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Walking technicolor and the $Zb\bar{b}$ vertex

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A slowly running technicolor coupling will affect the size of non-oblique corrections to the $Zb\bar{b}$ vertex from extended technicolor dynamics. We show that, while "walking technicolor" reduces the magnitude of the corrections, they generally remain large enough to be seen at LEP.

1. Introduction

The origin of the diverse masses and mixings of the quarks and leptons remains a mystery; most puzzling is the origin of the top quark's large mass. In technicolor models [1], the large top mass is thought to arise from extended technicolor (ETC) [2] dynamics at relatively low energy scales [#1]. Recent work [3] has shown that the dynamics responsible for generating the large top quark mass in extended technicolor models will produce potentially large "non-oblique" [4] effects at the $Zb\bar{b}$ vertex [#2]. In this note, we discuss what happens to these effects if the technicolor beta function is assumed to walk [5]. We show that the size of the signal is reduced but that it remains quite visible at LEP for many models.

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#1 So long as no additional light scalars couple to ordinary and techni-fermions [7,8].

#2 In contrast, the $Zb\bar{b}$ effects in models with additional light scalars (e.g. strongly coupled ETC models) are indistinguishable from those in the standard model [3].

2. ETC's effect on the $Zb\bar{b}$ vertex

We begin by reviewing the results of ref. [3]. Consider a model in which m_t is generated by the exchange of a weak-singlet extended technicolor gauge boson of mass $M_{\rm ETC}$ coupling with strength $g_{\rm ETC}$ to the current

$$\xi\bar{\psi}^i_L\gamma^\mu T^{ik}_L + \xi'\bar{t}_R\gamma^\mu U^k_R\,,$$
$$\psi_L = \begin{pmatrix} t \\ b \end{pmatrix}_L, \quad T_L = \begin{pmatrix} U \\ D \end{pmatrix}_L, \tag{2.1}$$

where U and D are technifermions, i and k are weak and technicolor indices, and the coefficients ξ and ξ' are extended technicolor Clebschs expected to be of order one. At energies below $M_{\rm ETC}$, ETC gauge boson exchange may be approximated by local four-fermion operators. For example, m_t arises from an operator coupling the left- and right-handed pieces of the current in

$$-\xi\xi'\frac{g^2_{\rm ETC}}{M^2_{\rm ETC}}\left(\bar{\psi}^i_L\gamma^\mu T^{iw}_L\right)\left(\bar{U}^w_R\gamma_\mu t_R\right) + \text{h.c.} \tag{2.2}$$

When this is Fierzed into a product of technicolor singlet densities, it is seen to generate a mass for the top quark after the technifermions' chiral symmetry breaking. We can use the rules of naive dimensional analysis [6] to estimate the size of m_t generated by eq. (2.2). Assuming, for simplicity, that there is only doublet of technifermions and that technicolor respects an $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ chiral symmetry (so that the technipion decay constant, F, is $v \approx 250$ GeV) we have

$$m_t = \xi\xi' \frac{g_{\mathrm{ETC}}^2}{M_{\mathrm{ETC}}^2} \langle \overline{U} U \rangle \approx \xi\xi' \frac{g_{\mathrm{ETC}}^2}{M_{\mathrm{ETC}}^2} (4\pi v^3) . \tag{2.3}$$

In the same language, we can also show that the extended technicolor boson responsible for producing m_t affects the $Zb\bar{b}$ vertex. Consider the four-fermion operator arising purely from the left-handed part of the current (2.1) – the only part containing b quarks,

$$-\xi^2 \frac{g_{\mathrm{ETC}}^2}{M_{\mathrm{ETC}}^2} \left(\overline{\psi}_L^i \gamma^\mu T_L^{iw}\right) \left(\overline{T}_L^{jw} \gamma_\mu \psi_L^j\right) . \tag{2.4}$$

