamma_\mu u(p_1)\bar{u}(p_4)\left(\frac{\not{q}\gamma^\mu}{p_4\cdot q} + \frac{\gamma^\mu\not{q}}{p_2\cdot q}\right)\gamma_+ - \gamma_+ u(p_1)\bar{u}(p_4)\not{q}\left(\frac{\gamma^-}{p_4\cdot q} + \frac{\gamma^+}{p_2\cdot q}\right)\right]\right]\Bigg\}u(p_2) - (p_1 \leftrightarrow p_2), \tag{3a}$$

$$M(\psi\psi \to \phi_1\phi_2\omega) = \frac{e^3}{\sqrt{2}}\bar{v}(p_1)\left(\frac{\not{p}_3\not{q}}{p_1\cdot p_3\, p_2\cdot q} - \frac{\not{q}\not{p}_4}{p_2\cdot p_4\, p_1\cdot q}\right)\gamma_+ u(p_2) + (p_1 \leftrightarrow p_2), \tag{3b}$$

$$M(\psi\phi_1 \to \psi\phi_1\omega) = \frac{e^3}{2\sqrt{2}}\bar{u}(p_3)\left(-\frac{1}{p_3\cdot q}\not{q}(\not{p}_2 + \not{p}_4) - \frac{1}{p_1\cdot q}(\not{p}_2 + \not{p}_4)\not{q}\right.$$
$$\left. - \frac{2\not{q}\not{p}_4}{p_3\cdot q\, p_1\cdot p_4} - \frac{2\not{p}_2\not{q}}{p_1\cdot q\, p_2\cdot p_3} + \frac{4}{p_1\cdot p_3}\right)\gamma_+ u(p_1), \tag{3c}$$

where q is the momentum of the ω. It can easily be shown that each of these amplitudes vanishes when $Q_i/p_i\cdot q$ are equal for all i, thus giving xeros in the same zone as zeros and szeros.

We can also express gauge scalar emission from an external fermion leg as a transformation of the leg, according to

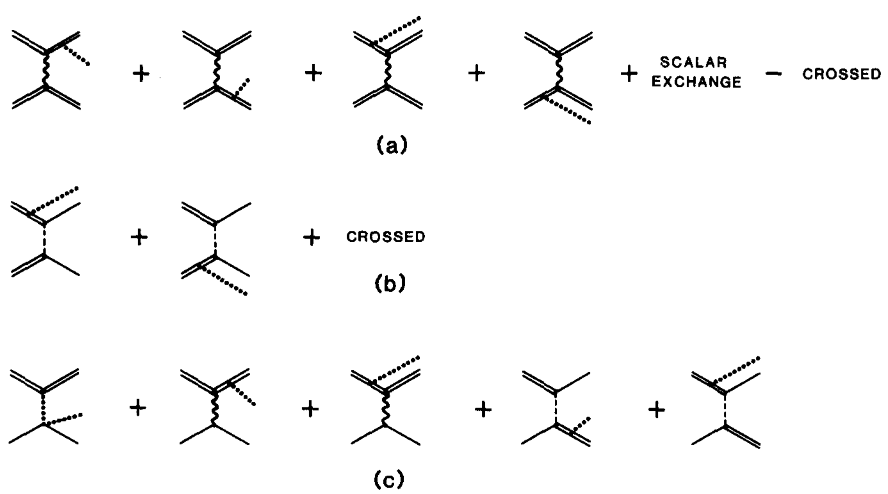

Fig. 1. First-order tree diagrams for three examples [corresponding to eqs. (3)] of gauge scalar emission from a scalar (solid line) or Dirac spinor (double line). Gauge particles are represented by a wavy line (photon), a dashed line (photino), or a dotted line (gauge scalar).

$$u(p) \to \not{k}\gamma_+ u(p) \tag{4a}$$

for incoming legs (momentum p) and

$$\bar{u}(p) \to \bar{u}(p)\gamma_+ \not{k} \tag{4b}$$

for outgoing legs, where

$$k_\mu = -\mathrm{i}\, e q_\mu/(\sqrt{2} p\cdot q). \tag{4c}$$

Thus emission of an ω particle generates a first-order chiral transformation of the associated external fermion leg. The absence of a trilinear $\phi^+\phi\omega$ vertex is consistent with the difference in the meaning of chirality for scalars and spinors, while a "chirally neutral" quadrilinear $\phi^+\phi\omega^+\omega$ vertex is allowed. We could also consider source graphs with external gauge-field legs in this analysis, noting that uncharged external legs severely restrict the null zone.

