Rayleigh-Taylor Unstable Flames: They’re Still Awesome!

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Type Ia Supernovae

Artist’s Depiction: NASA
Burning Begins

Nonaka et al. 2012
Rayleigh-Taylor Instability

The source of wrinkling and turbulence: the Rayleigh-Taylor instability

- The fuel is more dense than the ash
- The flame propagates upward against the direction of gravity

Image: Flash Center (University of Chicago)
Flame Speed Subgrid Model

A **Subgrid Model is Necessary** because

- White dwarf = Size of Earth
- Flame front width = 0.01 - 1 cm

**Two Types of Subgrid Model**

Flame speed set by:

- The Rayleigh-Taylor instability
- Turbulence
And Bob Said…

Let there be Rayleigh-Taylor unstable flame simulations!
From Complex to Simple: Model Flames

Simplifications:
- surface of burning bubble $\Rightarrow$ flame in box
- complex reaction network $\Rightarrow$ model reaction
- full Navier-Stokes equations $\Rightarrow$ Boussinesq equations
- conditions vary within the star $\Rightarrow$ parameter study
- 3D $\Rightarrow$ 2D

(Vladimirova & Rosner 2003, 2005)

Thanks to Natalia Vladimirova, Aleks Obabko and Paul Fischer I got started doing these simulations in **Nek5000**.
This problem is awesome!
I roved around…. but finally finished my thesis!
Result: The Rayleigh-Taylor model works!

2D
But what about 3D?
Rayleigh-Taylor Flame Speed Model: $s = \sqrt{1 + 0.125GL}$

The Rayleigh-Taylor model is fine at low $GL$, but too small at large $GL$. 
Turbulence Subgrid Model

**Physical Premise:** Turbulence controls the flame

The flame is moved ahead by the turbulent energy in the system:

\[ s \propto u' \]

(Niemeyer & Woosley 1997, Schmidt et al. 2006a,b, Jackson 2014)

**Major Assumption:** A flame disturbed by turbulence from behind behaves in a similar way to a flame moving through turbulence
Classical Turbulent Combustion: What is the speed of a flame moving through turbulence?

My question: What is the speed of a flame disrupted by its own self-generated turbulence?

Can classical turbulent combustion theory make predictions about RT-unstable flames?
Astrophysical Adaptation of Turbulent Combustion Theory

Three types of model:

1. Linear
   - \( s = u' \)

2. Scale Invariant
   - \( s = (1 + C_t u'^2)^{1/2} \)

3. Power Law / Bending
   - \( s \propto u''^n \) with \( n < 1 \)
The Models are the Wrong Shape!

Models
- Linear
- Concave down

Flame Speed Data
- Concave up

$$s = 1 + 0.366 u'^{1.277}$$
Turbulent Flame Width

Turbulence or RT seems to be **thinning** the flame, not thickening it!
Does classical turbulent combustion theory correctly predict the behavior of RT-unstable flames?

Flame Speed:
- Models do not capture the shape of the data curve

Flame Width:
- Flame is thin rather than thick when turbulence is strong

No, RT unstable flames are fundamentally different from turbulent flames!
Cusps: Pockets of Burning
Is the flame structure of RT unstable flames different than the flame structure of laminar flames?
Comparing Temperature Diffusion to the Laminar Value

Max: 9.691
Min: -3.986
Rayleigh-Taylor Flame Structure:

- Is generally different than the laminar flame structure
- Shows areas of strong diffusive focusing of temperature

Conclusion

Expect that the flame speed will vary from models that treat the flame as a thin surface with a laminar flame structure.

Question:

- Can changes to the flame speed be modeled by accounting for curvature? ⇒ No
Conclusions

Models that don’t work:

× Turbulence Based Models
  ▶ RT unstable flame data bends up, not down
  ▶ RT unstable flames are thin, not thick

× Fractal Models
  ▶ Flame Structure matters, and affects the flame speed

× Curvature Based Models
  ▶ Positive curvature doesn’t strongly outweigh negative curvature
Seeking a New Subgrid Model

**Strategy: build off of the RT model**
- Use the RT model when the instability is weak

**Keys to Success:**
- Flame Speed $\equiv$ Flame Surface Area & Flame Structure
- RT, KH, turbulence $\Rightarrow$ flame collisions $\Rightarrow$ local flame structure $\Rightarrow$ local flame speed $\Rightarrow$ global flame speed

**Next:**
- Models that account for flame surface density
It is well known that curvature changes the flame speed.  
- Positively curved flames burn faster  
- Negatively curved flames burn slower
Mean Curvature ($K_M$)

$L=32$
$G=4$

White = negative
Red = positive
The Second Question

Do regions of positive $K_M$ outweigh regions of negative $K_M$ and produce enhanced burning?
Percentage of Burning Generated above different $K_M$
Curvature isn’t Necessary for Burning Enhancement

Temperature

Max: 1.0
Min: 0.13