When Fierzed into a product of technicolor singlet currents, this includes [#3]

$$-\tfrac{1}{2}\xi^2 \frac{g_{\mathrm{ETC}}^2}{M_{\mathrm{ETC}}^2} \left(\overline{\psi}_L \gamma^\mu \tau^a \psi_L\right) \left(\overline{T}_L \gamma_\mu \tau^a T_L\right) , \tag{2.5}$$

where the τ^a are weak isospin Pauli matrices. As shown in ref. [3] this alters the Z-boson's tree-level coupling to left-handed bottom quarks

$$g_L = (e/s_\theta c_\theta)(-\tfrac{1}{2} + \tfrac{1}{3}s_\theta^2)$$

by

$$\delta g_L = -\tfrac{1}{2}\xi^2 \frac{g_{\mathrm{ETC}}^2 v^2}{M_{\mathrm{ETC}}^2} \frac{e}{s_\theta c_\theta} (I_3) \tag{2.6a}$$

$$= \frac{1}{4} \frac{\xi}{\xi'} \frac{m_t}{4\pi v} \frac{e}{s_\theta c_\theta} . \tag{2.6b}$$

Here eq. (2.6b) follows from applying eq. (2.3) to eq. (2.6a).

3. Measuring the effect at LEP

We now consider how to best measure experimentally the shift in g_L caused by extended technicolor. Altering the $Zb\bar{b}$ coupling will affect the decay width of the Z boson into b quarks. In addition, there are flavor universal (oblique) corrections to the width, coming from both technicolor and extended technicolor interactions. At one loop, the decay width of the Z is of the form

$$\Gamma_b^{\mathrm{corr}} \equiv \Gamma(Z \to b\bar{b}) = (1 + \Delta\Gamma)(\Gamma_b + \delta\Gamma_b) , \tag{3.1}$$

where Γ_b is the tree-level decay width, $\Delta\Gamma$ represents the oblique corrections and $\delta\Gamma_b$ represents the non-oblique (flavor-dependent) corrections. We will refer to the non-oblique effect of (2.6a) on the decay width as $\delta\Gamma_b^{\mathrm{ETC}}$. Ratios of Z decay widths into different final states are particularly sensitive to such effects; we suggest studying the ratio [#4] of the Z decay width into $b\bar{b}$ and the Z decay width into all non-$b\bar{b}$ hadronic final states: $\Gamma_b/\Gamma_{h\neq b}$. This is accessible to the current LEP experiments.

This particular ratio has several features to recommend it. First, since it is a ratio of hadronic widths, the leading QCD corrections cancel in the limit of small quark masses. Second, eq. (3.1) implies that the fractional change in this particular ratio is approximately the fractional shift in Γ_b:

$$\Delta_R \equiv \frac{\delta(\Gamma_b/\Gamma_{h\neq b})}{(\Gamma_b/\Gamma_{h\neq b})} \approx \frac{\delta\Gamma_b}{\Gamma_b} . \tag{3.2}$$

This is easily related to the change in g_L that extended technicolor effects cause:

$$\Delta_R^{\mathrm{ETC}} \approx \frac{\delta\Gamma_b^{\mathrm{ETC}}}{\Gamma_b} \approx \frac{2 g_L \delta g_L}{g_L^2 + g_R^2} . \tag{3.3}$$

For our benchmark ETC model with two technifermion flavors,

$$\Delta_R^{\mathrm{ETC}} \approx -3.7\% \cdot \frac{\xi}{\xi'} \left(\frac{m_t}{100\,\mathrm{GeV}}\right) . \tag{3.4}$$

There is also a fractional shift in Γ_b arising from one-loop diagrams involving longitudinal W-boson exchange and internal top quarks. This has already been calculated [9]; it is of order −0.7% (−2.5%) for $m_t = 100$ (200) GeV. This source of corrections to Γ_b (which we shall call Δ_R^W) occurs both in the standard model (where it is the dominant non-oblique correction to Γ_b) and in extended technicolor models. Note that both Δ_R^W and Δ_R^{ETC} act to *decrease* $\delta\Gamma_b/\delta\Gamma_{h\neq b}$.

[#3] The Fierzed form of (2.4) also includes operators that are products of weak-singlet left-handed currents; these will not affect the $Zb\bar{b}$ coupling.

[#4] This is simply related to the ratio Γ_b/Γ_h discussed in ref. [3]: $\Gamma_b/\Gamma_{h\neq b} = (\Gamma_b/\Gamma_h)/(1 - \Gamma_b/\Gamma_h)$ but is more convenient to work with. We thank A. Pich for pointing this out.

Then in comparing the size of Δ_R in the standard model with that in ETC models, we are comparing Δ_R^W to $\Delta_R^W + \Delta_R^{\rm ETC}$. The expected LEP precision of 2.5% for measurement of Δ_R [10] should suffice to distinguish them.