Solutions for gauge-scalar-interacting matter fields. The Volkov–Taub solutions of the Dirac and Klein–Gordon equations in the presence of an external photon plane wave [8] have long been known; these express the gauge-interacting matter fields in terms of free fields, by use of a local Poincaré transformation labeled ULT, where

$$\mathrm{D}^2 * ULT = ULT * \partial^2, \quad \not{\mathrm{D}} * ULT = ULT * \not{\partial} \tag{5}$$

as in ref. [4], with D_μ as above. When the photon plane wave is given by $V_\mu(n\cdot x)$ with $n^2 = n\cdot V = 0$, the transformations U, L, and T are functions of $n\cdot x$ and $n\cdot P$ only ($P \leftrightarrow \mathrm{i}\partial$). We now extend this method of solution to encompass gauge scalar fields as well.

Consider the matter field equations of motion derived from the lagrangian (2), namely

$$\mathrm{i}\not{\mathrm{D}}\psi + e\sqrt{2}[(\gamma_-\phi_1 + \gamma_+\phi_2)\lambda - \mathrm{i}(\gamma_-\phi_2 + \gamma_+\phi_1)p] - \mathrm{i}(\omega\gamma_- - \omega^+\gamma_+)\psi = 0, \tag{6a}$$

$$\mathrm{D}^2\phi_{1,2} - e\sqrt{2}(\bar{\lambda}\gamma_{+,-} + \mathrm{i}\bar{p}\gamma_{-,+})\psi + e^2(\Phi^+\Phi)\phi_{1,2} + 2e^2\omega^+\omega\phi_{1,2} = 0. \tag{6b}$$

By first applying ULT to the fields so that $\psi = ULT\psi'$, $\phi_{1,2} = ULT\phi'_{1,2}$, these equations are transformed into similar ones with ∂ replacing D everywhere, by virtue of (5). The non-photonic terms remain unchanged, since ω, λ, and p are required to be functions of $n\cdot x$.

Now introducing a local chiral transformation RA, where

$$R = \mathrm{e}^{\mathrm{i}e\xi}, \quad \xi = -\frac{e}{n\cdot P}\int^{n\cdot x} \mathrm{d}z\ [\omega^+\omega](z), \tag{7a}$$

$$A = \mathrm{e}^{\mathrm{i}C} = 1 + \mathrm{i}C, \quad C = \{0; (e/\sqrt{2}n\cdot P)\not{n}(\omega\gamma_- - \omega^+\gamma_+)\} \tag{7b}$$

for {scalars; spinors} respectively, the reader will observe that

$$\{\partial^2 + 2e^2\omega^+\omega; \mathrm{i}\not{\partial} - \mathrm{i}e\sqrt{2}(\omega\gamma_- - \omega^+\gamma_+)\}RA = R\{\partial^2; \mathrm{i}\not{\partial}\} \tag{8}$$

holds, so that $\psi' = RA\psi^0$, $\phi'_{1,2} = RA\phi^0_{1,2}$ further transforms the equations of motion into the equations for ψ^0 and ϕ^0 with no photon or gauge scalar interactions, using the supplementary conditions $\not{n}\lambda = \not{n}p = 0$. These conditions also ensure that ψ^0 and ϕ^0 have exactly the same interactions with λ and p as ψ' and ϕ'. This new $ULTRA$ transformation reduces the interacting lagrangian to a version with only photino fields left. It is also possible to transform away the photinos [5]. Thus the scalar-interacting matter fields are expressible in terms of the "free" fields by means of a chiral transformation, whose first-order term is now easily identified with the external leg factors (4) for gauge scalar emission.