4. Walking technicolor

The dimensional estimates employed in section 2 are self-consistent so long as the extended technicolor interactions may be treated as a small perturbation on the technicolor dynamics, i.e. so long as $g_{\rm ETC}^2 v^2/M_{\rm ETC}^2 < 1$ and there is no fine-tuning [7]. Note that the rules of naive dimensional analysis do not require that $M_{\rm ETC}$ be large, only that $g_{\rm ETC}^2 v^2/M_{\rm ETC}^2$ (or equivalently $m_t/4\pi v$) be small. However, these estimates [in particular, the relationship (2.3) between $(g^2v^2/M_{\rm ETC}^2)$ and $(m_t/4\pi v)$] are typically modified in "walking technicolor" models [5], where there is an enhancement of operators of the form (2.2) due to a large anomalous dimension of the technifermion mass operator.

Let us define what is meant by a "walking" technicolor coupling. The beta function for an SU(N) technicolor force has the same form as that for QCD. At leading order it is simply

$$\beta(\alpha_{\rm TC}) = -b\alpha_{\rm TC}^2 + {\rm O}(\alpha_{\rm TC}^3), \tag{4.1}$$

where (for technifermions in the fundamental representation) b is related to the technicolor group and the number of technifermion flavors (n_f) by

$$b = \frac{1}{2\pi}\left(\tfrac{11}{3}N - \tfrac{2}{3}n_f\right). \tag{4.2}$$

For our benchmark model with two technifermion flavors, setting $N = 2$ yields $b = 3/\pi$. Adding more flavors of technifermions to the model decreases b so the TC coupling falls off relatively slowly with increasing momentum scale (it "walks").

The expected effect of a walking technicolor coupling on $\Delta_R^{\rm ETC}$ can be outlined fairly briefly. When the technicolor coupling becomes strong and the technifermion condensate $\langle\overline{T}T\rangle$ forms, a dynamical mass $\Sigma(p)$ is also generated for the technifermions. As discussed in ref. [5], having $\alpha(p)$ fall off slowly with increasing p causes $\Sigma(p)$ to decrease more slowly with rising p than it would in a "running" TC theory. Since the technifermion condensate is

$$\langle\overline{T}T\rangle \sim \int_0^{M_{\rm ETC}^2} {\rm d}k^2\,\Sigma(k), \tag{4.3}$$

enhancing Σ increases $\langle\overline{T}T\rangle$. According to eq. (2.3) this means that a walking TC coupling increases m_t for a given ETC scale $M_{\rm ETC}$. The factor $(g^2v^2/M_{\rm ETC}^2)$ appearing in our expression (2.6a) for δg_L is therefore smaller than $(m_t/4\pi v)$ in an ETC model with walking TC. Thus, the expected size of $\Delta_R^{\rm ETC}$ is reduced.

5. Numerical results

To illustrate the effect of walking technicolor on the size of $\Delta_R^{\rm ETC}$, we have studied coupled ladder-approximation Dyson–Schwinger equations [5] for the dynamical technifermion and top quark masses, $\Sigma(p)$ and $m_t(p)$. The gap equations always possess a chiral symmetry preserving solution with m_t and Σ both equal to zero. Our interest is in finding chiral symmetry *violating* solutions with both m_t and Σ nonzero. We have focused on SU$(N+1)_{\rm ETC} \to$ SU$(N)_{\rm TC}$ models with a full family of technifermions.

In Landau gauge and after the angular integrations have been performed, we approximate the gap equations by [11]

$$\begin{aligned}\Sigma(p) = {} & C_2^{\rm TC}\int_0^\infty \frac{3\alpha_{\rm TC}(M[p,k])}{\pi M[p^2,k^2]}\frac{\Sigma(k)}{k^2+\Sigma^2(k)}k^2{\rm d}k^2 \\ & + c_1\int_0^\infty \frac{3\alpha_{\rm TC}(M[p,k,M_{\rm ETC}])}{\pi M[p^2,k^2,M_{\rm ETC}^2]}\frac{\Sigma(k)}{k^2+\Sigma^2(k)}k^2{\rm d}k^2 \\ & + c_2\int_0^\infty \frac{3\alpha_{\rm TC}(M[p,k,M_{\rm ETC}])}{\pi M[p^2,k^2,M_{\rm ETC}^2]}\frac{m_t}{k^2+m_t^2}k^2{\rm d}k^2 \\ & + C_2^{\rm QCD}\int_0^\infty \frac{3\alpha_{\rm QCD}(M[p,k,M_{\rm ETC}])}{\pi M[p^2,k^2,M_{\rm ETC}^2]} \\ & \times\frac{\Sigma(k)}{k^2+\Sigma^2(k)}k^2{\rm d}k^2,\end{aligned} \tag{5.1}$$

$$m_t = c_3 \int_0^\infty \frac{3\alpha_{\rm TC}(M[p,k,M_{\rm ETC}])}{\pi M[p^2,k^2,M_{\rm ETC}^2]} \frac{m_t}{k^2+m_t^2} k^2 {\rm d}k^2$$
$$+ c_4 \int_0^\infty \frac{3\alpha_{\rm TC}(M[p,k,M_{\rm ETC}])}{\pi M[p^2,k^2,M_{\rm ETC}^2]} \frac{\Sigma(k)}{k^2+\Sigma(k)^2} k^2 {\rm d}k^2$$
$$+ C_2^{\rm QCD} \int_0^\infty \frac{3\alpha_{\rm QCD}(M[p,k,M_{\rm ETC}])}{\pi M[p^2,k^2,M_{\rm ETC}^2]}$$
$$\times \frac{\Sigma(k)}{k^2+\Sigma^2(k)} k^2 {\rm d}k^2, \qquad (5.2)$$

where $M[x,y]$ signifies the greater of x and y, $C_2^{\rm TC} = (N^2-1)/2N$, $C_2^{\rm QCD} = \frac{4}{3}$ and the coefficients c_i are

$$c_1 = \frac{1}{2N(N+1)}, \quad c_2 = \tfrac{1}{2},$$
$$c_3 = \frac{N}{2(N+1)}, \quad c_4 = \tfrac{1}{2}N. \qquad (5.3)$$

We have ignored the mass splittings between the extended technicolor gauge bosons and used $M_{\rm ETC}$ to stand for the masses of all the heavy extended technicolor gauge bosons in the gap equations.

To integrate the gap equations, we use the following one-loop form for the running of the technicolor coupling:

$$\alpha_{\rm TC}(p) = 2\alpha_{\rm TC}^c, \qquad p < \Lambda_c,$$
$$= \frac{2\alpha_{\rm TC}^c}{1 + b\alpha_{\rm TC}^c \ln(p^2/\Lambda_c^2)}, \quad p \geqslant \Lambda_c, \qquad (5.4)$$

where $\alpha_{\rm TC}^c \equiv \pi/3C_2^{\rm TC}$ is the "critical" value for chiral symmetry breaking, and Λ_c is to be determined by requiring that the model reproduce the correct electroweak gauge boson masses. At energies below the extended technicolor scale $M_{\rm ETC}$, the beta-function parameter b is given by eq. (4.2); above $M_{\rm ETC}$, it is

$$b_{\rm ETC} = \frac{1}{2\pi}\left(\tfrac{11}{3}(N+1) - \tfrac{2}{3}n_f\right) = b + \frac{1}{6\pi}. \qquad (5.5)$$

We set the scale of the chiral symmetry breaking and the dynamical masses by using the calculated $\Sigma(p)$ to compute the technipion decay constant [12] [#5]

[#5] There is a typo in the final equation of the third of refs. [12], k^4 should be replaced by $k^4 - 2k^2\Sigma^2$.

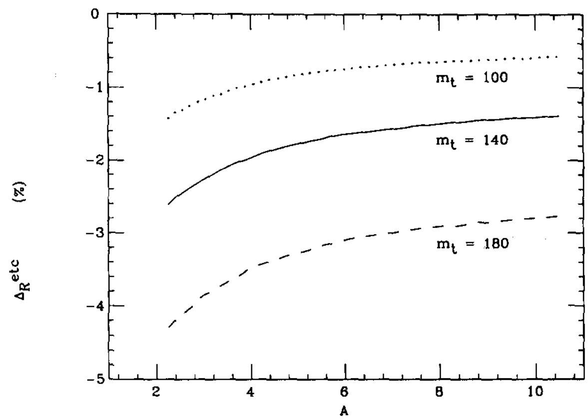

Fig. 1. Plot of $\Delta_R^{\rm ETC}$ as a function of walking parameter A in an $SU(3)_{\rm ETC} \to SU(2)_{\rm TC}$ model. The dotted (solid, dashed) curve is for a top quark mass of 100 (140, 180) GeV.

$$f^2 = \frac{N_{\rm TC}}{16\pi^2} \int_0^\infty \frac{4k^2\Sigma^2 + \Sigma^4}{(k^2+\Sigma^2)^2} {\rm d}k^2. \qquad (5.6)$$

In one-family technicolor models, $f \approx 125$ GeV.