The decoupling theorem [4] may also be extended to gauge scalars. We omit the technical details and merely sketch a proof here: first, consider an n-legged vertex, with all momenta incoming. In the null zone, all the RA transforms depend upon the same parameter $Q/n\cdot P$, and so by charge and chirality conservation, their product at each vertex reduces to the identity. Therefore tree amplitudes are independent of an external gauge scalar plane wave in the null zone, and the wave decouples from the matter fields.

Proof of the radiation theorem for scalars

Theorem. Consider a theory with global XSUSY and gauge-theoretic couplings. Then all tree-approximated amplitudes for gauge scalar emission have the same radiation zeros as those for photons and photinos.

Proof. Define the Majorana spinors $Q_i, i = 1, ..., N$, to be the conserved, translationally invariant fermionic charges that generate rigid XSUSY transformations, such that

$$[\bar{\alpha} Q_i, S] = 0, \tag{9}$$

where α is an arbitrary constant Majorana Grassman spinor and S is the scattering operator. Proceeding as in ref. [5], we now sandwich (9) between two *in* (or *out*) states that differ by one fermionic unit, with a photino ($\tilde{\gamma}$) in one of them. Since α and i are arbitrary, we get

$$\langle a_1 a_2 \ldots |S| b_1 b_2 \ldots \omega\rangle = \sum A_j \langle a_1 \ldots \tilde{a}_j \ldots |S| b_1 \ldots \tilde{\gamma}\rangle - \sum B_j \langle a_1 \ldots |S| b_1 \ldots \tilde{b}_j \ldots \tilde{\gamma}\rangle - C\langle a_1 \ldots |S| b_1 \ldots \gamma\rangle, \tag{10}$$

where γ is the photon, A_j, B_j, and C may depend upon the various momenta, and $\tilde{a}$ is the (possibly summed) conjugate of a by the Q_i. The matrix elements in (10) have matching sets of momenta and charges. From the extended radiation theorem (for photon and photino emission), all amplitudes on the right side of (10) have the same tree null zone, given above. Therefore the left side also vanishes there, and xeros are identical with zeros and szeros.

In conclusion, we observe that demanding radiation symmetries in lagrangians can be a powerful tool in model-building, since only gauge-theoretic interactions and supersymmetric versions thereof give rise to zeros/szeros/xeros and decoupling in the null zone [‡2]. Thus we can say that the forms of interactions for gauge particles are closely related to their basic chiral super-Poincaré symmetries. Conversely, we now have a reason for adding photinos and gauge scalars to the standard models: if we demand that symmetries other than simple Poincaré be incorporated, we are then led to include partners of the photon which employ these symmetries in their interactions.

Zeros/szeros/xeros also gain relevance because of their use as a quick and easy check on gauge particle emission (tree graph) calculations [‡3]. In this respect, we note the relevance of scalar emission calculations, such as in ref. [12], which expedite the finding of gluon and gluino scattering amplitudes. Also, radiation representations (as in ref. [1]) analogous to those for spinors and vectors apply for scalars.

It is interesting to compare the various transformations' mass requirements: the *ULT* transform for the photon does not care about the masses of the matter fields involved, while the *ULT*–SUSY transform for the photino requires that all the matter fields have the same mass, and the *ULTRA* transform in the model considered here requires masslessness for all (we could alternatively consider a massive multiplet possessing a central charge). This parallels models of no SUSY / $N = 1$ SUSY / $N = 2$ SUSY.

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[‡2] It should be noted that this scheme does *not* distinguish between renormalizable and certain non-renormalizable interaction/derivative terms; for that one needs to include assumptions like minimal derivative coupling. This scheme also allows for the inclusion of additional vector fields, whose equations of motion are solved just as scalars' with *ULTRA*.

[‡3] Note that graphs with internal loops do not acquire null zones, because it is impossible to pin down the circulating internal momenta to a specific value, but radiation symmetry is still defined for these graphs (see ref. [11] for a discussion of the general statement of radiation symmetry).

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