For a given value of b, we vary the ETC scale, $M_{\rm ETC}$, until we obtain a chiral symmetry violating solution to the gap equations with a particular value of m_t. Knowing $M_{\rm ETC}$ allows us to use equations (2.6a) and (3.3) to find the value of $\Delta_R^{\rm ETC}$ associated with our initial values of b and m_t. In applying (2.6a) we recall that $g_{\rm ETC}^2 \equiv 4\pi\alpha_{\rm TC}(M_{\rm ETC})$ and we set $\xi = \xi' = 1/\sqrt{2}$ as is appropriate for our ETC models.

Our numerical results for an $SU(3)_{\rm ETC} \to SU(2)_{\rm TC}$ model are shown in fig. 1. Here, $\Delta_R^{\rm ETC}$ is plotted as a function of $A \equiv (b\alpha_{\rm TC}^c)^{-1}$ for several values of the top quark mass. Similar results for an $SU(5)_{\rm ETC} \to SU(4)_{\rm TC}$ model are plotted in fig. 2. In the small-A ("running") regime of the plots, $\Delta_R^{\rm ETC}$ is of order a few percent, in good agreement with the estimates from naive dimensional analysis. As one moves towards the large-A ("walking") regime, the size of the effect decreases as we expected. Note that the decrease is *very* gradual. For the large top quark masses shown, $\Delta_R^{\rm ETC}$ generally remains big enough to be visible at LEP even if the TC coupling runs very slowly.

Fig. 3 compares the variation of $\Delta_R^{\rm ETC}$ with A found for several $SU(N+1)_{\rm ETC} \to SU(N)_{\rm TC}$ models with m_t set to 140 GeV. Note that the size of $\Delta_R^{\rm ETC}$ grows with N and that $\Delta_R^{\rm ETC}$ depends much less strongly on A as N increases.

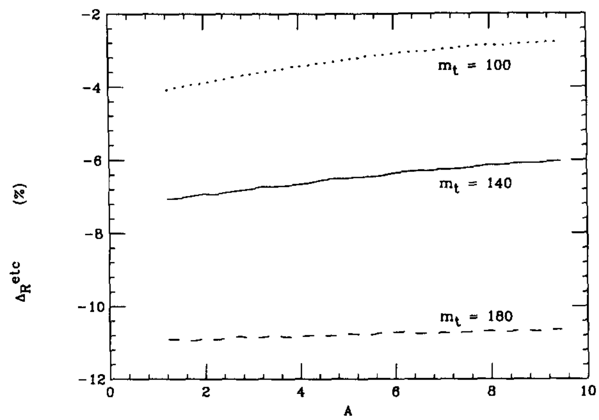

Fig. 2. Plot of $\Delta_R^{\rm ETC}$ as a function of walking parameter A in an $SU(5)_{\rm ETC} \to SU(4)_{\rm TC}$ model. The dotted (solid, dashed) curve is for a top quark mass of 100 (140, 180) GeV.

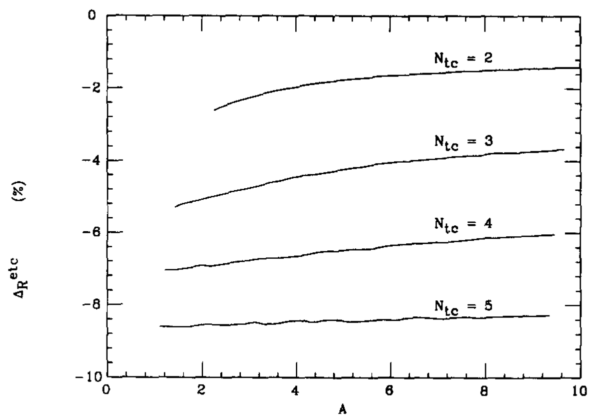

Fig. 3. Plot of $\Delta_R^{\rm ETC}$ as a function of walking parameter A for m_t = 140 GeV in several $SU(N+1)_{\rm ETC} \to SU(N)_{\rm TC}$ models.

6. Conclusions

In this note, we have discussed the degree to which extended technicolor effects reduce the $Zb\bar{b}$ coupling in models with a walking technicolor beta function. We chose the variable Δ_R (fractional shift in the ratio of Z hadronic widths $\Gamma_b/\Gamma_{h\neq b}$) as most suitable for measurement of the shift in the coupling. We indicated why one expects models with a slowly running technicolor beta function to have a smaller $\Delta_R^{\rm ETC}$ than models with a running TC beta function. Then we presented a numerical analysis of dynamical chiral symmetry breaking to illustrate how strongly the technicolor beta function affects the size of $\Delta_R^{\rm ETC}$. Our results show that, while $\Delta_R^{\rm ETC}$ is reduced in walking technicolor models, it generally remains large enough to be visible at LEP.

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References

[1] S. Weinberg, Phys. Rev. D 13 (1976) 974; D 19 (1979) 1277;
L. Susskind, Phys. Rev. D 20 (1979) 2619.

[2] S. Dimopoulos and L. Susskind, Nucl. Phys. B 155 (1979) 237;
E. Eichten and K. Lane, Phys. Lett. B 90 (1980) 125.

[3] R.S. Chivukula, S.B. Selipsky and E.H. Simmons, Phys. Rev. Lett. 69 (1992) 575;
E.H. Simmons, R.S. Chivukula and S.B. Selipsky, in: Proc. Beyond the standard model III (1992), to appear (1993).

[4] B.W. Lynn, M.E. Peskin and R.G. Stuart, SLAC-PUB-3725 (1985); in: Physics at LEP, Yellow Book CERN 86-02, Vol. I, p.90.

[5] B. Holdom, Phys. Lett. B 105 (1985) 301;
K. Yamawaki, M. Bando and K. Matumoto, Phys. Rev. Lett. 56 (1986) 1335;
V.A. Miransky, Nuovo Cimento 90A (1985);

T. Appelquist, D. Karabali and L.C.R. Wijewardhana, Phys. Rev. D 35 (1987) 389, 149;
T. Appelquist and L.C.R Wijewardhana, Phys. Rev. D 35 (1987) 774; 36 (1987) 568.
[6] A. Manohar and H. Georgi, Nucl. Phys. B 234 (1984) 189;
H. Georgi and L. Randall, Nucl. Phys. B 276 (1986) 241.
[7] T. Appelquist et al., Phys. Lett. B 220 (1989) 223;
T. Takeuchi, Phys. Rev. D 40 (1989) 2697;
V.A. Miranksy and K. Yamawaki, Mod. Phys. Lett. A 4 (1989) 129;
R.S. Chivukula, A.G. Cohen and K. Lane, Nucl. Phys. B 343 (1990) 554;
T. Appelquist, J. Terning and L.C.R. Wijewardhana, Phys. Rev. D 44 (1991) 871.
[8] E.H. Simmons, Nucl. Phys. B 312 (1989) 253;
C. Carone and E.H. Simmons, Nucl. Phys. B 397 (1993) 591.
[9] A.A. Akhundov, D.Yu. Bardin and T. Riemann, Nucl. Phys. B 276 (1986) 1 ;
W. Beenakker and W. Hollik, Z. Phys. C 40 (1988) 141;
J. Bernabeu, A. Pich and A. Santamaria, Phys. Lett. B 200 (1988) 569;
B.W. Lynn and R.G. Stuart, Phys. Lett. B 252 (1990) 676.
[10] J. Kroll, in: XXVIIth Rencontres de Moriond, Electroweak interactions and unified theories (1992), to be published.
[11] T. Appelquist and O. Shapira, Phys. Lett. B 249 (1990) 83;
T. Appelquist, Fourth Mexican School of Particles and fields (Mexico City, Mexico, 1990).
[12] R. Jackiw and K. Johnson, Phys. Rev. D 8 (1973) 2386;
H. Pagels and S. Stokar, Phys. Rev. D 20 (1979) 2947;
T. Appelquist et al., Phys. Rev. D 41 (1990) 3192.